Convergence of a Fully Overlapping Domain Decomposition Method

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Abstract

A fully overlapping domain decomposition method (DDM) is proposed for finite element modeling of small features within large domains. The approach decouples the fine mesh associated with antenna details from that of the background domain. This allows for unstructured meshing, providing great flexibility in designing *in situ* antennas. Another advantage of this algorithm is its faster convergence as compared to traditional non-overlapping domain decomposition methods of the same order. This is due to the smaller iteration matrix eigenspectrum. In this paper, the accuracy of the fully overlapping domain decomposition method is presented with h-refinement analysis.

1 Introduction

Design and optimization of small, conformal and ultra-wide-band antennas embedded within large non-metallic platforms are numerically demanding. Various intrinsically parallel domain decomposition (DDM) algorithms have been formulated to tackle this challenge. Recently, we proposed a novel fully overlapping domain decomposition scheme described in [1]. Based on this algorithm, the entire problem domain is divided into two regions: a finely meshed domain enclosing small features and their surrounding media, and a coarsely meshed background domain. Coupling from the background domain to the finely meshed detail domain is carried out via the conventional Robin Transmission Conditions. But reverse coupling is enforced by treating the detail domain as a volumetric source. A key aspect of the method is that the two domains are completely unstructured. Therefore, different and independent discretizations can be enforced for each domain. As such, our method does not require re-meshing, implying significant simplicity and time reduction for design optimizations.

The proposed fully overlapping domain decomposition algorithm avoids ill-conditioned matrices typically arising when small features are included in large domains. Thus, our approach achieves faster convergence than non-overlapping DDM schemes [2] with comparable accuracy. The formulation details can be found in [1] and are omitted here. In the next section, we directly compare their iteration matrix eigenspectra and their h-refinement convergence.

2 Iteration Eigenspectrum of Fully Overlapping DDM

As in [1], fully overlapping domain decomposition method decouples the entire problem into two sub-domains. The final accurate solution is achieved by iterating between these two sub-problems. The eigen-spectrum of the iteration matrix is directly related to the convergence speed. That is, iterative algorithms with smaller eigenspectrum radius converge faster. In this example, a traditional non-overlapping domain decomposition approach [2] and the proposed fully overlapping domain decomposition method [1] are employed for solving the radiation of a thick dipole. We used first order basis functions and first order mesh truncation conditions in both schemes for one-to-one comparison. The size of the detail domain and the distance of the radiation boundary were also maintained identical.

In this example, the total length of the dipole is $l = \lambda/1.5$. Both DDM schemes converged with similar accuracy. As depicted in Fig. 1, the 200 largest eigenvalues of the fully overlapping DDM iteration matrix (red) are compared with those of the non-overlapping DDM iteration matrix (blue). The latter has a larger eigenspectrum radius, and this is consistent with its slower convergence.

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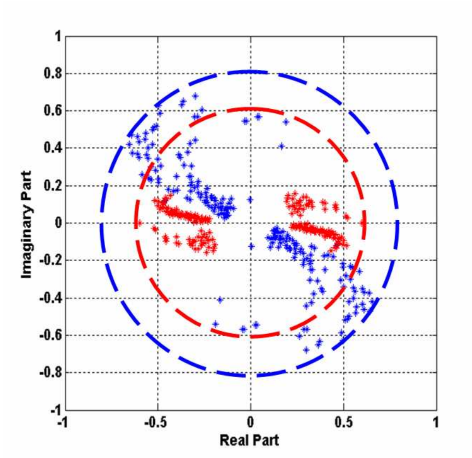

Figure 1: Plot of the 200 largest eigenvalues of the iteration matrices associated with fully overlapping DDM (red, within smaller circle) and the non-overlapping DDM (blue, larger circle).

3 h-refinement Analysis of Fully Overlapping DDM

For h-refinement analysis, we consider the scattering by a PEC sphere. For this example, the excitation was plane wave and we proceeded to compute the bistatic echowidth in the E plane, using the fully overlapping, non-overlapping and Chimera DDM schemes. Chimera DDM is a partially overlapping DDM scheme that relies on boundary couplings through iterations [3]. For the Chimera scheme, no auxiliary cement variables were used. Fig. 2 shows the h-refinement convergence of the fully overlapping, non-overlapping and Chimera DDM. First order vector edge elements were used along with the first order Robin condition.

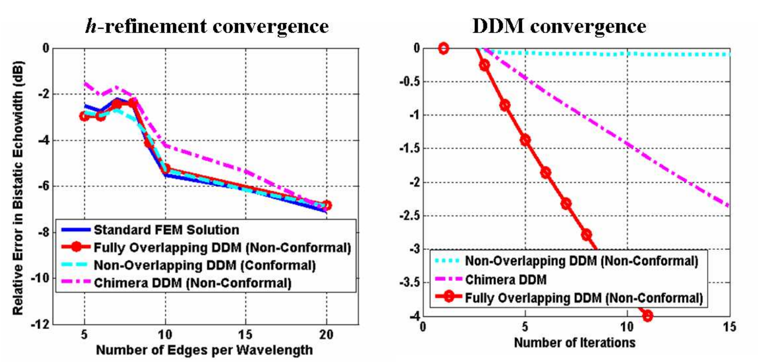

Figure 2: Left: convergence comparison with h-refinement for the three domain decomposition schemes. Right: iterative convergence of the same three DDM schemes.

When the electrical size of the object is very small (i.e., $R = \frac{\lambda}{8}$), Fig. 2 shows that all DDM solutions converge as the mesh density is increased at about the same rate as the standard FEM solution. That is, the completely non-conformal meshes in the fully overlapping DDM scheme do not have significant effect on accuracy. However, our fully overlapping DDM has the fastest convergence.

4 Conclusion

We presented the eigenspectrum and h-refinement convergence of a new fully overlapping domain decomposition method. Its smaller eigenspectrum is consistent with its faster DDM convergence without compromising accuracy. When h-refinement is adapted with our DDM method, similar accuracy is attained as compared to other schemes. A key aspect of the proposed fully overlapping DDM is its flexibility in meshing small features within large domains. This feature makes it attractive for *in situ* design and optimizations.

5 References

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