A Uniform Geometrical Theory of Diffraction (UTD) for Curved Edges Illuminated by Electromagnetic Beams

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(This paper dedicated to the memory of Prof. Robert. G. Kouyoumjian)

Abstract

A uniform geometrical theory of diffraction (UTD) is presented for an arbitrary curved edge in an otherwise smooth curved surface that is a perfect electric conductor (PEC), when the latter is illuminated by an electromagnetic (EM) beam. the beam type illumination may be generated by an EM point source positioned in complex space, or be due to an astigmatic Gaussian beam (ABG) incident on the edge. The UTD for an EM beam type illumination is developed from the asymptotic high frequency (HF) solutions to appropriate canonical problems of diffraction of a complex source beam (CSB) by a straight PEC wedge. The asymptotic saddle point evaluation of the canonical wedge diffraction integral is treated by the two available standard methods, namely the Pauli-Clemmow method (PCM) and the Vander Waerder Method (VWM), respectively. It is noted that the dominant terms in PCM are valid for wedge excitation by a source in real space (which produces real waves) but it is strictly not valid for a complex source (which produces a beam); however, it is the set of dominant terms in PCM that lead directly to the simple UTD format for the total field consisting of the geometrical (incident and reflected) field and diffracted field where the latter contains the UTD transition functions. Hence, it is not justifiable, apriori, to just analytically continue the previously well developed UTD solution based on PCM for PEC wedges excited by real sources and expect it to work directly for complex sources. The VWM, on the other hand, is a more general asymptotic procedure valid for complex waves, but when one retains the dominant VWM terms in their original form, it does not lead to the simpler UTD format for the total field (that is generally preferred in applications). Nevertheless, it is shown after some rearrangement of terms that it is indeed possible to write the dominant terms as VDM=PCM+Δ , where Δ is the missing correction to PCM for complex waves. Surprisingly, the Δ is found to be negligible for the present problem of beam diffraction by wedges thereby allowing the PCM to remain accurate and hence allow the solution to return to the simple UTD format even though it is strictly not expected to. The explanation for why PCM is generally not valid for complex waves, and also why it works in the present problem (where $\Delta \rightarrow 0$) will be given. An application of this work to the rapid analysis of large reflector antenna systems will be described.

1. Introduction

A UTD solution is presented for an arbitrary curved PEC wedge, when it is illuminated by an EM beam. The EM beam can be produced by a point current source located in complex space [1,2,3] or it could be an AGB [4]. The field due to a point source in complex space will be referred to as a complex source beam (CSB). The present UTD solution is constructed via a generalization, based on locality of high frequency (HF) wave phenomena, of the asymptotic HF solutions to appropriate canonical problems of diffraction of CSBs by a straight wedge with planar PEC faces. The solutions to the above canonical problems becomes necessary, because it is not possible to "apriori " justify an analytic continuation of the well known UTD for PEC curved wedges given previously for real sources [5] to directly arrive at the UTD solution for complex sources, due to the fact that the method used to obtain the real source solution is strictly not valid for complex waves (i.e. CSB illumination). However, in the final analysis, it surprisingly turns out that the results obtained for CSB excitation is the same as if the above analytic continuation was directly applied to the available UTD results in [5] for the real source case. The reason for this surprising result is explained. This unexpected outcome is extremely useful because the UTD form of the solution for wedge diffraction of CSBs is more convenient and thus preferred to other forms of asymptotic HF wedge diffraction solutions when it comes to applications. Other forms of asymptotic HF wedge solutions for real source excitation of PEC wedges are available in [6,7]; these are not in the simple UTD format, but it is possible to justify analytically continuing them for dealing directly with a CSB type illumination. A brief summary of the technical approach is presented in the next section which mentions the relevant canonical CSB excited wedge diffraction problems and their asymptotic HF solutions. Also a generalization of this solution to treat an arbitrary curved wedge is discussed. A further generalization to treat an AGB by an arbitrary curved

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wedge is also briefly discussed at the end. In the last section, some numerical results are presented to illustrate the accuracy of the present UTD for CSB illumination. A very short discussion is also included at the end of that last section to basically review and summarize the results of the present work. An $e^{j\omega t}$ time convention for the sources and fields is assumed and suppressed in the following development.

2. Summary of Technical approach

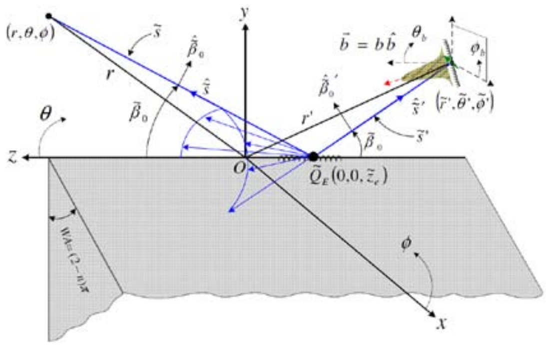


Figure. 1. Complex point source beam illumination of a PEC wedge. Here, b is the beam parameter and $\hat{b}$ is the direction of beam axis. $\tilde{Q}_E$ is a unique complex point of diffraction on the edge for a given source and observation point. Complex rays and source location are drawn only symbolically.

Consider an EM type CSB which excites a PEC wedge as shown in Fig. 1. Such a CSB is generated by locating an EM point (electric or magnetic) current source in complex apace at $\tilde{\overline{r}}'$ where

$$\tilde{\overline{r}}' = \overline{r}' - j\overline{b} \tag{1}$$

and $\overline{r}'$ is the location of the source in real apace. It is well known that such a source generates a CSB which propagates strongly in the $\hat{b}$ direction [1,2,3] where $\overline{b} = b\hat{b}$ and

$$\hat{b} = \sin(\theta_b)\cos(\phi_b)\hat{x} + \sin(\theta_b)\sin(\phi_b)\hat{y} + \cos(\theta_b)\hat{z} \ . \tag{2}$$

The angles θ_b and ϕ_b indicate the polar and azimuthal angles of the beam axis. If b is set to zero, then one obtains the real location of the source at $\overline{r}'$ whose spherical coordinates are denoted by (r', θ', ϕ'). The solution to the above problem begins by first writing the corresponding exact solution for illumination of that wedge by the same source when it is located in real apace at $\overline{r}'$; the latter exact solutions are available for both electric or magnetic current point sources in [8] and are given in terms of an integral in the complex ξ plane. Since the canonical solutions of [8] are exact, they can be analytically continued to accommodate a complex source location at $\tilde{\overline{r}}'$ of (1). The general form of the typical complex contour integral is of the type:

$$I(k) = \int_C g(\xi) e^{kf(\xi)} d\xi \tag{3}$$

where k denotes a large parameter in the asymptotic HF evaluation of (3); here k is real while g and f are complex. The saddle point of $e^{kf(\xi)}$ occurs at $f'(\xi_s) = 0$ (with $f''(\xi_s) \neq 0$). Also, g has simple poles at $\xi = \xi_p$ (which may come close to ξ_s). There is no exact closed form solution to the integral in (3). However, (3) can be approximated for large k by an asymptotic series expansion for $I(k)$ in inverse powers of k via the method of steepest decent; in the latter, C is first deformed into the steepest decent patch (SDP) to arrive at

$$I(k) = I_{SDP}(k) + 2\pi j R_p \Lambda \tag{4}$$

where

$$I_{SDP}(k) = \int_{SDP} g(\xi) e^{kf(\xi)} d\xi \tag{5}$$

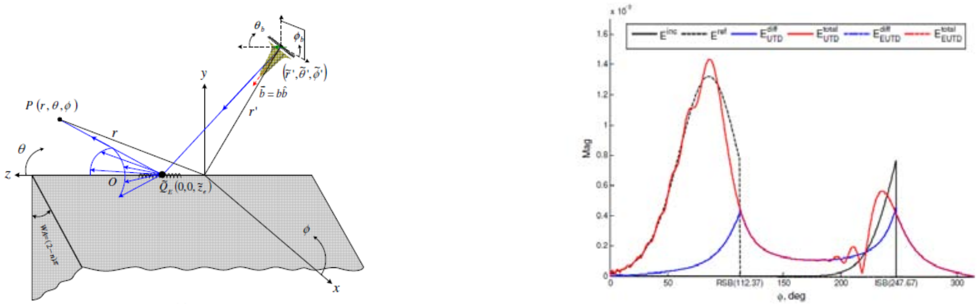
and R_p is the residue associated with the pole at ξ_p. It is noted that Λ in (5) is +1, if ξ_p is captured in counter clockwise (CCW) or -1, if ξ_p is captured in clockwise (CW), and zero if ξ_p is not captured when C is deformed into the SDP. It is assumed here that no branch cut singularities are crossed in this contour deformation. The latter is true for the present wedge diffraction problem of interest. For the real source case, the pole wave corresponds to the geometrical optic incident (GO) incident and reflected waves, whereas in the complex source case they are the incident and reflected CSBs. There are two well known methods for evaluating the SDP integral of (6) as summarized in [9]. One of these is the Pauli-Clemmow method (PCM), while the other is the Vander Waerder method (VWM). The UTD form for the canonical wedge diffraction problem is obtained for the real source case by evaluating the wedge diffraction integral (over SDP) via the PCM. Such a UTD form which does not modify the GO as is also true of Keller's geometrical theory of diffraction (GTD) [10] and is thus simpler, physically appealing, and more convenient for applications; in contrast, the VWM in its original form does not yield such a simple UTD form and it modifies the GO field. The GO field $O(k^{\circ})$ become discontinues across its respective incident and reflected rays shadow boundaries (SBs); in the UTD, the dominant field ($O(k^{-1/2})$ away from the SBs) diffracted from the edge then properly compensates for this jump to provide a continuous total field. However, for the CSB or the complex source case, the PCM is strictly not valid, and hence it is not possible to apriori expect a corresponding continuous total field across the CSB incident and reflection shadow boundaries. The dominant PCM which provides a UTD form for the field is valid whenever the GO poles cross the SDP though the saddle point as it does so for the real source case whenever a ray SB is crossed. In the complex source case, the CSB incident and reflected wave poles cross the SDP away from the saddle point whenever the CSB (or complex ray) SB is crossed; thus, the PCM is strictly not valid for the CSB case, and thus will not guarantee continuity of the total field across the shadow boundaries. As a result, the VWM which is valid even for the above CSB case is utilized to treat the canonical wedge diffraction problem for CSB illumination; i.e., $I_{SDP}(k)$ is evaluated asymptotically and dominant terms to $O(k^{-3/2})$ are retained. This VWM in its original form does not lead to a UTD field format. However, it is shown after some rearrangement that the VWM based result can be actually expressed as

$$VWM = PCM + \Delta \tag{6}$$

where, loosely put, the VWM and PCM in (6) simply denote the VWM and PCM based field contributions, respectively. Furthermore, it is surprising that the correction Δ to PCM consists of a factor of two terms; both of which are relatively small, but one of these tends to vanish if the CSB pole happens to be close to the saddle point as it crosses the SDP, while the other term tends to vanish if the pole CSB is not so close. Thus, the Δ remains vanishingly small for all cases as can be shown both, analytically, and numerically. The net outcome is that (6) reduces to $VWM \cong PCM$ for the CSB case. A result analogous to that in (6) was also observed in [11] for the problem of inhomogeneous plane wave diffraction by a wedge. What $\Delta \to 0$ implies is that the PCM results become valid in the practical sense for CSB illumination of wedges. The canonical CSB excited wedge diffraction solution thus obtained via VWM agrees for all practical purposes with the one based on PCM; this observation in turn allows one to simply and directly analytically continue the canonical UTD solution for a wedge excited by a real source as given in [8] to directly obtain the UTD for CSB excitation of the wedge (by replacing $\overline{r}'$ in [8] with $\overline{\tilde{r}}'$). More importantly, the UTD result in [5] for an arbitrary curved wedge based on a generalization, via the HF locality principle, of the canonical straight wedge solution of [8] for a real source can also be used to obtain a UTD for complex source or CSB illumination of the arbitrary curved wedge by analytic continuation. Equally, important is that the analytic continuation of the UTD for diffraction of an arbitrary (astigmatic) real ray field by an arbitrary curved wedge as also given in [5] to directly furnish the diffraction of an AGB incident on that curved wedge, It is noted that a CSB reduces to a rotationally symmetric beam within its paraxial region, but a CSB cannot be astigmatic and is hence less general than an AGB.

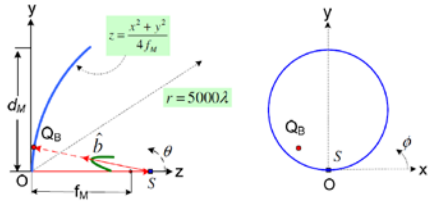
3. Numerical results and Discussion

A typical numerical example based on the results developed for the CSB excitation case is presented in Fig. 2. This solution based on VWM expressed as PCM+ Δ is designated by EUTD (for extended UTD), whereas, the analytic continuation based solution (which is essentially PCM) is simply designated as UTD, in Fig.2. It is seen that there is essentially no difference between the EUTD and UTD indicating the $\Delta \to 0$. The pole wave fields are discontinuous at the SB, but the total field is continuous everywhere. It is noted that points of reflection and diffraction are in general complex for the CSB case. In Fig. 2, the interior wedge angle is 45^{o} and the electric point source is $\hat{z}$-directed with ($r' = 20\lambda$, θ'=100^{o} and ϕ'=70^{o}) and ($b = 10\lambda$, θ_b=78.57^{o} and ϕ_b=258.57^{o}). Also the real observation point is at $r = 12\lambda$, $\theta = 40^{o}$ and ϕ varies around the wedge. The axis of the incident CSB hits the wedge face away from the edge in Fig.2. A typical numerical example of CSB illumination of an offset parabolic is shown in Fig. 3. Included therein are comparisons of the UTD, EUTD and the more approximate physical optics (PO) method. From the above results in Figs. 2 and 3 it is seen that the solution developed here for predicting the diffraction of CSBs by straight as well as curved edges can be utilized within the compact and simple format of the UTD for describing the resulting fields in an efficient and physically appealing manner. It is in this latter sense that, a UTD for CSB diffraction by edges has been obtained in this work.

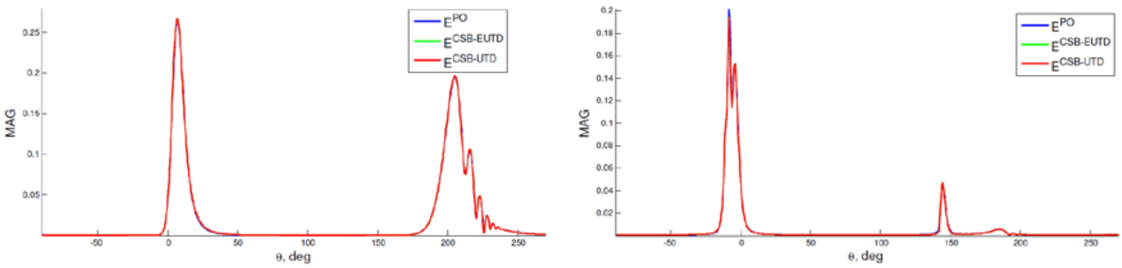


(a) (b)

Figure 2. (a) z-directed complex point source illumination of a wedge. $\tilde{Q}_E$ is a unique complex point of diffraction on the edge for a given source and observation point. Complex rays and source location are drawn only symbolically. (b) Comparison of CSB-UTD and CSB-EUTD are indicated.



(a) Incident CSB illumination away from the focus on a finite parabolic surface



(b) Transverse plane (c) Symmetry plane

Figure 3. Comparison of offset reflector antenna radiation, with CSB illumination away from the focus, via the numerical PO, CSB-EUTD, CSB-UTD. The incident CSB axis hits the actual surface at Q_b. Here, d_M =50λ, f_M=30λ, b=10λ, $\hat{p} = \hat{x}$, Q_b=(-17λ, 10λ), S=(40λ, 0°, 90°)

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