A Stochastic Extension of the Uniform Theory of Diffraction accounting for Geometrical Uncertainty or Surface and Edge Roughness

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Abstract

We present a stochastic extension of the Uniform Theory of Diffraction (UTD) which is capable to account for some uncertainty in the objects position or geometry, including roughness of surfaces or edges. Namely, we derive a solution for the electromagnetic field scattered by a perfectly conducting wedge whose faces are described as a statistical perturbation of a standard flat wedge. We give a uniform closed form expressions for the evaluation of the main statistical moments of the total electric field. The proposed statistical UTD formulation is suitable for engineering applications which involve UTD ray based codes. Some numerical examples highlight the accuracy and the effectiveness of the proposed ray description.

1. Introduction

The Uniform Geometrical Theory of Diffraction (UTD) is a very effective tool for modeling complex environments at high frequencies, which describes the electromagnetic field in terms of rays [1]. This technique has been thoroughly employed for the prediction of the radar cross section (RCS) of complex targets or for the computation of radiation characteristics of antennas in their operating environments (on board of aircraft, ships, satellites, etc.), [2], [3]. It is observed in the literature that, at high frequency, the provided results are very sensitive with respect to the exact scatterer geometry. Indeed, since the scattered field is dominated by few localized scattering centers, for relatively small changes of the geometry of an electrically large scatterer, the position of such scattering centers may significantly change in terms of a wavelength. As a consequence, the interference among the various contribution dramatically changes. This effect has a strong impact, for instance, in the RCS calculation of targets, in the field level prediction in propagation models, or in the accurate in situ antenna modeling, when the scenario exact geometry is not perfectly known or may change. Typical examples are: flap position and on flight wings deformation in aircrafts, uncertainty in the exact terrain or buildings profile, thermal blankets on satellites.

To allow a stochastic extension of UTD, i.e., the possibility of closed form calculation of field statistics in terms of scenario statistics, as suggested in [4]–[6], we solve in this paper the problem of the scattering by a rough perfectly conducting wedge, namely a wedge with rough faces and rough edge. The solution is cast in an asymptotic form similar to UTD but in terms of field mean and variance, and involves the wedges statistical roughness parameters. The final analytical diffracted ray field expression is very simple and provides a new effective engineering tool which is the basic block for the application of a statistic UTD. During the presentation, the formulation of the problem will be presented and its effectiveness and accuracy will be demonstrated by resorting to some numerical examples.

2. Formulation

Referring to Fig. 1, we consider a rough perfectly conducting half-plane defined by

$$\begin{cases} y'' = \eta(x,z) \\ x'' > \varepsilon(z) \end{cases} \tag{1}$$

where $\eta(x,z)$ and $\varepsilon(z)$ are random processes representing the roughness of the surface and the irregularity of the edge, respectively. Both the processes are described by zero mean Gaussian probability distributions

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$$p(\eta)=\frac{1}{\sigma_\eta\sqrt{2\pi}}e^{-\frac{\eta^2}{2\sigma_\eta^2}} \quad \text{and} \quad p(\varepsilon)=\frac{1}{\sigma_\varepsilon\sqrt{2\pi}}e^{-\frac{\varepsilon^2}{2\sigma_\varepsilon^2}} \tag{2}$$

with variances $\sigma_\eta^2,\sigma_\varepsilon^2$. Furthermore, we also suppose that each process is characterized by its own joint probability density function, which is here omitted for space limitation, involving a specific surface and edge correlation length L_η, L_ε. The same formulation comprises roughness or simply geometry uncertainty when $L_\eta, L_\varepsilon \to \infty$ and the stochastic processes reduce to random variables.

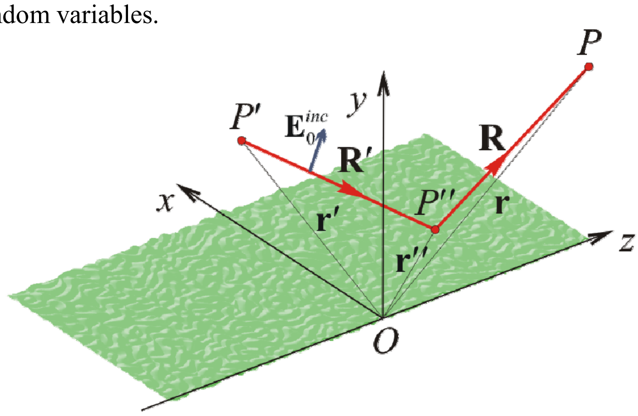

Fig. 1.: Rough half-plane reference geometry.

Under this assumptions a point on the rough half-plane is parameterized through

$$\mathbf{r}'' = x''\hat{\mathbf{x}} + \eta(x,z)\hat{\mathbf{y}} + z''\hat{\mathbf{z}}. \tag{3}$$

In order to provide a statistical UTD description of the scattering phenomena, we first analyzed the field scattered by the half-plane illuminated by an electric dipole under the Physical Optics (PO) approximation for $R, R' \gg \lambda$, as in [4]–[6],

$$\mathbf{E}^{PO}(P) = -2jk\iint_S \left(\underline{\mathbf{1}} - \hat{\mathbf{R}}\hat{\mathbf{R}}\right)\cdot\hat{\mathbf{n}}\times\left(\hat{\mathbf{R}}'\times\mathbf{E}_0^{inc}\right)\frac{e^{-jkR'}}{4\pi R'}\frac{e^{-jkR}}{4\pi R}dS'', \tag{4}$$

which is a stochastic process, and that can be characterized by its mean value and variance. In order to give such description we evaluated asymptotically the integrals involved in the calculation of the statistical moments of first and second order by assuming that $\sigma_{\eta,\varepsilon}$ are small or comparable to λ. The final expressions of the field mean value is

$$\left\langle \mathbf{E}^{PO}(P)\right\rangle \sim -\mathbf{E}^{inc}(P)\left(1-U^i\right) + \mathbf{E}^{r,smooth}(P)e^{-2\left(k\sigma_\eta\cos\theta_i\right)^2}U^r + \mathbf{E}^{d,smooth}(P)e^{-\frac{1}{2}k^2\sin^2\beta_d\left\{\sigma_\eta^2(\sin\phi+\sin\phi')^2+\sigma_\varepsilon^2(\cos\phi+\cos\phi')^2\right\}}. \tag{5}$$

The first two terms in (5) are the Geometrical Optics ray contribution; i.e., the negative of the incident field in the shadow region behind the half- plane where the unit step function U^i vanishes, and the mean reflected field which is present in its lit region where $U^r=1$ and has the same expression of the field reflected by a smooth (i.e., not stochastic) half-plane but dumped by the exponential factor, [7]. The last term in (5) is the expression for the mean diffracted field, which represents a stochastic extension of its standard deterministic well-known counterparts. On the basis of the PO stochastic diffracted field here derived, an analogous extension is introduced in the UTD wedge diffracted field thus providing its stochastic version, here omitted for the sake of brevity. A closed form expression is also derived but here not shown for space limitation for the PO scattered field variance, which represents the incoherent part of the scattered field. Again, by examining the transitional behavior of the solution obtained from the PO approximation, we derived variance expressions for the UTD wedge diffracted ray field. A detailed analysis of the solution comprising all the formulas for the statistical moments will be deeply discussed during the presentation, highlighting the transitional behavior of the both the coherent and incoherent part of the diffracted field and its capability of compensating for the discontinuity of the respective part of the GO field.

3. Numerical Results

In order to show the effectiveness of the proposed formulation, we considered a smooth perfectly conducting half-plane whose position is described by $y'' = \eta, x'' > \varepsilon$, with η, ε known Gaussian random variables with $\sigma_\eta = \sigma_\varepsilon = 0.5\lambda$ (Fig. 2). The half-plane is illuminated either by a vertical or an horizontal dipole at $r' = 56\lambda$ from the edge. The field is measured on a circular scan $r = 20\lambda$ around the edge. We considered 100 pairs of randomly generated η, ε and, for each pair, we calculated and collected the field predicted by standard UTD. Properly averaging this set of field samples we estimated the mean value and variance of the total field (GO + diffracted field) with a Montecarlo (MC) approach. Such estimations of the first and second order moments (represented in Fig. 4 by the dashed lines) are in complete agreement with those obtained by using the proposed formulation (continuous lines in Fig. 4). Moreover, in Fig. 4 the statistical distribution of the field collected in the 100 scenarios is also shaded revealing as the mean squared field, which is proportional to the mean power and simply given by $\left\langle |\mathbf{E}(P)|^2 \right\rangle = \left| \left\langle \mathbf{E}(P) \right\rangle \right|^2 + \mathrm{var}\{\mathbf{E}(P)\}$, i.e., by the sum of the coherent and incoherent field powers, is a good estimation of the median field amplitude.

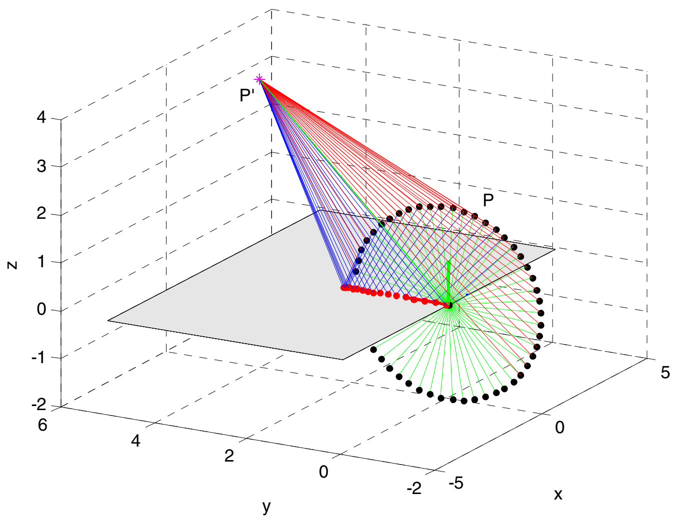

Fig. 2.: Half-plane reference geometry for a single scenario. The blue lines refer to the rays reflected by the half-plane, the red lines refer to the incident direct rays, while the green lines identifies the edge diffracted rays.

It is interesting to note that in the deep shadow region below the screen ($\phi < -150°$) the phase of the field, which is only due to the diffracted ray, is randomized by edge position uncertainty. Therefore the mean field vanishes ant the field is totally incoherent here. Conversely, when approaching the incident field shadow boundary ($\phi = -135°$) and entering in the incident field lit region, the field is dominated by the deterministic incident field which render the total field fully coherent. However, when crossing the reflected field shadow boundary ($\phi = -45°$) and entering in the reflected field lit region, again a fading effect is experienced because of the interference between the incident and the randomly phased reflected field. This effect is weak for the vertical dipole illumination (left) because the reflection point is illuminated close to the dipole null and the reflected field is much weaker than the direct one. Conversely for the horizontal dipole illumination (right) the interference of the reflected field result in an incoherent part of the field which is of the same order of the coherent one, that is the mean field.

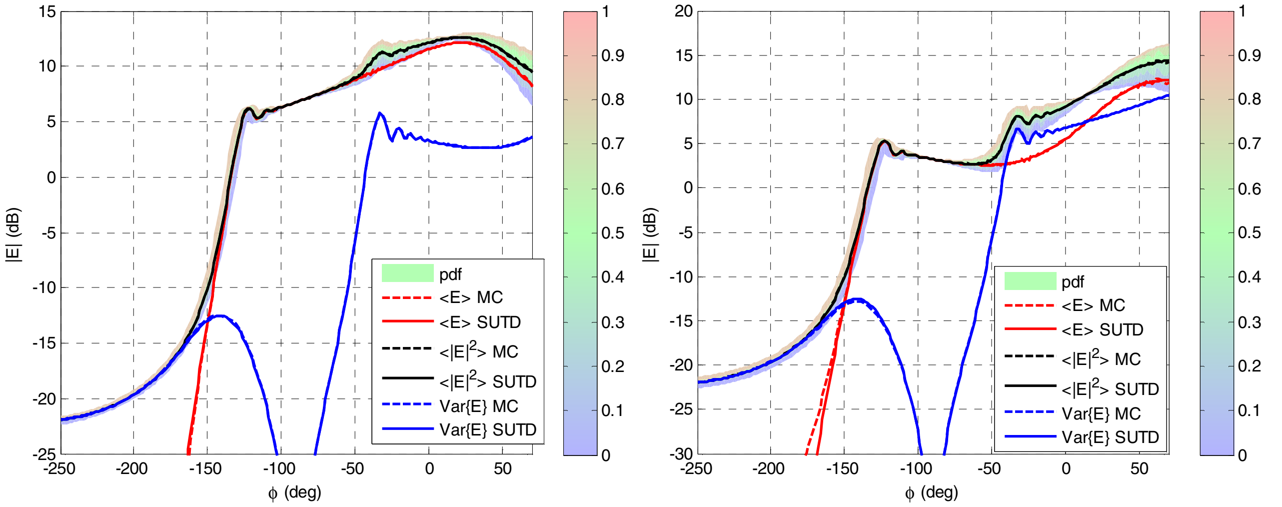

Fig. 3.: Amplitude of the statistical moments of the total electric field. Dashed lines represent the estimated statistical moments, while continuous lines identify the related values predicted by the presented formulation. Mean Field (red), Field variance (blue), mean squared field (black). The statistical distribution of the field samples collected over the 100 scenarios is also shaded. Vertical (left) and horizontal (right) dipole illumination.

4. Conclusion

We derived a solution for the field scattered by a rough wedge (rough faces & rough edge) by providing a stochastic extension of the UTD. In particular, we derived closed form uniform expressions which allow the calculation of the mean value, variance, and mean square value of the total field. Such formulation was validated by comparison with a statistical approach where the moments of the total field were obtained averaging the total field samples collected from 100 scenarios. The proposed statistical UTD formulation is suitable for engineering applications which involve UTD ray based codes.

5. References

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