Exact Radiation by a Line Source Located Inside a Confocal Elliptic Layer of DNG Metamaterial

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Abstract

The two-dimensional problem of radiation from a line source encased inside an elliptic-cylinder confocal sheath of DNG metamaterial is solved exactly by separation of variables, utilizing series expansions of Mathieu functions. The radiation pattern is studied as a function of frequency, location of the line source along the major axis of the elliptical cavity, and thickness of the confocal sheath. A comparison of the exact solution with the geometrical optics solution of the problem is performed.

1. Introduction

The electromagnetic radiation by a line source located at one of the focal lines of an elliptic-cylinder lens has been studied previously for the case of a lens made of double-negative (DNG) metamaterial, both in the optical limit and as an exact solution in terms of infinite series of elliptic-cylinder wave functions [1-2]. In this work, the line source is parallel to the generators of the cylindrical structure and is located anywhere in the plane containing the two focal lines. The source is shielded by a confocal elliptic layer made of lossless DNG metamaterial whose electric permittivity and magnetic permeability are real negative and the opposite of the corresponding real positive parameters characterizing both the space inside the coating and the infinite space outside the structure. As a consequence of causality, the refractive index inside the metamaterial sheath is real negative, and is the opposite of the refractive index outside the sheath, whereas the intrinsic impedance has the same real positive value both inside and outside the sheath. The analysis is conducted in phasor domain with the time-dependence factor exp(+jωt) omitted throughout.

2. Exact Solution

In a cross-sectional view perpendicular to the generators of the cylindrical structure, the electric or magnetic isotropic line source is a Hankel function of the second kind located anywhere along the major axis of the ellipse, and the DNG sheath occupies the region between two confocal ellipses. The primary field components are expressed as infinite series of products of angular Mathieu functions, Se_m , and radial Mathieu functions of the fourth kind, $Re_m^{(4)}$, in the Stratton-Chu notation [1-2]. Because of the location of the line source, only the even functions appear in the expansions. The total field inside the cavity is the sum of the primary field and of a scattered field whose infinite series consists of terms involving products of the same angular functions of the primary field times radial functions of the first kind, with as yet unknown modal expansion coefficients. The field outside the sheath as series expressions similar to those of the primary field (the radiation condition is thus satisfied), but with modal expansion coefficients to be determined. The fields inside the DNG sheath are series containing products of angular functions times linear combinations of radial functions of the first and fourth kind, thus containing two sets of unknown modal expansion coefficients. However, the parameter contained in the Mathieu functions inside the

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sheath is of opposite sign to the parameter in the functions outside the sheath. Since the angular Mathieu functions are invariant when such a parameter changes sign, it is possible to determine analytically the expressions of all modal expansion coefficients on a mode-by-mode basis by imposing the boundary conditions at the two surfaces of the sheath, resulting in an exact analytical solution of the boundary-value problem.

The obtained exact solution is evaluated numerically for different values of four parameters: the ratio of the interfocal distance to the wavelength, the eccentricity of the structure, the ratio of the maximum sheath thickness to the wavelength, and the position of the line source along the major axis of the cross-section. Particular attention is paid to the structure of the fields inside the sheath, and to the radiated far field.

For the specific case when the line source is located at a focal line of the structure, the geometrical-optics far field is evaluated and compared to the far field obtained from the exact solution.

The particular case in which there is no cavity and the DNG material occupies the entire cross-section of the cylindrical structure has been studied previously, both in the optical limit and as an exact boundary-value problem [3 - 4].

3. References

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