New type of gyrotropy in graphene - Comparison with gyrotropy in plasmas

Dimitrios L. Sounas and *Christophe Caloz*

Department of Electrical Engineering, École Polytechnique de Montréal, Montréal, QC, Canada H2T 1J4,
dimitrios.sounas@polymtl.ca, christophe.caloz@polymtl.ca

Abstract

The gyrotropic properties of magnetically biased graphene are compared to these of magnetically biased plasmas. Graphene exhibits a significant polarization rotation phenomenon at frequencies below the cyclotron resonance, where plasmas do not provide any polarization rotation. At frequencies above the cyclotron resonance the behavior of graphene is similar to this of plasmas. Graphene allows the control of its gyrotropic properties via a static electric field, while keeping the bias magnetic field constant, in contrast to plasmas where the variation of the bias magnetic field is required.

1 Introduction

Graphene, an arrangement of carbon atoms in a 2D honeycomb lattice, is a 2D plasma which has been gaining an ever increasing interest over the past years due its remarkable properties, such as field ambipolarity, electron relativistic behavior, and ballistic transport, which all originate from its unique gapless conical band structure [1]. Great theoretical and experimental research efforts have been dedicated to the study of its electronic properties [2], recently leading to the realization of the first graphene-based electronic devices [3, 4, 5]. Recently, magnetically biased graphene has been shown to exhibit significant gyrotropic properties for frequencies both below [6] and above the cyclotron resonance [7], and the first microwave devices based on this phenomenon have been proposed [8].

Herein the gyrotropic properties of graphene are compared to the corresponding of conventional 3D plasmas. Starting from the tensorial form of conductivity in the presence of a static magnetic field, the effective conductivities for the two types of circular polarization are derived. These conductivities are then used to derive the effective permittivities of 3D plasmas, through which the propagation characteristics are studied. In graphene, the propagation characteristics are examined through the transmission coefficient for a normally incident plane wave, since the definition of effective permittivity is meaningless due to the 2D nature of the material. The analysis shows that graphene gyrotropy at frequencies below the cyclotron resonance, which is absent in 3D plasmas, owes its existence to the zero thickness of graphene. Furthermore, it demonstrates the potential of controlling the amount and the direction of rotation in graphene through the chemical potential via a static electric field.

2 Conductivity of Magnetically Biased Plasmas and Graphene

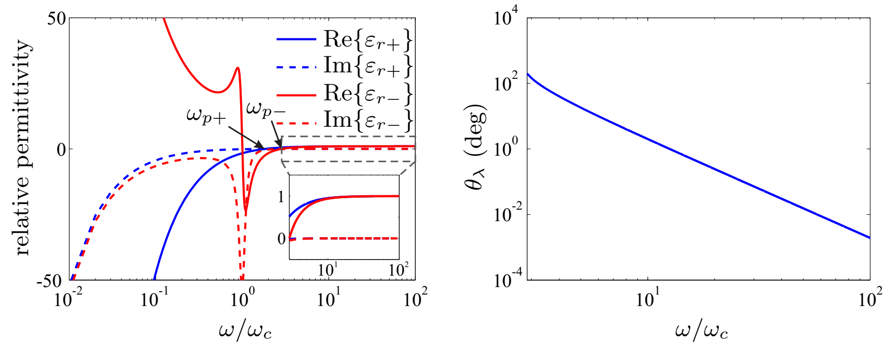
Consider a plasma biased with a static magnetic field $\mathbf{B}_0 = B_0\hat{\mathbf{z}}$. Its conductivity splits in two parts: a longitudinal scalar conductivity, parallel to $\mathbf{B}_0$, and a transverse tensorial one, perpendicular to $\mathbf{B}_0$. The latter can be expressed in the dyadic form

$$\bar{\bar{\sigma}}_t = \sigma_d \bar{\bar{I}}_t + \sigma_o \bar{\bar{J}}_t, \tag{1}$$

where $\bar{\bar{I}}_t = \hat{\mathbf{x}}\hat{\mathbf{x}} + \hat{\mathbf{y}}\hat{\mathbf{y}}$ and $\bar{\bar{J}}_t = \hat{\mathbf{y}}\hat{\mathbf{x}} - \hat{\mathbf{x}}\hat{\mathbf{y}}$. In (1) σ_o is the so-called Hall conductivity, which is related to a current density perpendicular to the applied electric field. Following the classical free electron Drude model, σ_d and σ_o are given by

$$\sigma_d(\omega, B_0) = \sigma_0 \frac{1 + j\omega\tau}{(\omega_c\tau)^2 + (1 + j\omega\tau)^2}, \quad \sigma_o(\omega, B_0) = \sigma_0 \frac{\omega_c\tau}{(\omega_c\tau)^2 + (1 + j\omega\tau)^2}, \tag{2}$$

978-1-4244-6051-9/11/$26.00



(a) (b)

Figure 1: Propagation in InSb plasma with electron density $n = 10^{15}$ cm^{-3}, electron mobility $\mu = 20000$ cm^2/Vs and biasing magnetic field $B_0 = 0.5$ T. (a) Relative permittivity versus frequency for clockwise and counter-clockwise circular polarizations. (b) Rotation angle per wavelength versus frequency.

where $\sigma_0 = ne\mu$ is the magnetically unbiased DC conductivity, μ the DC mobility, n the free electron density, m^* the effective electron mass and $\omega_c = eB_0/m^*$ the cyclotron frequency [9].

Graphene is a 2D plasma and therefore it also follows (1) and (2). The classical Drude model accurately describes graphene under the following twofold condition: $\mu_c \gg \hbar\omega_c$, corresponding to a negligible Landau quantization of the band diagram, and $\mu_c \gg \hbar\omega$, corresponding to negligible interband transitions between the valence and conduction bands [10]. Due to the conical band structure of graphene, the effective mass is given by $m^* = \mu_c/v_F^2$, where μ_c is the chemical and v_F the Fermi velocity, and the magnetically unbiased DC conductivity by $\sigma_0 = (2e^2\tau/\hbar^2)k_BT\ln\{2\cosh[\mu_c/(2k_BT)]\}$. If $\mu_c \gg k_BT$, then $\sigma_0 = n_se\mu$, where $n_s = \mu_c^2/(\pi\hbar^2v_F^2)$ is the free electron surface density and $\mu = e\tau/m^*$ is the DC mobility.

Assume now a circularly polarized wave with electric field (phasor) intensity $\mathbf{E}_\pm = \hat{\mathbf{x}} \pm j\hat{\mathbf{y}}$, where the + and − signs correspond to clockwise and counter-clockwise polarizations, respectively, with respect to the receiver assuming the same $\exp(+j\omega t)$ time dependence as in (2). The current induced in a magnetically biased plasma or graphene is, according to (1),

$$\mathbf{J}_\pm = \bar{\bar{\sigma}} \cdot \mathbf{E}_\pm = (\sigma_d \mp j\sigma_o)(\hat{\mathbf{x}} \pm j\hat{\mathbf{y}}). \tag{3}$$

For plasmas, $\mathbf{J}_\pm$ is a volume current density whereas for graphene it is a surface current density. Equation (3) allows the definition of a clockwise circular polarization conductivity $\sigma_+ = \sigma_d - j\sigma_o$ and a counter-clockwise one $\sigma_- = \sigma_d + j\sigma_o$, which are found, upon insertion of (2) into these expressions, as

$$\sigma_\pm = \frac{\sigma_0}{1 + j(\omega \pm \omega_c)\tau}. \tag{4}$$

The fact that the medium responds differently to clockwise and counter-clockwise polarization through $\sigma\pm$ is a clear manifestation of gyrotropy.

3 Gyrotropy in Magnetized Plasmas and Graphene

Wave propagation in plasmas is better described in terms of an effective permittivity, defined as $\varepsilon_{r,\pm} = 1 + \sigma_\pm/(j\omega\varepsilon_0)$ for circularly polarized waves, than by the conductivity (4). This permittivity is found by inserting (4) into this expression as

$$\varepsilon_{r,\pm} = 1 - \frac{\omega_p^2}{\omega(\omega \pm \omega_c - j\tau^{-1})}, \tag{5}$$

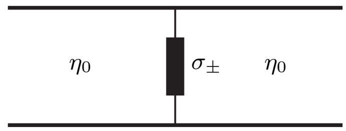
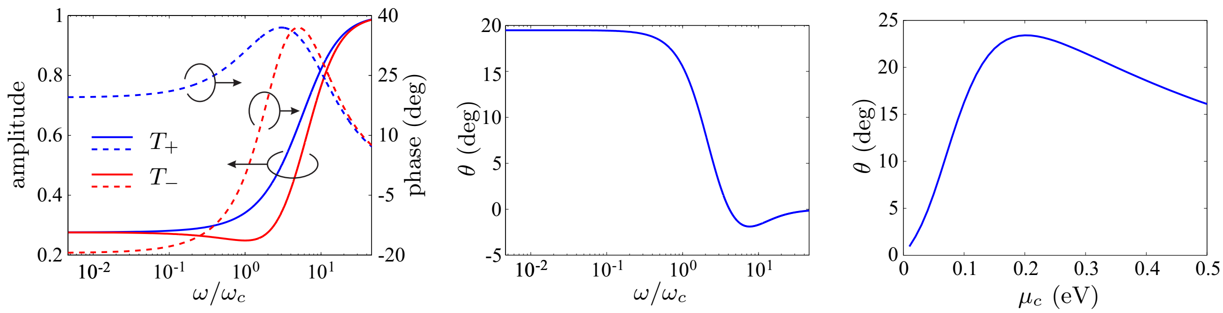


Figure 2: Transmission line model for the problem of normal incidence of a circularly polarized wave on a free-standing graphene sheet in free space.



(a) (b) (c)

Figure 3: Normal incidence on a free-standing graphene sheet in free space with $\tau = 0.369$ ps and $B_0 = 0.5$ T. (a) Transmission coefficient versus frequency for $\mu_c = 0.37$ eV. (b) Rotation angle versus frequency for $\mu_c = 0.37$ eV. (c) Rotation angle versus μ_c at 10 GHz.

where $\omega_p^2 = \sigma_0/(\varepsilon_0\tau) = ne^2/\varepsilon_0$ is the square of the plasma frequency of the unmagnetized plasma. In the presence of the bias magnetic field, however, the plasma frequency is different for the two types of circular polarization and it is derived by solving $\varepsilon_{r,\pm} = 0$ under the assumption $\omega_c\tau \gg 1$. As an example, consider InSb with electron density $n = 10^{15}$ cm^{-3}, electron mobility $\mu = 20000$ cm^2/Vs and bias magnetic field $B_0 = 0.5$ T, for which $\omega_{p,+} = 1.86\omega_c$ and $\omega_{p,-} = 2.86\omega_c$. The relative permittivity for this material is shown in Fig. 1(a). For $\omega < \omega_c$, $\mathrm{Re}\{\varepsilon_{r,+}\} < 0$ and $\mathrm{Re}\{\varepsilon_{r,-}\} > 0$, therefore allowing propagation only for counter-clockwise circularly polarized waves (helicon waves) [9]. On the other hand, for $\omega > \omega_{p,-} > \omega_{p,+}$, $\mathrm{Re}\{\varepsilon_{r,+}\} > 0$ and $\mathrm{Re}\{\varepsilon_{r,-}\} > 0$, allowing propagation for both circular polarizations. The difference in $\varepsilon_{r,\pm}$ results in different propagation constants for the two types of circular polarizations, which in turn induces a rotation in the polarization of a linearly polarized incident wave. The rotation angle per wavelength is given by $\theta_\lambda = 2\pi(\sqrt{\varepsilon_{r,+}} - \sqrt{\varepsilon_{r,-}})$ and it is plotted in Fig. 1(b). It is worth noticing that for frequencies greater than $10\omega_c$, $\mathrm{Re}\{\varepsilon_{r,\pm}\} \approx 1$, resulting in almost zero reflection. The price to pay for this low reflection is a small value for θ_λ, as seen in Fig. 1(b), which requires a thickness of many wavelengths to achieve a large rotation angle.

The zero thickness of graphene (0.34 nm, corresponding to an electrical thickness of less than $\lambda/882$ for frequencies up to 1 PHz) prohibits any physically meaningful definition of electric permittivity. Instead, its gyrotropic characteristics can be examined through the transmission coefficient for a normally incident plane wave. For circular polarization and for a free-standing graphene in free space the transmission coefficient is found, through the transmission line model of Fig. 2, as

$$T_\pm = \frac{2}{2 + \eta_0\sigma_\pm}, \tag{6}$$

where η_0 is the free-space wave impedance. Figure 3(a) shows the amplitude and the phase of $T_\pm$ versus frequency for a graphene sheet with $\mu = 10000$ cm^{-2}, $n_s = 10^{13}$ cm^2/Vs and $B_0 = 0.5$ T. For frequencies below the cyclotron resonance $|T_\pm|$ is small – less than 0.3 – indicating that most of the incident power is reflected. This is similar to what happens in 3D plasmas, as can be seen from the negative $\mathrm{Re}\{\varepsilon_{r+}\}$ and the high positive $\mathrm{Re}\{\varepsilon_{r-}\}$ (Fig. 1(a)). However, in contrast to what happens in plasmas, the zero thickness of graphene allows both circular polarizations to pass through it with almost the same amplitude, which in combination with the large phase difference between T_+ and T_- yields the extremely high – considering the thickness of graphene – rotation angle shown in Fig. 3(b). For frequencies above the cyclotron resonance, $T_\pm \approx 1$, meaning that both circular polarizations traverse graphene with almost zero reflection. However, similarly to 3D plasmas, this close to zero insertion loss is accompanied by a small phase difference between T_+ and T_- and, subsequently, to a small rotation angle, as seen in Fig. 3(b).

One of the most remarkable features of gyrotropy in graphene, and simultaneously one of the most fundamental differences with gyrotropy in plasmas, is the possibility of controlling the amount of rotation through the chemical potential via a static electric field. This is seen in Fig. 3(c), which presents the rotation angle versus B_0 and μ_c for a graphene sheet with $\tau = 0.369$ ps and $B_0 = 0.5$ T. For small μ_c, when the conductivity is very small and graphene is almost transparent, $\theta \approx 0$. As μ_c increases, θ also increases until a critical value of μ_c (here 0.15 eV), where θ takes its maximum value. For a further increase of μ_c, θ decreases until it becomes 0 for very high μ_c, when graphene is almost a perfect conductor. Furthermore, μ_c can also control the direction of polarization rotation. Specifically, by reverting the sign of μ_c, the dominant charge carrier type changes from electrons to holes. Holes have the opposite charge of electrons and, therefore, they feel an opposite Lorentz force, providing a reversal in the polarization rotation direction.

4 Conclusions

A comparison between the gyrotropic properties of graphene and 3D plasmas has been performed. Above the cyclotron frequency, 3D plasmas and graphene exhibit the same kind of polarization rotation effect. However, below the cyclotron resonance, graphene exhibits a strong gyrotropic behavior – much stronger than above the cyclotron resonance – which is totally absent in 3D plasmas. This new kind of gyrotropy is attributed to the zero thickness of graphene. Furthermore, gyrotropy in graphene can be controlled by a static electric field under a constant magnetic field.

References

[1] A. K. Geim and K. S. Novoselov, "The rise of graphene," *Nature Materials*, vol. 6, pp. 183–191, 2007.

[2] A. H. C. Neto, F. Guinea, N. M. R. Peres, K. S. Novoselov, and A. K. Geim, "The electronic properties of graphene," *Rev. Mod. Phys.*, vol. 81, pp. 109–162, 2009.

[3] H. Wang, D. Nezich, J. Kong, and T. Palacios, "Graphene frequency multipliers," *IEEE Electron Device Lett.*, vol. 30, pp. 547–549, 2009.

[4] Y.-M. Lin, K. A. Jenkins, A. Valdes-Garcia, J. P. Small, D. B. Farmer, and P. Avouris, "Operation of graphene transistors at gigahertz frequencies," *Nano Lett.*, vol. 9, pp. 422–426, 2009.

[5] M. Dragoman, D. Dragoman, F. Coccetti, R. Plana, and A. A. Muller, "Microwave switches based on graphene," *J. Appl. Phys.*, vol. 105, p. 054309, 2009.

[6] D. L. Sounas and C. Caloz, "Electromagnetic non-reciprocity and gyrotropy of graphene," *Appl. Phys. Lett.*, vol. 98, p. 021911, 2011.

[7] I. Crassee, J. Levallois, A. L. Walter, M. Ostler, A. Bostwick, E. Rotenberg, T. Seyller, D. van der Marel, and A. B. Kuzmenko, "Giant faraday rotation in single- and multilayer graphene," *Nature Phys.*, vol. 7, pp. 48–51, 2011.

[8] D. L. Sounas and C. Caloz, "Gyrotropy and non-reciprocity of graphene for microwave applications," *International Microwave Symposium*, 2011, accepted.

[9] C. Kittel, *Introduction to solid state physics.* John Wiley & Sons, Inc., 2005.

[10] V. P. Gusynin, S. G. Sharapov, and J. P. Carbotte, "On the universal ac optical background in graphene," *New J. Phys.*, vol. 11, p. 095013, 2009.