Does evanescent gain exist?

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Abstract

We have investigated the situation where light incident from a passive high-refractive-index medium is totally reflected off an infinite half space with gain. The question of whether or not evanescent gain can prevail in this case, has been at issue for 40 years. We argue that the controversy can be resolved for week gain media using the Laplace transform, combined with a detailed analysis of analytic and global properties of the permittivity function of the active medium.

Paper

Light incident from a high-refractive-index medium onto a low-index medium, undergoes total internal reflection provided the angle of incidence is larger than a certain critical angle. Total internal reflection is a fundamental physical phenomenon with several famous applications; in particular modern telecommunications rely on optical fibers based on this phenomenon.

Since the tangential electric and magnetic fields must be continuous at the interface, there must be nonzero fields in the low-index medium, even though the incident wave is totally reflected. For lossless/gainless media, these evanescent fields decrease exponentially away from the interface. The presence of evanescent fields in the low-index medium suggests that the reflected wave will sense any perturbation induced there. In particular, if the low-index medium has gain, the reflection response will change compared to the lossless/gainless case. The problem of determining the correct electromagnetic response in the case of an active low-index medium is far from trivial, and has been discussed for 40 years without reaching consensus [1-4]. A key issue is whether the reflectivity may exceed unity (i.e., evanescent gain exists) when the active medium fills the entire half-space. Experiments have indicated that evanescent gain exists [5]. However, it has been argued that the amplified reflection may be due to back reflection from e.g., the boundaries of the active medium [4].

Since there are no gain media with infinite thickness, why do we choose to examine this case? Assuming darkness for time $t < 0$, the solution to Maxwell's equations for an infinite gain medium equals that of a finite slab for times t less than d/c, where d is the slab thickness and c is the vacuum velocity of light. Hence, understanding this situation, can help explain transient phenomenons, and it is clearly interesting from a fundamental point of view.

The existing controversy is described in a paper recently submitted for publication [6]. A summary will follow here. Assuming well defined frequency-domain fields, we solve Maxwell's equations in the frequency-domain, using the sign convention $\exp(-i\omega t)$. The transversal wavenumber (spatial frequency of the source) k_x is defined as illustrated in Fig. 1. We further let both media be nonmagnetic with permittivities ϵ_1 and ϵ_2, where $\mathrm{Re}\,\epsilon_1 > \mathrm{Re}\,\epsilon_2$. For plane waves, Maxwell's equations require the longitudinal wavenumbers in the high-index and low-index media to be

$$k_{1z} = \pm\sqrt{\epsilon_1\omega^2/c^2 - k_x^2}, \tag{1a}$$

$$k_{2z} = \pm\sqrt{\epsilon_2\omega^2/c^2 - k_x^2}. \tag{1b}$$

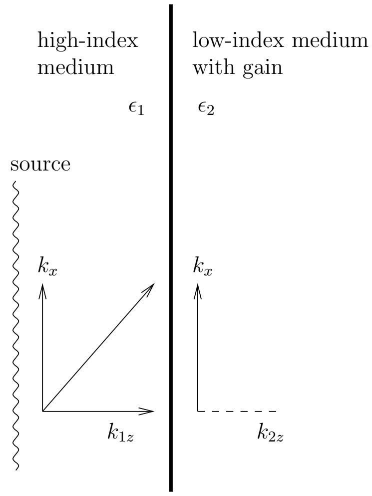

Figure 1: A wave is incident from a high-index medium onto a low index medium with gain. The source produces a single, spatial frequency k_x. The spatial frequency of the fields in the low-index medium must be k_x as well, to satisfy the electromagnetic boundary conditions. The longitudinal wavenumbers are denoted k_{1z} and k_{2z}. Note that since the excitation is assumed to be causal, it contains an infinite frequency band.

At some observation frequency $\omega = \omega_1$, we assume $k_x^2 < \mathrm{Re}\,\epsilon_1\,\omega_1^2/c^2$ while $k_x^2 > \mathrm{Re}\,\epsilon_2\,\omega_1^2/c^2$. Since the high-index medium is passive, we may readily determine the correct sign of the square root in (1a). For the low-index medium, we assume $\mathrm{Im}\,\epsilon_2 < 0$ and $|\,\mathrm{Im}\,\epsilon_2| \ll 1$ (i.e., small gain). The correct sign for the square root in (1b) is far from obvious: Either $\mathrm{Im}\,k_{2z} > 0$ and $\mathrm{Re}\,k_{2z} < 0$, or $\mathrm{Im}\,k_{2z} < 0$ and $\mathrm{Re}\,k_{2z} > 0$. We will show that the right choice depends on the analytic and global properties of the permittivity, ϵ_2. In other words, two media with the same permittivity at some frequency can behave differently, even in the limit of (quasi-)monochromatic fields.

To determine the correct solution, we must be certain that we consider the real, physical situation. In the time-domain, the electromagnetic fields are the real, physical fields. By requiring the fields to be zero for $t < 0$ we obtain the causal solution to Maxwell's equations. When there is gain in the system, using the Fourier transform can be perilous, since the field may increase with time. Hence, within a linear medium framework, Fourier transforms do not necessarily exist. Therefore, as in electronics and control engineering, we generalize the analysis by using the Laplace transform,

$$E(\omega) = \int_0^\infty \mathcal{E}(t)\exp(i\omega t)\mathrm{d}t. \tag{2}$$

In (2) a sufficiently large value of $\mathrm{Im}\,\omega$ will quench an exponential increase in the time-domain electric field $\mathcal{E}(t)$, such that the integral converges. The inverse transform is given by

$$\mathcal{E}(t) = \frac{1}{2\pi}\int_{-\infty+i\gamma}^{+\infty+i\gamma} E(\omega)\exp(-i\omega t)\mathrm{d}\omega. \tag{3}$$

The integral is taken along the line $\omega = i\gamma$, for a sufficiently large, real parameter γ, above all non-analytic points of $E(\omega)$ in the complex ω-plane.

For simplicity, we further restrict ourselves to TE polarization. The Fresnel reflection coefficient ρ and the transmission coefficient τ (including the propagation factor $\exp(ik_{2z}z)$ become [7,8]

$$\rho = \frac{k_{1z} - k_{2z}}{k_{1z} + k_{2z}}, \tag{4a}$$

$$\tau = \frac{2k_{1z}}{k_{1z} + k_{2z}}\exp(ik_{2z}z), \tag{4b}$$

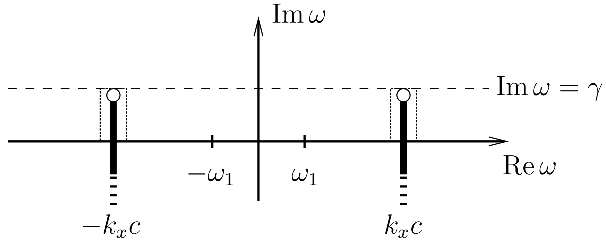

Figure 2: The complex ω-plane. For conventional, weak gain media, there are branch cuts in the vicinity of $\omega = \pm k_x c$. These branch cuts can be chosen arbitrarily; however, the shown, vertical cuts minimize the integral around the part of the branch cuts in the upper half-plane.

provided the sign of k_{2z} is determined such that $k_{2z} \to +\omega/c$ as $\operatorname{Im}\omega \to \infty$, and k_{2z} is an analytic function of ω. The reflected and transmitted frequency-domain fields are given by (4) multiplied by the Laplace transform of the incident field. The associated, physical, time-domain fields are obtained by the inverse transform (3). Now, by analytic continuation, we can reduce γ until we reach a non-analytic point of (4), without altering $\mathcal{E}(t)$. If the expressions (4) are analytic in the entire, upper half-plane, we can set $\gamma = 0$ and interpret ρ and τ for real frequencies. On the other hand, if there are non analytical points, the time-domain fields diverge. In that case, real frequencies are not meaningful in general.

To find the actual reflection and transmission response, we first consider a conventional, weak, gain medium, described by an inverted Lorentzian resonance:

$$\epsilon_2(\omega) = 1 - \frac{F\omega_0^2}{\omega_0^2 - \omega^2 - i\omega\Gamma}. \tag{5}$$

Here F, ω_0, and Γ are positive parameters, describing the resonance strength, frequency, and bandwidth, respectively. Solving the equation $\epsilon_2\omega^2/c^2 = k_x^2$ for $F \ll 1$ and $\Gamma \ll \omega_0$, we find 4 branch points of the function $k_{2z} = k_{2z}(\omega)$. Two of them are located in the vicinity of the frequencies ω_0, both with imaginary parts $-\Gamma/2$. Since these are located in the lower half-plane, they do not lead to instabilities. The last two branch points are located right above the real frequencies $k_x c$ and $-k_x c$. For simplicity we assume that $k_x c > \sqrt{2}\omega_0$. Then they have imaginary parts less than $F\Gamma$. Thus, there are necessarily two branch cuts in the upper half-plane. If we want to interpret frequency-domain fields at a real frequency ω, we want to use the inverse Fourier transform instead of (3). However, from the above discussion, to obtain the right result we must add the integrals around the branch cuts: By path deformation the original path from $-\infty + i\gamma$ to $+\infty + i\gamma$ is the same as the path from $-\infty$ to ∞ plus the paths around the branch cuts in the upper half-plane, see Fig. 2. Due to the exponential factor $\exp(-i\omega t)$, the integrals around the branch cuts diverge and dominate after some time.

If the excitation frequency ω_1 is sufficiently far away from the branch points, the instability associated with the branch cut at $k_x c$ is excited very weakly, and can be neglected up to a certain time. This instability can in practice be interpreted as an unbound wave traveling at the surface, with wave vector $k_{2z} = 0$, growing exponentially with time. The condition that the excitation frequency is away from $k_x c$ is quite natural; it means that the incident angle is not close to the critical angle.

To examine the time-domain behavior in detail, it is useful to evaluate (3) analytically for the excitation $u(t)\exp(ik_x x - i\omega_1 t)$, with Laplace transform $\exp(ik_x x)/(i\omega_1 - i\omega)$ [9]. The reflected field at $z = 0$ is given by

$$\mathcal{E}_\rho(x,t) = \frac{1}{2\pi}\int_{-\infty+i\gamma}^{+\infty+i\gamma} \frac{k_{1z} - k_{2z}}{k_{1z} + k_{2z}} \frac{\exp(ik_x x - i\omega t)}{i\omega_1 - i\omega} \mathrm{d}\omega, \tag{6}$$

The integral (6) can be evaluated by a generalized version of the residue theorem, in which we find the contour integral around all poles and branch cuts of the integrand in half-plane $\operatorname{Im}\omega < \gamma$. Simplifying by

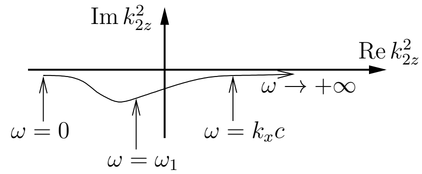

Figure 3: The function $k_{2z}^2(\omega)$ for a typical gain medium, plotted in the complex k_{2z}^2-plane. To identify k_{2z}, we require it to be $+\omega/c$ at $\omega = \infty$, continuous as ω decreases towards zero, except at the branch cut at $\omega = k_x c$ where it changes sign.

assuming $x = 0$, $F \ll 1$, $\Gamma \ll \omega_0$, $k_x c > \sqrt{2}\omega_0$, and $k_x c - \omega_1 \gg \sqrt{F\Gamma}$, we can show that

$$\mathcal{E}_\rho(0,t) = \frac{k_{1z} - k_{2z}}{k_{1z} + k_{2z}} \exp(-i\omega_1 t) + \mathcal{E}_{\mathrm{bc}}(0,t), \tag{7}$$

where the integral contribution around the branch cuts can be bounded

$$|\mathcal{E}_{\mathrm{bc}}(0,t)| \leq \frac{\Gamma^{3/2}}{(k_x c - \omega_1)\sqrt{k_x c(\epsilon_1 - 1)}} \left(e^{F\Gamma t} - e^{-\Gamma t/2}\right) \tag{8}$$

for $t \gtrsim 2/\Gamma$. In other words, for $2/\Gamma \lesssim t \lesssim 1/F\Gamma$, we can ignore the branch cuts contribution such that the reflected field is well described by the first term in (7).

We can now answer the question about existence of evanescent gain. To obtain (7), we have only considered two branch cuts in the upper half-plane; these are the necessary branch cuts due to the zeros of $\epsilon_2\omega^2/c^2 - k_x^2$. Therefore, we must ensure that the integrand in (6) is analytic elsewhere in the upper half-plane. That is, the sign of k_{2z} must be determined such that k_{2z} is analytic everywhere away from the two mentioned branch cuts, in the upper half-plane. Since $k_{2z} \to +\omega/c$ as $\omega \to \infty$, we conclude that $\mathrm{Im}\, k_{2z} > 0$ at the observation frequency ω_1, see Fig. 3. For the conventional gain medium (5) we conclude that, provided the "reflected" field from the side wave can be ignored, evanescent gain is possible.

More sophisticated gain media can be constructed, at least in principle, that behave differently compared to the conventional weak gain media. In fact, the longitudinal wavenumber can be tailored to have a negative imaginary part at an observation frequency $\omega_1 < k_x c$ [6]. Thus both solutions (k_{2z} in the second and fourth quadrant of the complex plane) are possible. In other words, whether or not evanescent gain prevails, depends on the detailed permittivity function. It cannot be determined from the electromagnetic parameters at a single frequency, but must be identified from the entire frequency domain dependence.

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9. Any physical excitation is real and can be represented by two complex terms, e.g., $u(t)exp(-i\omega_1 t) + u(t)exp(i\omega_1^* t)$. For clarity it is often convenient to consider the positive frequency excitation separately.