FULL WAVE THEORY OF WHISTLER DUCTING,

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ABSTRACT

It has been known for many years [1,2] that whistlers can be guided along field lines in the magnetosphere by field-aligned variations in the plasma density. However, it appears that the extensive literature in this area is based on ray optics, which is strictly applicable to situations in which the channels are broad compared to the spatial scale of the whistler wavelength. We are interested in situations where the channel diameter may be comparable to the wavelength, and many years ago Stenzel [3] reported ducting experiments in this regime that have never been fully explained. To address this situation, we have developed a full-wave analytic theory of whistler ducting.

The starting point for our calculation is the cold fluid model with the "quasi-longitudinal" approximation [2], which essentially involves neglecting the displacement current and assuming quasineutrality. In typical situations where $\omega < \Omega_e << \omega_{pe}$, this approximation is accurate except for wave propagation vectors **k**nearly perpendicular to the magnetic field $\mathbf{B}_0$. In particular, it is valid up to and beyond the resonance cone, and gives exactly the same resonance cone as the exact cold fluid / Maxwell's equations. This is therefore quite a reasonable approximation, which simplifies the problem enormously.

We have used two approaches to the analytic solution of the problem. In the first of these, we assume that we have a channel of uniform density n_c embedded in a background medium with a different uniform density n_0, with a sharp density jump at the edge of the channel. For a specified values of $k_{||}$, and for an assumed (but as yet unknown) value of ω, we then solve the dispersion relation analytically for the component $k_\perp$ (perpendicular to **B**) of **k**, both inside and outside the channel. It turns out there are always four solutions for $k_\perp$, but two of these can be ruled out as unphysical at either zero or infinity. Matching solutions at the channel edge leads to a 4×4 determinant that we solve for ω. Ducting is indicated if $k_\perp$ outside the channel is evanescent, either pure imaginary or complex; both cases are found to occur. We find ducted solutions for reduced-density channels ($n_c < n_0$), at all frequencies in the range $0<\omega<\Omega_e$. In addition, we find ducted solutions for enhanced-density channels ($n_c > n_0$), but in this case the ducting is somewhat lossy, since the guided whistler couples (to varying degrees) to a propagating wave as well as an evanescent wave outside the channel.

In the second approach, which can be used for smooth density profiles n(r), we again specify $k_{||}$, but then simply solve a set of ordinary differential equations (which are fourth-order) for E, applying boundary conditions at zero and infinity to determine ω as an eigenvalue. We shall discuss the ways in which the results depend on the shape of the density profile.

We have also performed two-dimensional simulations over a wide range of parameters, using the simulation code HEMPIC [4]. The simulations agree in detail with the analytic calculations, and vividly illustrate the ducting phenomena. Our results also appear to be in good agreement with Stenzel's experiments, as will be discussed.

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