# Ionospheric Total Electron Content (TEC) from the Global Positioning System

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## INTRODUCTION

A huge quantity of ionospheric measurements is presently provided by the Global Positioning System (GPS). As any other precision positioning system GPS must correct for ionospheric effects, and to do this it uses the classical approach of transmitting two coherent frequencies. As ionosphere is a dispersive medium, it affects in a different amount (but in a exactly known way) the signals at the two frequencies: so, taking proper combinations of the propagation delays, the navigator will get rid of ionospheric errors, as the ionospheric investigator will extract ionospheric information only. In practice, the situation is not so good as it could appear. GPS measurements of propagation delays are affected by several factors, the effects of which are not a priori known, so that proper procedures are needed to correct for them. In practice, GPS does not provide direct measurements of ionospheric parameters, but measurements from which ionospheric parameters can be estimated. In the following, a description of what outlined above will be performed, namely the characteristics of actual GPS measurements, the way to extract the ionospheric information, the errors affecting them and the possible ways to get rid of them. In these terms, some of the possible advantages of the single station solution will be described.

## GPS MEASUREMENTS

GPS data are now available at different centers, national and international, many of them participating the International GPS Service (IGS), some of them providing with huge amounts of data densely distributed in space and time, such as the sserver "*lox*" (University of California, San Diego; Scripps Institution of Oceanography; Institute of Geophysics and Planetary Physics) which are here acknowledged. Data are available in RINEX (Receiver Independent EXchange Format) files each one containing the data of one station for one UTC day sampled (generally) every 30$^{th}$ second.

In the following, only items that will be needed for the following discussion will be outlined: the situation with GPS is actually much more complex. The basic observables used by GPS to estimate the distance between satellite and receiver are propagation delays, which are affected by many contributions other than distance itself, namely the contributions from troposphere and ionosphere and instrumental errors. In order to correct for ionospheric contribution, GPS uses two carriers $f_1$ (1575.42 MHz) and $f_2$ (1227.6 MHz). The two carriers are modulated by a pseudo random code **P** that enables the estimation of the distance between the satellite and the receiver by a one-way technique: a local replica of the **P** code is shifted in time at the receivers until it matches the code of the incoming signal. If the time scales of transmitter and receiver coincide, the time shift will provide with the group propagation delay ***GD*** between satellite and receiver: actually this is impossible, therefore the measured group delay will be affected by the offset $\Delta\tau$ between the scales of time. This quantity $DG+\Delta\tau$, multiplied by ***c***, the speed of light, is known as pseudo-range ***P***. With similar techniques, the phase delay ***PD*** of the incoming signal is measured, but also in this case with an offset $\Omega$ that is the ambiguity of the initial phase count. There is a basic difference between the time offset and phase offset. Time offset is a quantity that can vary with time but is the same for all simultaneous measurements. Phase offset is constant for any continuously observed arc, but arbitrarily varying from arc to arc. Offsets are not the only contributions to the measurements of propagation delays: one must taken into account also thermal noise ***n***, multi-path ***m*** and delays introduced by electronic circuits of transmitters $\beta$, and receivers, $\gamma$ (known as biases). See now in detail the actually available GPS observations.

According to the universally used terminology, using the numbers 1 and 2 to specify quantities relative to the frequencies $f_1$ and $f_2$, the measurable delays available from GPS are phase delays ***L1*** and ***L2*** (cycles), and pseudo-ranges ***P1*** and ***P2*** (meters), which will be written in terms in terms of phase propagation delay ***PD*** and group propagation delay ***GD*** and the above reported disturbances (offsets, noise, multi-path, biases).

$$P1 = GD1 + \Delta\tau \cdot c + n1 + m1 + \beta 1 + \gamma 1 \qquad (1)$$
$$P2 = GD2 + \Delta\tau \cdot c + n2 + m2 + \beta 2 + \gamma 2$$

As concerns phase, both noise and multi-path can be neglected in ionospheric work, and instrumental biases can be included in the ambiguity $\Omega$

$$L1 = PD1 + \Omega 1 \qquad (2)$$
$$L2 = PD2 + \Omega 2$$

At GPS frequencies, propagation delays are derived by the optical path $\Lambda$ of Geometrical Optics, using a first order approximation for the path of the ray. The ray is assumed to be a straight line for the two wavelengths (valid for ray elevations greater than 10°). The resulting optical path $\Lambda$ can be written as $\Lambda = D + T + I$, where ***D*** is the geometrical distance between transmitter and receiver, ***T*** and ***I*** the tropospheric and ionospheric contributions obtained integrating along the path the refractivity of the medium. In these assumptions, $D$ and $T$ are the same for both frequencies (troposphere is not a dispersive medium). The ionospheric contribution, using a first order expansion of Appleton-Hartree formula) is given by.

$$I = -\frac{40.3\, TEC}{f^2} \qquad \textbf{(3)}$$

Where ***TEC***, the total electron content, is the integral of electron density along the ray path: this is the ionospheric information to extract. Phase delay $L$ in cycles is given directly by the optical path divided by the wavelength of the signal, $L = \Lambda/\lambda$: in the following, the optical path in meters will be used as measurement instead of phase, so $\Lambda = L \cdot \lambda$ . Group delay ***GD*** in seconds is obtained taking the derivative of phase with respect to frequency, $GD = dL/df$. To write pseudo-range, it is needed multiplying by $c$. Expanding the derivatives and multiplying by $c$ it results that optical path and code path are the same apart the ionospheric contribution which has opposite sign.

$$\begin{aligned} \Lambda &= D + T + I + \Omega \\ P &= D + T - I + \Delta\tau + n + m + \beta + \gamma \end{aligned} \qquad \textbf{(4)}$$

Consider that in the above formulas no new symbol is used for "optical path offset" $\Omega$ which is actually the former phase offset of $L$ multiplied by $\lambda$,

## THE DIFFERENTIAL PROPAGATION DELAYS

The differential propagation delays are formed taking the difference of code paths (pseudo-range) and optical paths (phase times the wavelength) at two frequencies $f_1$ and $f_2$. In the above assumptions, the contributions from geometrical path length, troposphere and GPS-User clock offset are the same for the two channels and will cancel out. All other contributions will give a difference $\delta n$ (noise), $\delta m$ (multi-path), $\delta\beta$ and $\delta\gamma$ (electronic delays in transmitter and receiver circuitry) and $\delta\Omega$ (phase). Taking the differences in reverse order, the sign of ionospheric contribution will be the same, and positive is operating as follows

$$\begin{aligned} \Lambda 1 - \Lambda 2 &= L1 \cdot \lambda 1 - L2 \cdot \lambda 2 = I1 - I2 + \delta\Omega \\ P2 - P1 &= I1 - I2 + \delta n + \delta m + \delta\beta + \delta\gamma \end{aligned} \qquad \textbf{(5)}$$

Consider now the various terms appearing in the above equations.
The term $I1 - I2$ is written as $k \cdot TEC$: in the following, all terms of above formulas will be divided by $k \cdot 10^{16}$ in order to have all quantities in TEC units ($10^{16}$ electrons/square meter). Writing $S_G$ for code delay in TEC units and $S_\Phi$ for phase (optical path) delay, the observations equation so far obtained are (still not changing the symbols for offset, bias, multi-path and noise)

$$\begin{aligned} S_\Phi &= TEC + \delta\Omega \\ S_G &= TEC + \delta n + \delta m + \delta\beta + \delta\gamma \end{aligned} \qquad \textbf{(6)}$$

The two quantities are indicated also as *slants*, as they provide biased, affected by errors estimates of slant total electron content ***TEC***: as they are affected by unknown contributions, they are named as un-calibrated slants. Let's consider the various terms to be evaluated in order to calibrate the slants, taking into account that they preserve the characteristics of the parent terms.
The offset $\delta\Omega$ is arbitrary but constant over a continuously observed arc. In a global analysis with hundreds of stations observing tens of arcs the number of unknowns to determine becomes very large.
The biases $\delta\beta$ and $\delta\gamma$ are delays originating in electronic circuits: it is expected that in environments properly preserved by thermal effects they should undergo only some aging. If the set of data processed covers reasonable spans of time (some day, some month) they can be considered constant in the analysis, i.e. with one $\delta\beta$ for satellite and one $\delta\gamma$ for station.
The noise $\delta n$ is rather strong as the non linear techniques used to get rid of W code, used for anti spoofing, severely degrade the signal to noise ratio. It is a stochastic quantity that will limit the accuracy of the results.
The multi-path $\delta m$ is an instantaneous disturbance as noise, but it is not stochastic: first its distribution has no zero mean. If the location of reflecting objects does not change, its effects repeat day after day shifting about 4 minutes, the difference between solar and sidereal time (periods of GPS orbits are 12 sidereal hours) [1].
Taking a proper combination of code and phase slants it is possible to build another observable, the leveled slant $S$, in the following way. Consider the weighted mean over each continuously observed arc $\langle S_\Phi - S_G \rangle$. As any mean of constant quantities is the quantity itself, it results from (6)

$$\langle S_\Phi - S_G \rangle = \delta\Omega - \langle \delta n \rangle - \langle \delta m \rangle - \delta\beta - \delta\gamma \qquad \textbf{(7)}$$

So, still for each arc, subtracting to $S_\Phi$ the constant quantity $\langle S_\Phi - S_G \rangle$, we obtain the leveled slant $S$

$$S = TEC + \delta\beta + \delta\gamma + <\delta n> + <\delta m> \quad (8)$$

$S$ is affected by the same noise of $S_\Phi$, it is free from offset, affected by the hardware biases $\delta\beta$, $\delta\gamma$, independent from the individual arc, and by the quantities, constant arc by arc, $<\delta n>$ and $<\delta m>$. Besides caring the environment of the antenna, avoiding the presence of reflections, also the way to perform the average affects $<\delta n>$ and $<\delta m>$ and great care must be taken in this operation: data at low elevation angle should be given small weights, short arcs should be avoided [2].

## THE CALIBRATION BY THE THIN SHELL METHOD

In the following the mapping function approach will be discussed: anyway most of the problems related to the observations described above will hold for all other approaches too. According to the thin shell method, the ionosphere is confined in an in infinitely thin shell at a given reference height. For each point $P$ of the shell, individuated by two (horizontal) coordinates, i.e. geographical, geomagnetic, sun-synchronous etc, it is defined a vertical total electron content ***VEC***: slant ***TEC*** of rays crossing the shell in $P$ shall be given by

$$TEC = VEC \sec\chi \quad (9)$$

where $\chi$ is the angle between the vertical in $P$ and the ray. Note that this approximation does not take into account of gradients: all ***TEC***s of rays having the same $\chi$ will have the same ***TEC*** whichever be their direction. ***VEC*** can be written as a proper expansion of functions $p_n(l,f)$ of horizontal coordinates $l, f$ with one set of coefficients $c$ at each time $t$

$$VEC(l,f) = \Sigma_n c_n p_n(l,f) \quad (10)$$

Horizontal coordinates can be geographical or geomagnetic latitude and longitude, or any other set better describing the behavior of ionosphere, such as local time and modified (according to Rawer) dip latitude. The equations of observation will therefore be written, using (10) and indicating by $O_{ijt}$ one of the observations $S_\Phi$, $S_G$ or $S$ to the satellite $i$ from the station $j$ at time $t$

$$O_{ijt} = \Sigma_n c_{tn}\, p_n(l_{ijt}, f_{ijt}) \sec\chi_{ijt}\ [\delta\beta_\iota,\ \delta\gamma j,\ <\delta n>_{Arc},\ <\delta m>_{Arc},\ \Omega_{Arc}] \quad (11)$$

where one will select the proper set of quantities (biases, offsets, possibly average multi-path and noise) in the square bracket according to the observation used. These quantities, together with the set of coefficients $c$ (which is the same for all types of observation) will be the unknowns of the problem. After the solution, one will get the quantities enabling to evaluate ***TEC*** from the observations in (6) or (8): the calibration is obtained. Errors affecting this procedure come from the following causes:

1) Errors in the observations
2) Errors due the mapping function approximation
3) Inadequacy of model (10) describing the ***VEC*** behavior

The best choice of the observable to use is determined by the resulting observations/unknowns budget and by their noisiness. Noisiness excludes the use of code slants, affected by strong noise and multi-path. For phase and leveled slants, noisiness is the same and very low for ionospheric work. As outlined, using phase slants there is an unknown offset for each continuous arc. Some techniques use as observables the first differences in time of phase slants: this eliminates the arc offsets and reduces the solution to the coefficients of TEC expansion only. The offsets for the calibration are then evaluated after the solution as $S_\Phi - TEC_{Model}$: the results will be strongly dependent on the model used. Most of techniques use the leveled slants $S$: in this case the unknowns, other than the coefficients $c$, are one bias per satellite and one bias for station, a very reduced set if one considers a global or regional analysis. The terms $<\delta n>$ and $<\delta m>$ are assumed to be disturbances affecting the ultimate accuracy of the results. This assumption is satisfactory for $<\delta n>$, but for $<\delta m>$ the situation can be problematic. At each time, the set of observations from station $j$ to satellite $i$ is described by

$$S_{ij} = TEC_{ij} + <\delta n>_{ij} + <\delta m>_{ij} + \delta\beta_\iota + \delta\gamma \quad (12)$$

If the differences of slants to satellite $i$ for two close stations $j$ and $k$ are taken, given that $TEC_{ij} \approx TEC_{ik}$, (12) will give

$$S_{ij} - S_{ik} = <\delta n>_{ij} - <\delta n>_{ik} + <\delta m>_{ij} - <\delta m>_{ik} + \delta\gamma_j - \delta\gamma_k \quad (13)$$

If the multi-path contribution is not significant, all the quantities for a given station pair would provide, apart from noise, the difference in station biases $\delta\gamma_j - \delta\gamma_k$. An experimental analysis of calibrated slants from two close stations has been carried out in collaboration with C. Brunini and F. Azpiliqueta of La Plata (Argentina), and shows a remarkable dispersion of data arc by arc (up to ±7 TEC units, still more for other pairs of stations). The dependence on multi-path is confirmed indirectly by the fact that day by day the results repeat almost systematically, due to the GPS constellation shift of about 4 minutes. There is also a direct confirmation of the role of multi-path by the strongly reduced dispersion (±1.5 TEC units) obtained using one only antenna feeding two receivers: this ensures the same multi-path for the two receivers. Therefore, using the leveled slants, the model of the equations of observation should take into account of an extra term per station per arc, affecting heavily the observations/unknowns budget in global and regional solutions. The problem is reduced, but not solved, for the single station solution, as in the solution the multi-path contribution can be included in the satellite bias, preserving the traditional form of the equations. This holds for long arcs occurring only once in a day, as for satellites with two arcs at the beginning and the end of the UTC day (the typical set of RINEX data) the problem is still present.

The other factor affecting accuracy is the use of the mapping function approximation: but also in this case the single-station solution has some advantage. The assumption of the thin shell can be reviewed in the following way: we have to compute the integral of electron density, which can be rewritten using the sec $\chi$ of the ray at a given reference height $h_{Ref}$, sec $\chi_{Ref}$

$$TEC = \int N_e(h,l,f)ds = \int N_e(h,l,f)\sec\chi ds = \sec\chi_{\mathrm{Re}f}\int N_e(h,l,f)\frac{\sec\chi}{\sec\chi_{\mathrm{Re}f}} = \sec\chi_{\mathrm{Re}f}V_{Eq} \qquad \textbf{(14)}$$

In (14), $h$ is the height and $l,f$ the horizontal coordinates used. This is formally identical to (9), with the difference that to map ***TEC***, it is not used the actual vertical content ***VEC***, but an equivalent content $V_{Eq}$ which is the vertical content at zenith, but in general different from it at other zenith angles. This concept does not apply to multi-station solutions, as $V_{Eq}$ is evaluated in a different way by different stations. Not differently by other methods, care must be taken in using observations above 10° elevation, as simulations carried out using ionospheric models show that it becomes very difficult to find an expansion like (10) reproducing correctly $V_{Eq}$. This leads to the last problem of the calibration, just the expansion of $V_{Eq}$ in the single station solution, which is its weakest points in special circumstances. For the single station, a polynomial expansion is generally satisfactory, but in some area (presence of equatorial anomaly) this could require a high order for the polynomial, higher than the number of available observations. This problem can be circumvented with a proper use of horizontal coordinates.

## CONCLUSIONS

Having in mind what reported before, a summary of the steps of the single station solution presently used by the writer to calibrate GPS data follows

RINEX files are downloaded and decompressed. Differential delays are computed as in (5) transforming phases ***L1*** and ***L2*** in optical paths ***Λ1, Λ2.*** A raw filtering of code slants is performed to reject unreliable points. A recovery of phase jumps occurring during one arc is attempted. Phase slants are then leveled to code slants as in (8). The equations of observations (11) are written using local time and Rawer [3] modip angle using a reference height of 400 km. Plasmaspheric contributions (contrarily to former solutions) are not taken into account, as in (14) electron density is assumed to include plamaspheric density. The order of the polynomial varies inversely to the modip angle: from 2nd to 3rd order for local time, from fourth to 6th order for modip. The solution for the biases is carried out as follows: consider (11) in terms of leveled slants, local time and modip at one time or a set iof adjacent times and rewrite it matrix form

$$S_{ijt} = \Sigma_n c^{t}_n p_n(l_{ijt}, f_{ijt}) \sec\chi_{ijt} + \beta_i + \gamma_j \qquad \textbf{(15)}$$

$$S = Ac + B\beta + C\gamma \qquad \textbf{(16)}$$

The LSQ formal solution for $c$ is $c = (A^TA)^{-1}A^T(S - B\beta - C\gamma)$. Reintroducing $V$, a set of equations containing <u>only the biases</u> is obtained

$$S = A(A^TA)^{-1}A^T(S - B\beta - C\gamma) + B\beta + C\gamma \qquad \textbf{(17)}$$

The set of column $S$ become a new set of observation at time $t$, provides with the full set of equations to solve by standard LSQ.

No more care is obviously given to the fact that "satellite biases" are different from station to station, as they include the multi-path contribution.

The standard deviation of residuals for slants after the computation ranges from tenths of TEC unit at middle latitudes to about TEC at low latitudes.

## REFERENCES

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