Empirical Study on The Rain Drop-Size Model for Rain Attenuation Calculations

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ABSTRACT

The attenuation due to rain has been recognized as one of the major cause of unavailability of radio communication systems operating above about 10 GHz. To design radio links for telecommunications and to evaluate the attenuations due to rainfall, it is important to have a good model for raindrop-size distribution(DSD). In this paper, the extended generalized gamma distribution is introduced to represent the raindrop-size distribution based on the measurement performed in Chungnam National University. Analysis of the fitted parameters of this distribution revealed that the lognormal distribution is the most adequate than other distributions. The rain attenuation has been also calculated using the DSD and compared with the measurement data at 40 Ghz. The result shows a good agreement with the measurements.

Index Terms : Rain attenuation, drop size distribution , DSD, rain.

INTRODUCTION

To design radio links for telecommunications and to evaluate the attenuations due to rain, it is important to have a good model for raindrop-size distributions. However, natural rain drop size distributions are highly variable in different climatic zones and, they are modeled using distributions like exponential, gamma, lognormal, and weibull. Many researches have been perform, but still there is no accepted judgment which of above mentioned distributions describes more accurately the raindrop-size distributions. In this paper, we propose to the extended gamma distribution function to describe a raindrop-size distribution, because widely used distributions like exponential, gamma, lognormal, and Weibull distributions can be considered as special cases.

The probability density function for the extended gamma distribution is given by

$$p(t; \mu, \sigma, \lambda) = \frac{|\lambda|^{1-2\cdot\lambda^{-2}}}{\Gamma(\lambda^{-2})\cdot\mu\cdot\sigma} \cdot \left(\frac{t}{\mu}\right)^{(1/\sigma\cdot\lambda)-1} \cdot \exp\left[-\lambda^{-2}\cdot\left(\frac{t}{\mu}\right)^{\lambda/\sigma}\right]$$ (1)

where μ and σ are scale and shape parameters, respecting. Relations of the some widely used distributions to the extended gamma distribution are demonstrated in Table 1.

Table.1   Relations of the parameters in DSD models

Case	λ	Scale	Shape
Gamma	$\lambda = \sigma$	$\mu\cdot\sigma^2$	σ^{-2}
Exp	$\lambda = \sigma = 1$	μ	-
Lognormal	$\lambda = 0$	μ	σ
Weibull	$\lambda = 1$	μ	σ^{-1}

EXTENDED GAMMA DISTRIBUTION MODEL FOR THE RAIN DROP SIZE DISTRIBTION

Assuming the extended gamma distribution, the raindrop-size distribution can be formulated as

$$N(D; N_0, \mu, \sigma, \lambda) = N_0 \cdot \frac{|\lambda|^{1-2\cdot\lambda^{-2}}}{\Gamma(\lambda^{-2})\cdot\mu\cdot\sigma} \cdot \left(\frac{D}{\mu}\right)^{(1/\sigma\cdot\lambda)-1} \cdot \exp\left[-\lambda^{-2}\cdot\left(\frac{D}{\mu}\right)^{\lambda/\sigma}\right]$$ (2)

For a given DSD, the rainrate is calculated as ;

$$R = 6\cdot\pi\cdot10^{-4}\cdot\int_{D_{\min}}^{D_{\max}} D^3\cdot v(D)\cdot N(D; N_0, \mu, \sigma, \lambda)\cdot dD$$ (3)

where $v(D)$ is the terminal velocity of a raindrop with diameter D.

Method for Parameter Estimation

The parameters μ, σ and λ were estimated from the distrometer data, through the maximum likelihood estimation method. Since the raindrop-size distribution N is also a function of R, the above relation (consistency relation(3)) must be satisfied in order for N to the valid.

All the analyses given in this paper are based on the data collected from July 2003 to November 2003 in Chungnam National University. The Joss-Walvogel distrometer (RD-80, Distromet Co. Ltd) has been used for measurement. In the JW distrometer, the measurement data are classified into 20 channels. The binned data for rainrate and raindrop-size were recorded in 10 sec interval. Assuming a Poisson distribution for the number of drops in each bin, n_b, the maximum likelihood function is given as follows :

$$L(\mu,\sigma,\lambda) = -\sum_{b=1}^{M} n_b \cdot \ln(f_b) + \sum_{b=1}^{M} f_b \qquad (4)$$

where f_b is the fitted bin contents

$$f_b = \int_{D(b_{i-1})}^{D(b_i)} f(D;\mu,\sigma,\lambda)dD \qquad (5)$$

To check the shape of the raindrop-size distributions we have selected the data, with integration time 60s, for which the calculated rainrate lies within 10% of the corresponding measured rainrate. We supposed here that selected data are less affected by sampling error and represent real raindrop-size distributions. Scattergrams of the selected data are shown in Fig 1. If we look at the scattergrams, one can see that exponential, Weibull, or gamma distribution is not a good candidate for raindrop-size distribution. On the other hand, application of the extended generalized gamma distribution with a negative λ parameter is not desirable because some moments are not existent.
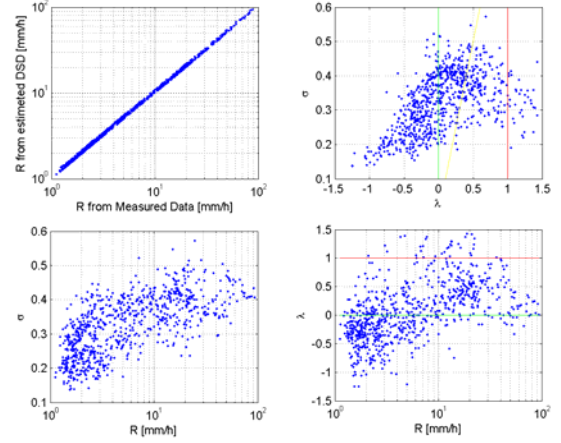

Fig. 1 Scattergrams of the parameters μ, σ and λ for the generalized gamma distribution. The integration time is 60sec.

For such reasons, the lognormal distribution is chosen in this paper for the raindrop-size distribution, the expression for lognormal distribution function is given below

$$p(t;\mu,\sigma) = \frac{1}{\sqrt{2\cdot\pi}\cdot\mu\cdot\sigma}\cdot\left(\frac{t}{\mu}\right)^{-1}$$
$$\cdot\exp\left\{-\frac{1}{2}\cdot\left[\frac{1}{\sigma}\ln\left(\frac{t}{\mu}\right)\right]^2\right\} \qquad (6)$$

The parameter scattergrams for lognormal distribution are shown in Fig. 2.
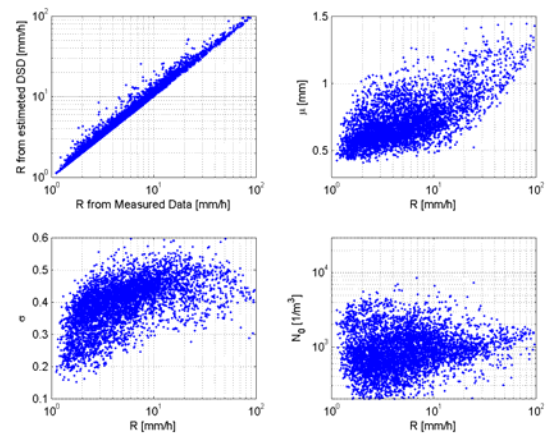

Fig. 2 Scattergrams of the lognormal distribution as a function of rainrate. The integration time is 60s.

The parameters of the lognormal distribution are

estimated by the maximum likelihood method mentioned before. The results are :

$$N_0 = \exp(0.04533 \cdot \ln(R)^3 - 0.4187 \cdot \ln(R)^2$$
$$+ 1.48 \cdot \ln(R) + 4.929)$$
$$\mu = 0.01709 \cdot \ln(R)^2 + 0.09389 \cdot \ln(R) + 0.5215 \tag{7}$$
$$\sigma = -0.01818 \cdot \ln(R)^2 + 0.1108 \cdot \ln(R) + 0.2705$$

In Fig. 3 the proposed DSD model is compared with measured DSD data at four different rain rates 5, 10, 40 and 80 mm/h. We can see a good agreement between the two except at the peak of the distribution.
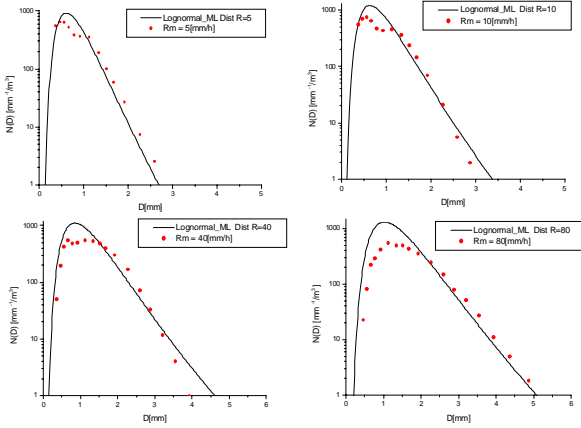

Fig. 3 Comparison of rain drop size distribution and measurement data

RAIN ATTENUATION MODELING

To verify the DSD model, we calculated the specific attenuation using our DSD model and compared the results with measurement data. Fig. 4 shows the measurement system for rain attenuation in Chungnam National University. The frequency is 44 GHz and the distance between the Tx and Rx module is about 500 meters.
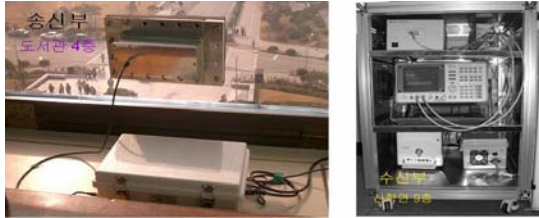
(a) Transmitter   (b) Receiver
Fig. 4 Measurement system for rain attenuation.

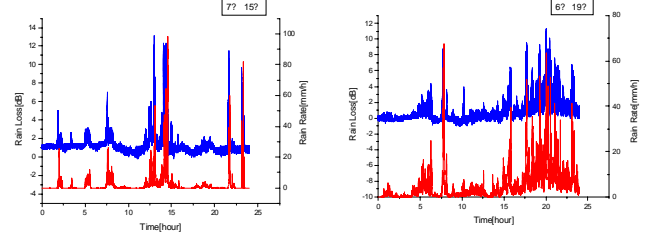Fig. 5 shows the measured rainrate and rain attenuation.

Fig.5   Comparison of the rain rate and rain attenuation

The specific attenuation due to rain for a given DSD model can be obtained

$$\gamma = 20 \log(e) \times 10^3 \int_0^{D_m} \frac{2\pi}{k^2} \mathrm{Re}\{S(0)\} N(D) dD$$
$$= 4.343 \times 10^3 \int_0^{D_m} Q(D) N(D) dD \quad [dB/km] \tag{8}$$

where $S(0)$ is the forward scattering amplitude and $Q(D)$ is the total extinction cross section in mm^2 of a raindrop with diameter D. Fig. 6 shows the specific attenuation calculated from the our DSD model and the measurement data. We can see an excellent agreement between the measured data and the calculated result, which reveals that our empirical DSD model works good in millimeter-wave frequency band.
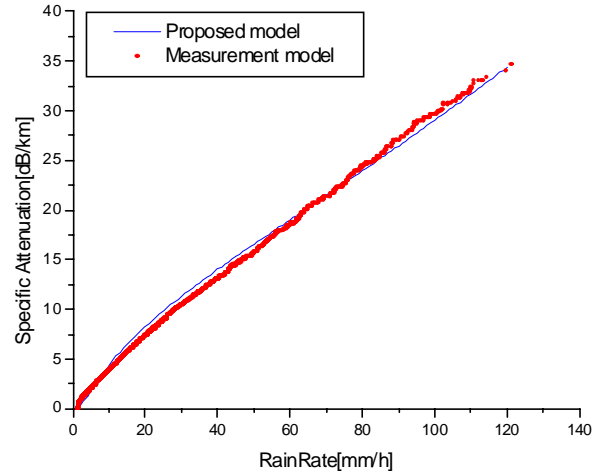

Fig. 6 Comparison of the calculated result with the measurement data.

CONCLUSION

In this paper, we introduced a new model for raindrop-size distribution, based on the measurement performed in Chungnam National University. It is concluded that the lognormal distribution is the most adequate candidate for describing rain drop size distribution than other distribution models. The results of the calculated rain attenuation using the proposed DSD model revealed an excellent agreement with the measurement data. Further study and measurement are on going, and the results will be published in other paper. Also a prediction model for rain attenuation in a 44 GHz satellite link, which includes an effective path length model, is currently under development.

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