# RECENT DEVELOPMENTS OF TRANSMISSION LINE MODELS FOR FULL-WAVE ANALYSIS OF INTERCONNECTS

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## ABSTRACT

The paper presents a complete review of a recently proposed *enhanced* transmission line (TL) model, which extends the validity of the popular TL model to frequency ranges where the transverse characteristic dimension of the interconnect is no longer electrically short. While retaining the same mathematical simplicity of a TL model, the model presented here is derived from a full-wave integral formulation, in terms of electromagnetic potentials. Case-studies and benchmarks show its ability to foresee high-frequency effects like radiation or dispersion. Furthermore, two-port representations of the interconnects are easily obtained by the solution of the model: this allows coupling the interconnect to the terminal devices and performing time-domain analysis, *e.g.* the typical signal-integrity tests.

## INTRODUCTION

In many practical VLSI applications, the bandwidth of signals carried by the electrical interconnects extends to frequency ranges where the quasi-TEM hypothesis of propagation no longer holds. In such cases, the popular transmission line model is inadequate to catch those high-frequency effects, like radiation and long-range non-local coupling, which may affect the correct operation of these circuits. These effects make the electrical design of such circuits extremely challenging and highlight the need of accurate and efficient electromagnetic models (*e.g.*,[1]).

An efficient way to analyze interconnects with homogeneous dielectrics is achieved by means of a full-wave integral formulation, based on the vacuum Green function, in which the current and charge distributions are the unknowns. This approach requires the solutions to be evaluated only in the conductor domain (the conductor surfaces if they were ideal). In addition, the boundary conditions at infinity may be imposed automatically. However, the full-wave numerical analysis of interconnect structure may be very costly, and sometimes unnecessary. For this reason, many authors are proposing the extension of the transmission line theory to such frequency ranges, through models which retain the same simplicity of the *standard* transmission line (*STL*) model. In this way it is possible to overcome at least three of the main problems associated to a brute-force full-wave numerical simulation: the computational cost, the qualitative insight and the field/circuit coupling. (*e.g.*, [2]-[6]).

In this paper we review a recently proposed model which is directly derived from an integral formulation of the full-wave electromagnetic problem and extends the validity of the *STL* model to cases where the smallest characteristic wavelength of the signals is comparable to, or smaller than, the distance between the conducting wires. Such an *enhanced* transmission line (*ETL*) model, describing finite-length interconnects with uniform sections, has the same structure of the *STL* model but is able to foresee high frequency effects like radiation or dispersion with a computational cost sensibly lower than that required by a full-wave numerical simulation [7]-[9].

Since its mathematical structure is similar to that of the classic transmission line model, its computational cost is relatively low. In addition, the *ETL* model provides accurate results, because it solves the field problem with the most correct approach, letting both the "field" and the "source" points to lie on the conductor surfaces. In this way it is able to catch all the singularities associated to the surface distributions, and to account naturally for the proximity effects. These two features are extremely important in IC and computer-chip problems, where the traces are very close to each other.

Case-studies and benchmark tests are presented showing the ability of the model to predict the frequency-domain behavior of the solution. In addition, the possibility to use the model to characterize the interconnect as a two-port, along with a suitable model-order-reduction, allows performing an efficient time-domain analysis and studying the influence on the signal integrity of the effects of high operating frequencies.

## DERIVATION OF THE ETL MODEL

Let us consider an interconnect of length $2l$, made of two perfectly conducting parallel conductors with arbitrary cross-sections, embedded in a homogeneous dielectric, Fig.1a. Here $\Sigma_1$, $\Sigma_2$ are the lateral surfaces of the wires and $\Gamma_1$, $\Gamma_2$ the contours of the respective transverse cross-sections, of length $c_1$ and $c_2$, Fig.1b.

Faraday's equation is automatically satisfied posing the electric field **E** equal to:

$$\mathbf{E} = -i\omega\,\mathbf{A} - \nabla\varphi \tag{1}$$

where the magnetic vector potential **A** and the electric scalar potential φ are expressed (using *Lorenz gauge*) as a function of the surface current density $\mathbf{J}_s$ and the surface charge density σ by:

$$\mathbf{A}(\mathbf{r}) = \frac{\mu_0}{4\pi}\iint_\Sigma G(|\mathbf{r}-\mathbf{r}'|)\mathbf{J}_s(\mathbf{r}')dS', \qquad \varphi(\mathbf{r}) = \frac{1}{4\pi\varepsilon_0}\iint_\Sigma G(|\mathbf{r}-\mathbf{r}'|)\sigma(\mathbf{r}')dS' \tag{2}$$

where $G$ is the Green's function of free space $G(r) = e^{-ikr}/r$, $k$ is the wave number and $\Sigma = \Sigma_1 \cup \Sigma_2$. The latter position is true if we can assume that the interaction between the terminal devices and the interconnect itself is mainly due to the terminal currents and voltages, hence their contribution to the potentials may be neglected. To close the mathematical model, we must impose the *charge conservation law* and the *boundary condition*:

$$\nabla_s \cdot \mathbf{J}_s\big|_\Sigma = -i\omega\sigma\big|_\Sigma, \qquad \mathbf{E}\times\mathbf{n}\big|_\Sigma = 0, \tag{3}$$

where **n** is the unit vector normal to Σ.

In order to derive a transmission line model we have to assume that the current is directed along **z**: $\mathbf{J}_s = J_s(\mathbf{r}_\perp, z)\hat{z}$, and that the *common mode* variables are equal to zero. In these conditions, by applying (3) we get two relations between the voltage and the per-unit-length (*p.u.l.*) magnetic flux, and between the current and the *p.u.l.* electrical charge:

$$-\frac{dV(z)}{dz} = j\omega\Phi(z), \qquad -\frac{dI(z)}{dz} = j\omega Q(z). \tag{4}$$

In addition we assume a separable-type dependence of $J_s$, $\sigma$ on the transverse and longitudinal coordinates:

$$J_s(\mathbf{r}) = \begin{cases} F_1(s_1)I(z) & \text{on } \Sigma_1, \\ F_2(s_2)I(z) & \text{on } \Sigma_2, \end{cases} \qquad \sigma(\mathbf{r}) = \begin{cases} F_1(s_1)Q(z) & \text{on } \Sigma_1, \\ F_2(s_2)Q(z) & \text{on } \Sigma_2, \end{cases} \tag{5}$$

and that the transverse 2D problem may be solved in the quasi-static limit.

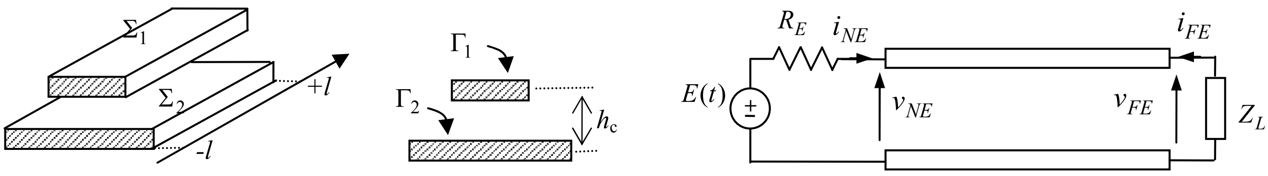
Once the *shape* functions $F_1$ and $F_2$ are computed, by applying (2) and (5) we may obtain the constitutive relations:

$$\Phi(z) = \mu\int_{-l}^{l} H(z-z')I(z')dz', \qquad V(z) = \frac{1}{\varepsilon}\int_{-l}^{l} H(z-z')Q(z')dz' \tag{6}$$

where the kernel is given by

$$H(z) = \frac{1}{c_1}\oint_{\Gamma_1} ds_1\left[\oint_{\Gamma_1} G(r_{P_1Q_1})F_1(s_1)ds_1 + \oint_{\Gamma_2} G(r_{P_1Q_2})F_2(s_2)ds_2\right] - \frac{1}{c_2}\oint_{\Gamma_2} ds_2\left[\oint_{\Gamma_1} G(r_{P_2Q_1})F_1(s_1)ds_1 + \oint_{\Gamma_2} G(r_{P_2Q_2})F_2(s_2)ds_2\right]. \tag{7}$$

In the particular case of widely-separated equal conductors with circular cross-section of radius $a$ the kernel $H(z)$ may be evaluated analytically [7]. In the general case $H(z)$ can be only evaluated numerically, and attention must be paid to




(a) (b) (c)

Figure 1. (a): Reference geometry; (b): cross-section; (c) reference network..

its singularities. In fact, the kernel may be divided into a static part, that is, the limit value for $k \to 0$, and a dynamic part strongly dependent on $k$. The static part has the typical logarithmic singularity of surface distributions at $z = 0$:

$$H(z) \approx -\frac{\ln(z)}{2\pi^2 a} \text{ for } z \to 0, \qquad a = \frac{1}{\pi}\left(\frac{1}{c_1} + \frac{1}{c_2}\right)^{-1}. \tag{8}$$

The *ETL* longitudinal full-wave 1D propagation model is given by (4), (6), where (6) makes the difference between such a model and the. In fact, in the additional hypothesis that the transverse dimensions are electrically short, (6) reduce to:

$$\Phi(z) = LI(z), \; V(z) = Q(z)/C, \tag{9}$$

where the *p.u.l.* parameters $L$ and $C$ are solutions of the static 2D problem in the transverse plane. Substituting (9) in (4) we obtain the popular Telegrapher's Equations for ideal lines, hence the *STL* model is a particular case of *ETL* one.

Summarizing, the *ETL* model is valid in the following limits: (i) the characteristic dimensions of the terminal devices are small compared to the interconnect length; (ii) the sources and the interconnect geometry do not excite common-mode currents; (iii) the characteristic transverse dimensions of the conductors and of the cross-section are electrically short; (iv) $kh_c \leq 1$, where $h_c$ is the distance between the conductors (Fig.1b).

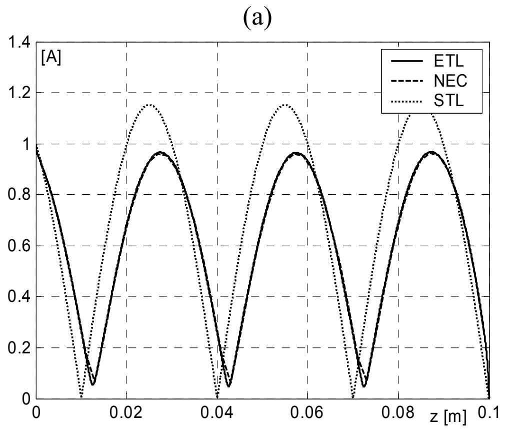
## ANALYSIS OF CASE-STUDIES

Let us first refer to a simple case of a two-wire interconnect, with radius $a = 1$mm, separation $h_c = 10$ mm and length 100 mm (Case 1). Figure 2a shows the distribution along the interconnect axis of the amplitude of the current obtained assuming as boundary conditions: $I(z=0) = 1\ A$, $I(z = 100mm) = 0\ A$. The solution is computed at 5 GHz, hence it is $kh_c \approx 1$ (the interconnect transverse dimension is electrically long). The results provided by the *ETL* model are compared to those obtained by the *STL* model and by a commercial full-wave numerical code (*NEC*, Numerical Electromagnetic Code). It is evident that *ETL* model is able to predict correctly both the amplitude and the nodal position of the high-frequency distribution.

Case 2 refers to an interconnect made of two conductors with rectangular cross-sections (1 mm x 2 mm), assuming a separation of $h_c = 2$ mm (the proximity effect is not negligible). The interconnect, whose length is 30 mm, has been modeled as a two-port through the Z-matrix representation. Figure 2b shows the amplitude of the self impedance $Z_{self}$, compared to that obtained by the *STL* model and to that provided by a full-wave numerical code (Surfcode [10]). Here again the frequency range is such that $kh_c \approx 1$, and it is evident that *ETL* model provides a very good approximation of the actual solution, while the *STL* model is very far from it. In particular, *ETL* model is able to foresee the finite-amplitude of the resonance peaks and their shift to lower values with respect to the ideal case. These effects are clearly due to radiation losses and to the dispersion effect introduced by the finite length of the interconnect.

In order to check the time-domain effects due to such operating conditions, the Z-matrix entries have been approximated by a reduced-order model, identified through the Vector Fitting algorithm [11]. Figure 3a shows the frequency behavior of the amplitude of $Z_{\text{mutual}}$, along with that of an approximating 25-pole version.

Using such reduced-order model we may perform a time-domain analysis of the system behavior at high operating



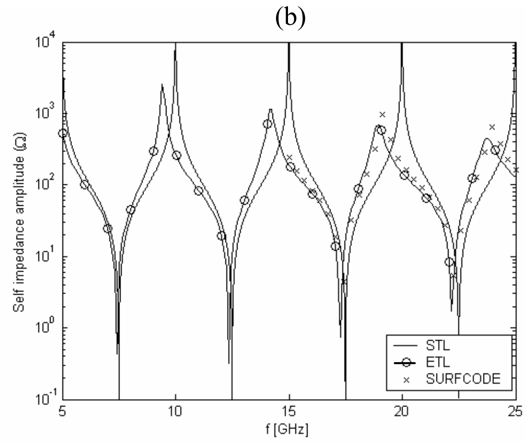




Figure 2. (a): Current distribution for Case 1; (b): Amplitude spectrum of $Z_{\text{self}}$ for Case 2.

(a) (b)

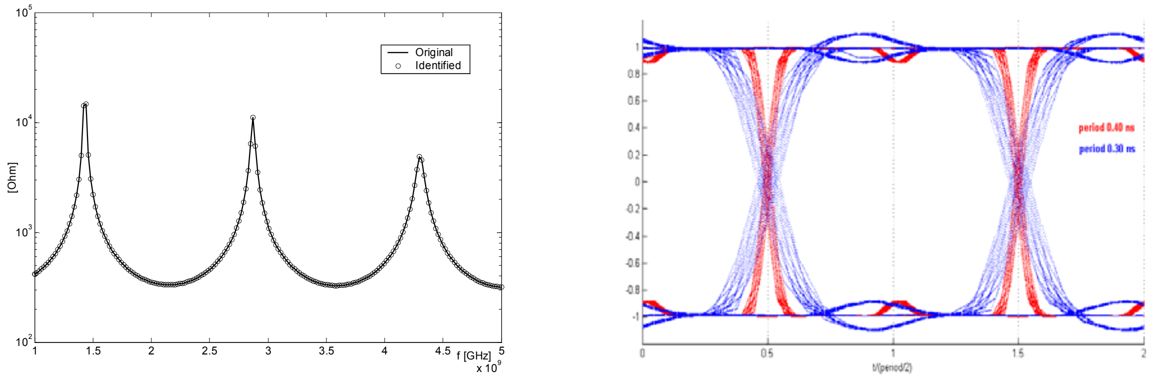



Figure 3. (a) Original and identified $Z_{mutual}$ for Case 2: (b) signal integrity analysis through the eye-diagram for Case 2.

frequencies: Fig.3b shows the eye-diagram obtained analyzing the simple signaling system depicted in Fig.1c, where the interconnect of Case 2 is used. In the *STL* limit, such an interconnect is modeled by a lossless transmission line with characteristic impedance $Z_0 \cong 96.9\,\Omega$ and delay time $T_{delay} \cong 0.10\,ns$. The terminal voltage source is a train of trapezoidal pulses with rise-time of 0.1 ns, period of 0.30 ns and duty-cycle of 0.5. In addition we have $R_E = Z_0$, while $Z_L$ is a capacitive load of 0.01 pF. The signal spectrum ranges over frequencies such that $kh_c \approx 1$, hence the results may be sensibly different from those predicted by the *STL* model. In particular a degradation of the eye-diagram is observed (Fig.3b), due to the high-frequency effects discussed above. However, by analyzing the transfer function of the system, it is possible to predict an optimal position for the signal carrier, in order to minimize such effects: in our case, the optimal period of the source is 0.40 ns (Fig.3b). This prediction is made possible by the accurate knowledge of the frequency-behavior of the structure, and in particular of the exact actual positions of the resonance frequencies.

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