# Coupled Mode Theory for Long Period Grating written in 980nm pumped Erbium Doped Fiber

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**ABSTRACT**

We describe the behavior of long period gratings (LPG) in 980nm pumped Erbium doped optical fibers. We have reformulated the coupled mode analysis of long period gratings to incorporate the gain term. Our results show that varying the gain coefficient can control the transmission spectrum of the grating by appropriate choice of pump power levels.

## 1. INTRODUCTION

Wavelength-dependent loss elements perform a simple function that is required in many optical fiber systems. Long period fiber gratings have emerged as an important wavelength-dependent loss element, as they are suitable for splicing in a fiber link because of their low insertion loss and low back-reflection. A long period grating couples light from the fundamental guided core mode to the forward propagating cladding modes. In its usual application the light coupled to the cladding at the end of grating length decays due to scattering losses leaving loss bands in the guided core mode observed at the output. The transmission spectrum of long period grating therefore consists of distinct resonant peaks [1].

In this paper, we report a quantitative description of LPG in amplifying environment. This essentially implies that the grating is written in an Erbium doped fiber [2] and this leads to distinctive new features. The attenuation of this grating at the resonance wavelength can be controlled by pump power of the EDF.

In order to study the characteristics of such an LPG, we have reformulated the conventional coupled mode equations used for the analysis of the long period grating to include the EDF gain term. It may be mentioned that the EDF gain is a function of the pump power and signal strength and, hence, varies along the propagation length. For completeness, we have also included in our analysis an extraneous attenuation factor to account for the scattering loss of the cladding mode.

## 2. COUPLED MODE ANALYSIS FOR LONG PERIOD GRATING IN EDF

A long period grating fabricated in a 980nm pumped EDF would exhibit gain in addition to the usual grating characteristics. Hence, we have reformulated the coupled mode analysis of the gratings to include the gain term of EDF. Further, the power coupled to the cladding decays due to scattering losses. Hence, we have also included in our analysis an extraneous attenuation factor for the cladding mode. The modified total field at any wavelength $\lambda_j$ can be written as

$$\psi^{(j)} = A_j(z)\, \psi_1^{(j)} e^{-i\bar{\beta}_{1j} z} + B_j(z)\, \psi_2^{(j)} e^{-i\beta_{2j} z} e^{-\alpha z} \qquad (1)$$

where $\Psi_1^{(j)}$ and $\bar{\beta}_{1j}$ represents the normalized modal field and propagation constant of the core mode of the EDF, and $\Psi_2^{(j)}$ and $\beta_{2j}$ represents the normalized modal field and propagation constant of the cladding mode of the fiber for the wavelength $\lambda_j$. The propagation constant, $\bar{\beta}_{1j}$, is complex, i.e., $\bar{\beta}_{1j} = \beta_{1j} + i\gamma_j(z)$, where $\gamma_j(z)$ is the corresponding gain coefficient and $\alpha$ is the extraneous scattering loss. The reformulated equations for evolution of amplitude of core mode as well as cladding mode are

$$\frac{d\bar{A}_j}{dz} = \frac{(\gamma'^2_j z^2 - 2i\beta_{1j}\gamma'_j z + 2\gamma'_j(\gamma_j z + 1))\bar{A}_j}{2(i\beta_{1j} - \gamma_j - \gamma'_j z)} + \frac{4e^{i\Delta\beta_j z}\beta_{1j}\kappa_{12}^{(j)} \sin(Kz)\bar{B}_j}{2(i\beta_{1j} - \gamma_j - \gamma'_j z)} + (\gamma'_j z + \gamma_j)\bar{A}_j \qquad (2)$$

$$\frac{d\bar{B}_j}{dz} = \frac{(2i\beta_{2j}\alpha + \alpha^2)\bar{B}_j}{2(i\beta_{2j} + \alpha)} + \frac{4e^{-i\Delta\beta_j z}\beta_{2j}\kappa_{21}^{(j)} \sin(Kz)\bar{A}_j}{2(i\beta_{2j} + \alpha)} - \alpha\bar{B}_j \qquad (3)$$

In the above equations $\Delta\beta_j = \beta_{1j}-\beta_{2j}$, $\kappa_{12}^{(j)}$ and $\kappa_{21}^{(j)}$ are the coupling coefficients for wavelength $\lambda_j$ and $K = 2\pi/\Lambda$, where $\Lambda$ is the grating period and satisfies the phase matching condition $\Lambda = 2\pi/\Delta\beta_j$ for coupling to a particular cladding mode. The new amplitudes $\overline{A}_j = A_j e^{\gamma_j z}$ and $\overline{B}_j = B_j e^{-\alpha z}$ take into

$$A(z) = \left[\cos(\Gamma z) - i\frac{\delta}{\Gamma}\sin(\Gamma z)\right]\exp(i(\delta)z) \qquad B(z) = \left[-\frac{\kappa}{\gamma}\sin(\Gamma z)\right]\exp(-i\delta z)$$

account the gain of the core mode and loss of the cladding mode. The gain coefficient of the EDF core mode is a function of z since it depends on the power in each signal wavelength and pump power level (which is depleted along *z*) in addition to the modal profile of the core mode as discussed in the following section. If gain coefficient $\gamma$ is independent of z, amplitude of core mode and cladding modes attain the following analytical form [4]

where $\kappa = \sqrt{\kappa_{12}\kappa_{21}}$, $\delta = \frac{1}{2}(\overline{\beta}_1 - \overline{\beta}_2 - \frac{2\pi}{\Lambda})$, $\Gamma = \sqrt{(\delta^2 + |\kappa|^2}$, $\overline{\beta}_1 = \beta_1 + i\gamma$ and $\overline{\beta}_2 = \beta_2 - i\alpha$

### 3. Evaluation of Gain Term $\gamma(z)$ in the EDF

In order to evaluate the modal gain term $\gamma(z)$ in the EDF, we follow the approach developed in [3] in which an EDF can be considered as a fiber with a complex refractive index profile. The analysis in [3] considered propagation of only one signal wavelength. Here, we have extended the analysis to include simultaneous propagation of a number of wavelengths ($\lambda_j$,j=1 to N) leading to the following expression for the gain coefficient of signal wavelengths and attenuation of the pump wavelength

$$\gamma_j(z) = \frac{1}{2}\frac{\Gamma_j\rho_0\sigma_a`(\lambda_j)[\eta_j(q+\sum_k \frac{1}{\eta_k+1}p_k) - (1+\sum_k \frac{\eta_k}{\eta_k+1}p_k)]}{(1+q+\sum_k p_k)} \tag{4}$$

$$\gamma_p(z) = -\frac{1}{2}\frac{\Gamma_p\rho_0\sigma_a`(\lambda_p)(1+\sum_k \frac{\eta_k}{\eta_k+1}p_k)}{(1+q+\sum_k p_k)} \tag{5}$$

$p_k$ is the normalized signal power at $\lambda = \lambda_k$ defined as $p_k = P_k(z)/P_{sat}(\lambda_k)$ and q is the normalized pump power, given as $q = P_p(z)/P_{sat}(\lambda_p)$, where $P_{sat}(\lambda)$ is the saturation power.

The decay of pump power (which is not affected by the presence of the grating) is hence described by the following equation

$$\frac{dq}{dz} = -\gamma_p(z)q = -\Gamma_p\sigma_a(\lambda_p)\rho_0 \frac{(1+\sum_k \frac{\eta_k}{1+\eta_k}p_k)}{(1+q+\sum_k p_k)}q \tag{6}$$

The (2N+1) coupled equations (2) and (3) corresponding to N signal wavelengths and equation (6) corresponding to one pump wavelength can be solved, typically, by the fourth order Runge-Kutta procedure to obtain the gain characteristics of the EDF with a LPG written in it.

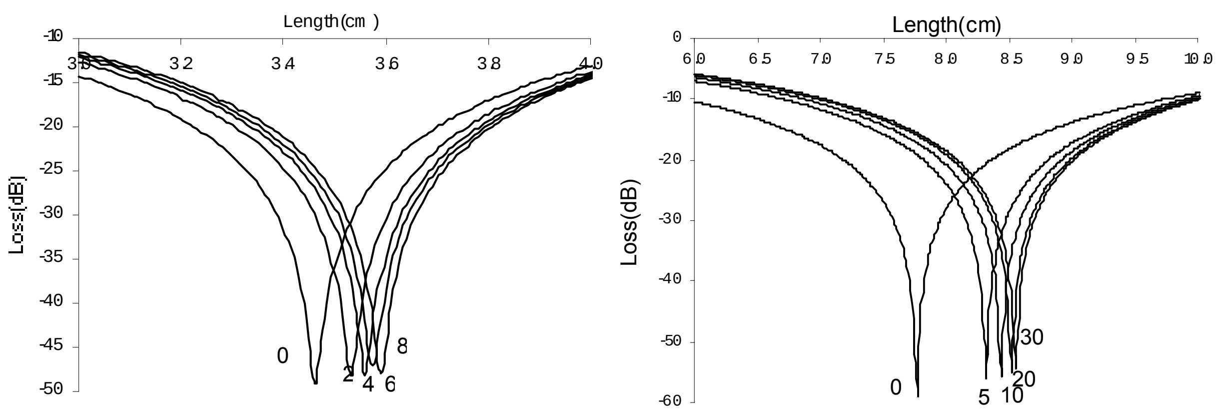
## 4. RESULTS AND DISCUSSION

For the typical fiber parameters tabulated in Table I with a grating of $\Delta n^2 = 2\text{x}10^{-4}$, a grating, which couples complete power from fundamental core mode to $LP_{03}$ cladding mode at 1.53 $\mu$m, requires $\Lambda = 671.47\,\mu$m and coupling length of about 8cm. While, a grating, which couples complete power from fundamental core mode to $LP_{08}$ cladding mode, requires $\Lambda = 230.64\,\mu$m and coupling length of about 3.5cm. The coupling length $l_c$ is approximately 100 $\Lambda$. Thus as order of mode coupling m increases grating period $\Lambda$ decreases and coupling length lc increases. For a typical EDF with parameters tabulated in Table 1, the gain coefficient is relatively low, i.e.,$\gamma \sim$ 0.03cm$^{-1}$. Increasing the Erbium ion concentration can increase the value of gain coefficient.

We now take a look at the behavior of the grating in an Erbium doped Fiber. In this direction, we first investigate the nature of the attenuation curves of the core mode with length at different pump powers. The attenuation curves in Fig.1 show that as pump power level increases; the coupling length for complete power transfer from core mode to cladding mode increases due to increase in $\gamma$. Fig. 1(a) shows that for grating of length 3.46cm (which couples power from core mode to $LP_{08}$ cladding mode), an increase in pump power levels beyond 10mW, $\gamma$ saturates for the chosen Erbium ion concentration, and hence, there is no change in coupling length. However if grating length is chosen to be 7.77cm (which couples power from core mode to $LP_{03}$ cladding mode), the coupling length increases upto pump power level ~ 20 mW as depicted in Fig. 1 (b).

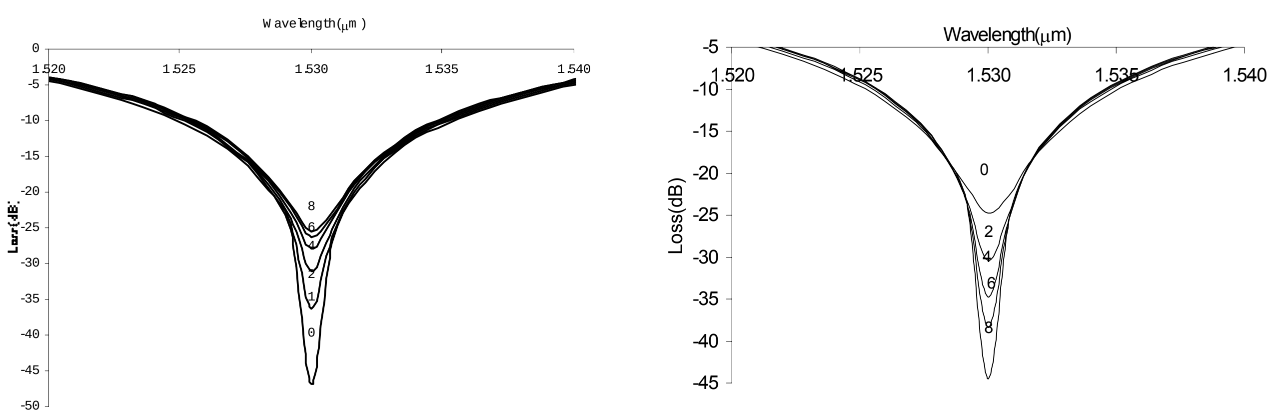
**Table I**

| **EDF Parameters** | | **Grating Parameters** |
|---|---|---|
| Core index ($n_1$) | : 1.458 | $\Delta n^2 = 2x10^{-4}$ |
| Cladding index ($n_2$) | : 1.45 | Mode Coupling : $LP_{01}$-$LP_{09}$ |
| Core radius (a) | : 2.5μm | Grating Period ($\Lambda$) : 230.64μm |
| Erbium ion density ($\rho_0$) | : $1.62x10^{25}/m^3$ | |
| $\sigma_a$(1.53 μm) | : $7x10^{-25}m^2$ | |
| $\sigma_e$(1.53 μm ) | : $6.4x10^{-25}m^2$ | |
| $\sigma_a$(0.98μm) | : $2x10^{-25}m^2$ | |




(a) (b)

Fig.1: Attenuation of core mode with length of the grating corresponding to different pump powers in mW as marked on the curves for grating length of (a) 3. 46cm (b) 7.77 cm

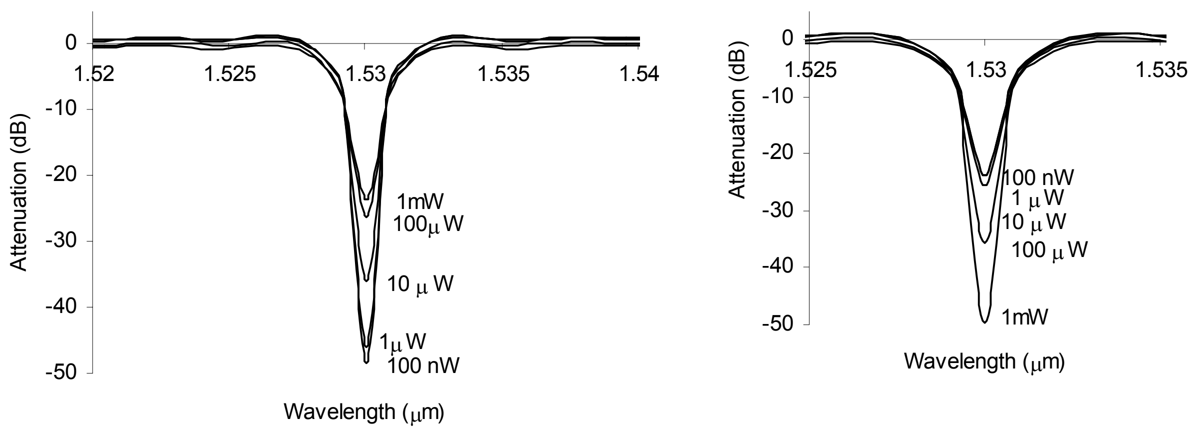



(a) (b)

Fig.2: Transmission spectrum of LPG for different pump powers in mW for grating length of (a) 3.46cm. (b) 3.59cm.

As the coupling length is different for different values of pump powers, we now delve into the transmission characteristics of the grating at different pump powers for a given grating. When grating length is 3.46cm (i.e., coupling length corresponding to zero pump power), the increase in pump power leads to decrease of core mode attenuation at the resonance wavelength as depicted by curves in Fig.2 (a). If the grating length is chosen as 3.59cm (i.e., coupling length corresponding to 8mW-pump power), the dip in the transmission spectrum decreases with decreasing pump power as shown by the curves in Fig.2 (b). Thus attenuation in the grating at resonance wavelength can be controlled by the pump power.

Finally, we also studied the influence on the transmission spectrum of the grating due to different signal power levels at a given pump power of 20mW. As signal power level increases, the coupling length for complete power transfer from core mode to cladding mode decreases due to decrease in $\gamma$. If we choose a grating of length 8.52cm (which couples power from core mode to $LP_{08}$ cladding mode coupling length corresponds to 100nW signal at 20mW pump power) the attenuation of core mode decreases with increasing signal power as shown in Fig. 3(a). The curves in Fig.3 (b) exhibit an opposite nature to that of curves in Fig.3 (a). If we choose a grating of length 8.19cm (which couples power from core mode to $LP_{08}$ cladding mode, coupling length corresponds to 1mW signal at 20mW pump power) the attenuation of core mode decreases with decreasing signal power. In fact, the phenomenon in Fig.3 (b) can be considered for use in automatic gain control.




(a) (b)

Fig. 3: Transmission spectrum of LPG corresponding to a pump power of 20 mW for different signal powers for (a) grating length corresponding to 100nW signal power (b) grating length corresponding to 1mW signal power.

## 5. CONCLUSION

We have shown that LPGs written in an Erbium doped fiber lead to distinctive new features. The attenuation of the grating at the resonance wavelength can be controlled by pump power of the EDF.

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