Principles and applications of dual-band operation in composite right/left-handed metamaterials

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1 Introduction

Over the last five years, a number of novel guided-wave, radiated-wave and refractedwave microwave applications based on electromagnetic metamaterials (MTMs) have been developed [1]. In the context of the first category, this paper explains the concept of composite right/left-handed (CRLH) transmission line (TL) *dual-band* operation and presents a number of applications illustrating this concept.

2 Dual-Band Components

A dual-band (DB) component is a component accomplishing the same function at two different arbitrary frequencies ω_1 and ω_2 . Such a component is therefore constituted of TL sections inducing equivalent phase shifts $\phi_1 = -\beta_1 \ell$ and $\phi_2 = -\beta_2 \ell$, where ℓ is the length of the TL sections, at these two frequencies [2]. In other words, a DB component should exhibit a dispersion relation $\beta(\omega)$ satisfying the double condition

$$\beta(\omega_1) = \beta_1 \quad \text{or} \quad \phi(\omega_1) = \phi_1,$$
 (1a)

$$\beta(\omega_2) = \beta_2 \quad \text{or} \quad \phi(\omega_2) = \phi_2,$$
 (1b)

where the two frequencies (ω_1, ω_2) and the two propagation constants (β_1, β_2) or phases (ϕ_1, ϕ_2) are arbitrary. A CRLH TL possesses this DB property and can therefore be used to transform virtually any microwave TL-based component into a DB component. This DB property of CRLH TL is extremely beneficial to modern wireless communication systems covering two bands because it reduces the number of required circuit components for a given functionality.

3 Principle of CRLH TL Dual-Band Operation

A CRLH TL (Fig. 1) has the balanced propagation constant and characteristic impedance [3]

$$\beta = \frac{1}{p} \left[\omega \sqrt{L_R C_R} - \frac{1}{\omega \sqrt{L_L C_L}} \right], \text{ i.e. } \phi(\omega) = -\frac{\ell}{p} \left[\omega \sqrt{L_R C_R} - \frac{1}{\omega \sqrt{L_L C_L}} \right], \quad (2a)$$

$$Z_c = \sqrt{\frac{L_R}{C_R}} = \sqrt{\frac{L_L}{C_L}},\tag{2b}$$

where L_R/C_R are the series/shunt right-handed (RH) reactances, L_L/C_L are the shunt/series left-handed (LH) reactances, and p ($p \ll \lambda_{guided}$ or $\Delta \phi = \beta p \ll 2\pi$) is the size of the unit cell which is cascaded periodically to constitute the artificial line.



Figure 1: CRLH TL, (unit cell) circuit model and dispersion relation. The dispersion curve of a purely RH (PRH) TL is also shown for comparison.

The four CRLH LC parameters can be determined for the arbitrary dual-band operation by solving the system constituted by Eqs. (1) with Eq. (2a) multiplied by N (number of cells constituting the line) and Eqs. (2b) set to the ports termination Z_t to ensure matching. This yields

$$L_{R} = \frac{Z_{t} \left[\phi_{1} \left(\omega_{1}/\omega_{2}\right) - \phi_{2}\right]}{N\omega_{2} \left[1 - \left(\omega_{1}/\omega_{2}\right)^{2}\right]},$$
(3a)

$$C_R = \frac{\phi_1 \left(\omega_1/\omega_2\right) - \phi_2}{N\omega_2 Z_t \left[1 - \left(\omega_1/\omega_2\right)^2\right]},\tag{3b}$$

$$L_{L} = \frac{NZ_{t} \left[1 - (\omega_{1}/\omega_{2})^{2} \right]}{\omega_{1} \left[\phi_{1} - (\omega_{1}/\omega_{2}) \phi_{2} \right]},$$
(3c)

$$C_{L} = \frac{N \left[1 - (\omega_{1}/\omega_{2})^{2} \right]}{\omega_{1} Z_{t} \left[\phi_{1} - (\omega_{1}/\omega_{2}) \phi_{2} \right]}.$$
 (3d)

4 Example of Applications

Fig. 2 presents a number of practical applications of CRLH TL dual-band components. In these applications, alternating RH TL and LH lumped network sections are used instead of real distributed CRLH TLs [1] for simplicity.

The same design procedure, which consist in synthesizing line sections with the CRLH parameters given by Eqs. (3), applies in all cases. In the case of the "quarter-wavelength" transformer and stub, the choice of phases $(\phi_1, \phi_2) = (-\pi/2, -3\pi/2)$ was made. In the Willkinson divider, the conventional quarter-wavelength sections where replaced by CRLH sections with $(\phi_1, \phi_2) = (+\pi/2, -\pi/2)$. The branch line couplers uses also $(\phi_1, \phi_2) = (-\pi/2, -3\pi/2)$. Finally, the quadrature subharmonic mixer includes seven CRLH components (two Willkinson dividers, one $\pm 45^{\circ}$ phase advance/delay line, and two subharmonically pumped mixers containing each two dual-passband dual-stopband shunt terminations).

5 Conclusion

A novel technique for designing dual-band components with CRLH TLs has been described and illustrated by several practical examples. The CRLH TL allows arbitrary dual-band operation as a benefit of its four degrees of freedom L_R, C_R, L_L, C_L , whereas, as seen in Fig. 1, a PRH TL exhibits the second band of a desired phase at a fixed and uncontrollable frequency. The CRLH TL dual-band principle is completely general and can be applied to any microwave component or circuit.

References

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Figure 2: Example of CRLH TL dual-band component applications. (a) Quarterwavelength transformer at $(f_1, f_2) = (0.88, 1.67)$ GHz. (b) Corresponding quarterwavelength open stub. (c) Willkinson power divider at $(f_1, f_2) = (1.0, 3.1)$ GHz. (d) Branch-line coupler at $(f_1, f_2) = (0.93, 1.78)$ GHz [4]. (e) Quadrature subharmonic mixer at $(f_{RF,1}, f_{LO,1}) = (1.0, 0.51)$ GHz and $(f_{RF,2}, f_{LO,2}) = (3.06, 1.56)$ GHz.