# Super-Compact Composite Right-/Left-Handed Transmission Line with Vertically Stacked Left-Handed Unit Cells

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## ABSTRACT

Over the last decade, left-handed (LH) meta-materials, characterized by anti-parallel phase and group velocities, have drawn considerable interest in the microwave community due to their potential for developing new types of devices and components. In such a movement of this research, we have proposed a super-compact multi-layered (ML) composite-right/left-handed (CRLH) transmission line (TL). This TL consists of U-shaped parallel plate LH unit cells connected to a ground enclosure by meander lines. In this architecture, the coupling between stacked U-shapes provides the LH series capacitance $C_L$, and the meander line provides the LH shunt inductance $L_L$, while the right-handed (RH) series inductance $L_R$ and the RH shunt capacitance $C_R$ are generated by the metallic connections in the direction of propagation and by the voltage gradient from the TL to the ground enclosure, respectively. In contrast to previously reported planar CRLH TLs, the multi-layered TL has its direction of propagation along the *vertical* direction, perpendicular to the plane of the substrates. Therefore, the large electrical length can be achieved over an extremely short TL length and small transverse footprint. In addition, it has a specific feature in the transmission parameter $S_{21}$ that the *transmission zero* can be created just above the LH passband, which characteristic is quite useful for the design of narrow passband response. However, this response could not be explained by the conventional CRLH equivalent circuit model.

In this paper, the effect of a capacitive coupling $C_P$ between the meander line and the U-shaped metallizations is newly introduced to the conventional equivalent circuit, and a new circuit model is developed for the ML CRLH TLs. When the $C_P$ is small enough, the behavior of the circuit is almost the same as the conventional CRLH TL. However, by applying the larger $C_P$, the shunt components of the TL makes a short circuit and consequently the transmission zero is created above the LH passband. In the demonstration with the circuit parameters, $C_L$=10.0pF, $L_{L1}$=6.5nH, $L_{L2}$=3.5nH, $C_R$=0.5pF, $L_R$=1.8nH, and $C_P$=5.5pF, the LH passband is obtained at the frequency range from 0.25GHz to 0.6GHz and around 1.05GHz. Also, the transmission zero can be seen around 1.0GHz.

This newly estimated result shows good agreement with the experimental demonstration and the full-wave FEM and FDTD simulations.

## ACKOWLEDGEMENTS

This research is financially supported by the Kansai University Special Research Fund 2005, "Development of multi-layered left-handed meta-materials for microwave and millimeter-wave applications".

## I. INTRODUCTION

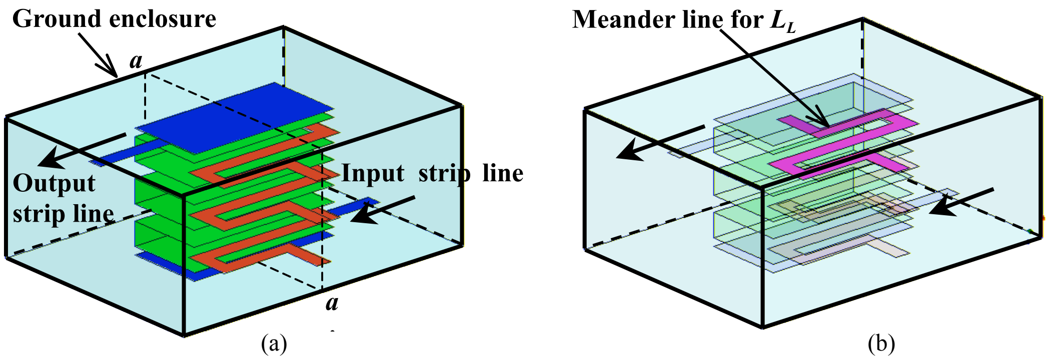
A composite right/left-handed (CRLH) transmission line (TL) theory, developed by Caloz *et al.* and Eleftheriades *et al.* [1-2], has been successfully applied to various kinds of LH structures, and high-performance applications have been produced in the fields of microwave couplers, resonators, antennas and dual-band components. Thus, this TL theory is quite powerful and useful for planar LH structures based on microstrip lines or coupled mushrooms. However, the conventional theory is still insufficient to describe the whole mechanism of newly proposed multi-layered (ML) CRLH TLs [3-5]. A LH unit cell of the ML CRLH TL is composed of U-shaped parallel plates and a meander line sandwiched by those plates. Therefore, a strong coupling between them is occurred just inside the U-shape. In this paper, the effect of this coupling is newly introduced to the conventional equivalent circuit, and a new circuit model is developed for the ML CRLH TLs. The basic behavior of this circuit is studied and the results are compared with the FEM and FDTD full-wave simulations and experiments.

## II. MULTI-LAYERED CRLH TRANSMISSION LINE

### A. Architecture and Equivalent circuit

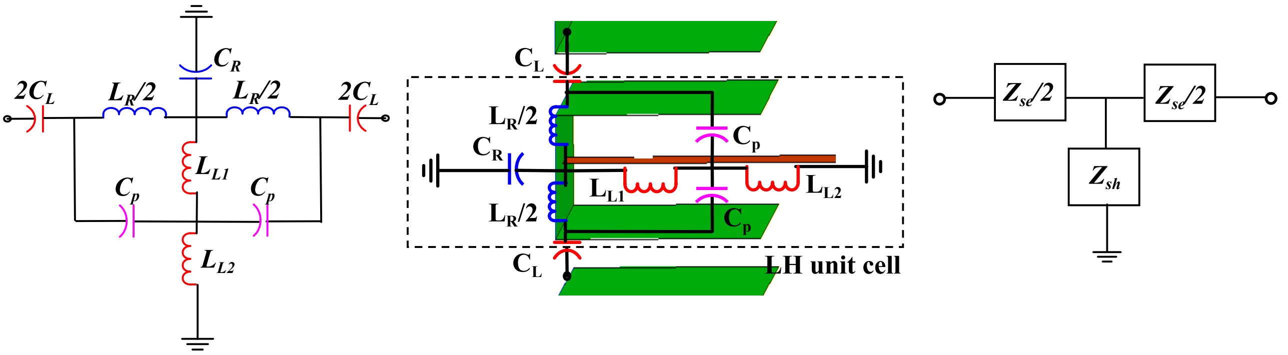
Fig.1 shows geometry of the ML CRLH TL. The TL is composed of periodically stacked LH unit cells, each of which is constituted by a pair of U-shaped parallel plates (shown by "green" color in the figure) connected to a ground enclosure by a meander line ("red"). Another two plates ("blue"), placed at the top or the bottom of the stacked U-shapes, couple to the LH body and give an input and an output ports of this TL.

An equivalent circuit model corresponding to this architecture is shown in Fig. 2 (a) and (b). In this architecture, the coupling between stacked U-shapes provides the LH series capacitance $C_L$, and the meander line provides the LH shunt inductance $L_L$. On the other hand, the right-handed (RH) series inductance $L_R$ and the RH shunt capacitance $C_R$ are



(a) (b)

Fig. 1 Geometry of a ML CRLH TL with three LH unit cells.



(a) (b) (c)

Fig. 2 A new equivalent circuit model for a ML CRLH TL.
(a) Equivalent circuit of a unit cell. (b) Arrangement of circuit elements on a U-shaped geometry, observed at *a-a*' section in Fig.1 (a).
(c) T-type unit cell in a ladder network.

generated by the metallic connections in the direction of propagation and by the voltage gradient from the TL to the ground enclosure, respectively.

In contrast to previously reported CRLH TLs based on a planar structure, it should be noted that the ML structure has a strong coupling between the meander line and the parallel plates occurred just inside the U-shape. To express this effect, the LH inductance $L_L$ is divided into two elements, $L_{L1}$ and $L_{L2}$, and a new capacitance $C_P$ is introduced as illustrated in Fig.2 (b).

T-network expression shown in Fig.2 (c) is useful to evaluate the behavior of the CRLH TLs in respects of a series and a shunt impdedances $Z_{se}$ and $Z_{sh}$. This expression can be derived by applying $Y-\nabla$ and $\nabla-Y$ transformations several times to the original equivalent circuit shown in Fig.2 (a).

### B. Equivalent Circuit Analysis

As a demonstration of the new equivalent circuit, the magnitude of the transmission coefficient $|S_{21}|$ and the reflection coefficient $|S_{11}|$ are calculated for a ML CRLH TL with three LH unit cells, assuming the circuit parameters as $C_L$=10.0pF, $L_{L1}$=6.5nH, $L_{L2}$=3.5nH, $C_R$=0.5pF, $L_R$=1.8nH, and $C_P$=5.5pF. Also, the series impedance $Z_{se}$ and the shunt impedance $Z_{sh}$ are evaluated and these results are shown in Fig.3.

Fig.3 (a) clearly indicates that the TL has a passband from 0.25GHz to 0.6GHz and around 1.05GHz. This response can be explained in respect of $Z_{se}$ and $Z_{sh}$, based on the conventional CRLH TL theory [1-2]. As the imaginary part of the series impedance $Im(Z_{se})$ is capacitive and the imaginary part of the shunt impedance $Im(Z_{sh})$ is inductive at these frequency range, the TL can work as a LH TL. While at the frequency range from 0.6GHz to 1.0GHz or above 1.15GHz, since both the $Im(Z_{se})$ and $Im(Z_{sh})$ become capacitive or inductive simultaneously, a band gap is yielded and the wave propagation is prohibited.

Another unique feature is that it has a transmissin zero around 1GHz. This is because $Im(Z_{sh})$ becomes zero when it changes from capacitive to inductive, leading to make a short circuit between the vertical metallic connections and the ground enclosure. This frequency is quite sensitive to the parameters $L_{L1}$ and $C_P$, and it can be shifted to the lower frequency by setting larger values for them. In the real ML CRLH structure, it can be done by placing the U-shaped parallel plates closer to each other or by giving a higher inductance to $L_L$ using a dense meander line or a large-scale spiral coil.

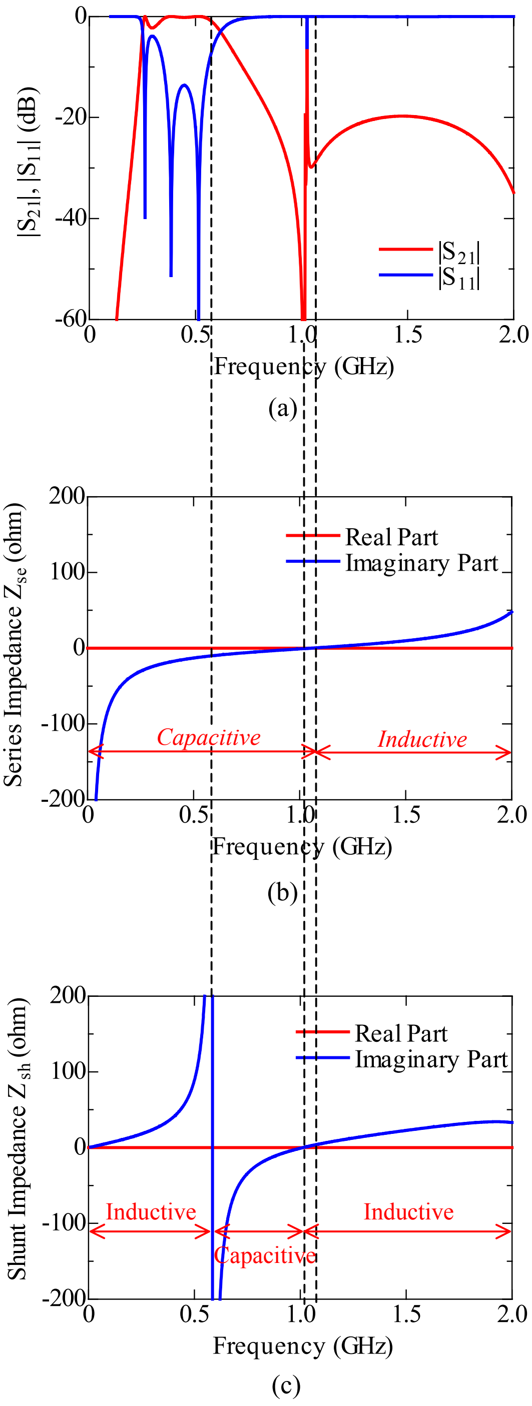


Fig. 3 Frequency response of a ML CRLH TL with three LH unit cells, calculated by an equivalnet circuit model shown in Fig.2. Parameters are $C_L$=10.0pF, $L_{L1}$=6.5nH, $L_{L2}$=3.5nH, $C_R$=0.5pF, $L_R$=1.8nH, and $C_P$=5.5pF.

(a) Scattering characteristics $|S_{21}|$ and $|S_{21}|$.
(b) Reactance of a series impedance $Z_{se}$.
(c) Reactance of a shunt impedance $Z_{se}$.

### C. Experiment

To confirm the theoretical predictions, experiments have been carried out by building a hand-made prototype [5] as shown in Fig.4. This TL consists of three LH unit cells, and the structure is the same as illustrated in Fig.1. The measured scattering parameters of this TL are presented in Fig.5 in comparison with the FEM (Ansoft HFSS 9) and the FDTD (in-house code) results. Though the experimental result has some errors due to misalignment and air-gaps included in the assembling process of different twelve layers, these results show good agreement with those of FEM and FDTD as a whole. Fig.6 illustrates the time variation of the 0.4GHz electric field distribution in the TL observed at the cross section *a-a*', calculated by FDTD. A backward wave propagation can be seen clearly in this structure.

## III. CONCLUSION

Taking a strong coupling between the parallel plates and the inductive line of each LH unit cell into consideration, a new equivalent circuit specialized for the ML CRLH TL was developed and demonstrated in this paper. This equivalent circuit has a potential to predict transmission zeros and pulse-like response observed just above the LH passband. The estimated result by the circuit analysis showed good agreement with the experimental results and the FEM and FDTD simulation results.

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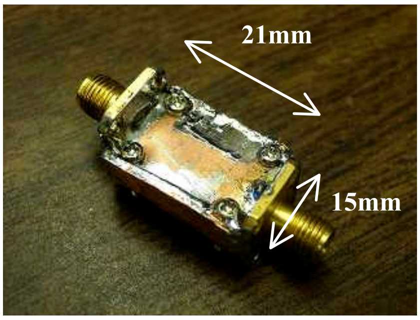
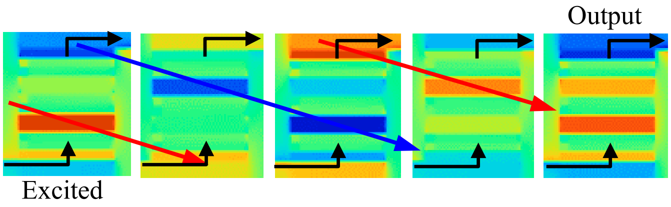


Fig. 4. Prototype of a ML CRLH TL [5].



(a) (b) (c) (d) (e)

Fig. 6. Time variation of the electric field distribution in the ML CRLH TL, observed at *a-a'* cross section in Fig. 1 (a). Calculation is carried out at 0.4 GHz by FDTD. (a) $t = 0$, (b) $t = 1/4T$, (c) $t = 1/2T$, (d) $t = 3/4T$, (e) $t = T$ (period $T = 2.5 \times 10^{-9}$ sec).

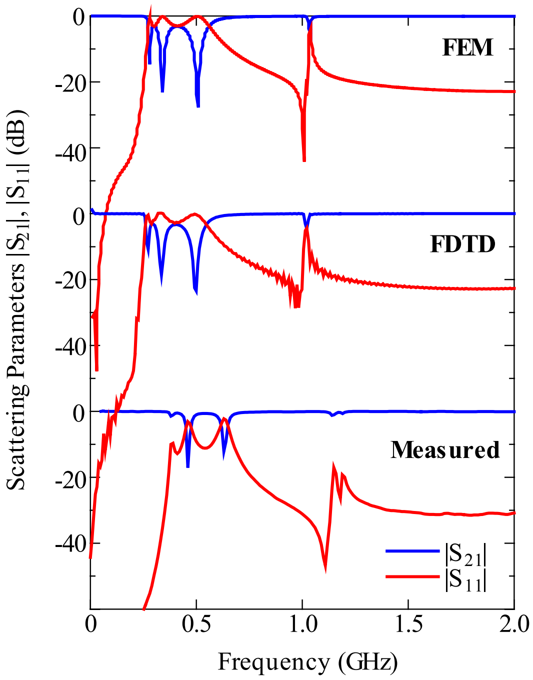


Fig. 5 Scattering characteristics of the ML CRLH TL shown in Fig. 4.