THE ORTHOGONAL COMPONENTS METHOD AND ITS APPLICATION FOR THE RESEARCH OF PSK SIGNALS PASSING THROUGH THE SELECTIVE FILTER

I.D.Zolotarev[1], Ya.E.Miller[2], T.O. Pozharsky[3]
1: Omsk State University (OMGU), Prospect Mira 55a Omsk 644077 Russian Federation
e-mail: izolotarev@bear.ru
2: Academy MBF, Ulitsa Unnatov 18 Moscow 127083 Russian Federation
e-mail: YakovMiller@ambf.ru
3: Omsk State University (OMGU), Prospect Mira 55a Omsk 644077 Russian Federation
e-mail: a_pozharsky@mail.ru

At present the communications electronics systems with the application of a fine phase structure of the radio signal are finding an increased application. The tendency of raising the data transmission rate leads to a high radar operation dynamism of the above-mentioned systems when the data are acquired and processed in presence of the transients. The danger of the given operation mode for the phase systems is that the transient can easily destruct the phase informative attribute.

The method of orthogonal components (MOC) [1-3] ensures a high level of visualization when making research of the dynamic operation mode of the phase systems. But the frequency characteristics of the system in all the indicated works are approximated by piecewise polygonal lines; that is why the solutions found in case of such a coarse approximation turn out to be unfit for getting reliable recommendations on building modern phase systems. The beauty of the method of orthogonal components for making research of the systems with a radio signal phase acting as an informative parameter determined the expediency of the given method development in the aspects of using the actual characteristics of the signal and the system but not the approximated ones. This is achieved in the given work by making use of MOC combined with the method of the fast inverse Laplace transform (FILT) [4-6]. First, let's suppose that the system lacks the initial energy reservoirs. Then if $x(t)$ is an input signal and $y(t)$ is the system response, the image equation is going to take the following form

$$Y(s) = X(s)k(s), \tag{1}$$

where $k(s)$ - the system function, $X(s) = L\{x(t)\}$, $Y(s) = L\{y(t)\}$, L - the direct Laplace transform operator.

Appealing to the equation (1), one will notice that multipliers $X(s)$ and $k(s)$ enter it on the equal terms which allowed to prove FILT for the fractional rational functions in [5]. According to the given method, the output signal image can be written in the following form

$$\dot{Y}(s) = \dot{Y}_{forced}(s) + \dot{Y}_{free}(s). \tag{2}$$

In the equation (2) $\dot{Y}_{forced}(s)$ and $\dot{Y}_{free}(s)$ are the images of the forced and free components of the transients. The form of the equation (2) in the space of the originals will be the following

$$\dot{y}(t) = \dot{y}_{forced}(t) + \dot{y}_{free}(t) \tag{3}$$

Let's find the real-valued response of the system $y(t)$ by performing a symbolic operation of taking the real part of the complex signal $\dot{y}(t)$, i.e. $y(t) = \mathrm{Re}\{\dot{y}(t)\}$ or $y(t) = \mathrm{Im}\{j\dot{y}(t)\}$.

Let's transform the expression (3) to the form:

$$\dot{y}(t) = \dot{y}_{stat}(t)[\dot{y}_{forced}(t)/\dot{y}_{stat}(t) + \dot{y}_{free}(t)/\dot{y}_{stat}(t)] = \dot{y}_{stat}(t)\dot{N}(t). \tag{4}$$

Here the complex function $\dot{N}(t) = N(t)e^{j\delta(t)}$ determined by the square brackets in the expression (4), characterizes the transients and that is $N(t)$ which determines the envelope curve behavior at the transient presence and $\delta(t) = \arg\dot{N}(t)$ - the phase behavior.

Let's represent the $\dot{N}(t)$ as the sum of the orthogonal components

$$\dot{N}(t) = P(t) + jR(t),\quad P(t) = \mathrm{Re}\{\dot{N}(t)\},\quad R(t) = \mathrm{Im}\{\dot{N}(t)\}. \tag{5}$$

Proceeding from (5), we will have the following in the image space

$$\dot{N}(s) = P(s) + jR(s).$$

Let's apply the direct Laplace transform to the right and left parts of the formulae (5)

$$P(s) = L\{P(t)\} = L\{\mathrm{Re}\,\dot{N}(t)\} = \widetilde{\mathrm{Re}}\{L[\dot{N}(t)]\} = \widetilde{\mathrm{Re}}\,\dot{N}(s),$$

$$R(s) = L\{R(t)\} = L\{\mathrm{Im}\,\dot{N}(t)\} = \widetilde{\mathrm{Im}}\{L[\dot{N}(t)]\} = \widetilde{\mathrm{Im}}\,\dot{N}(s). \tag{6}$$

Here the commutative property of the symbolic operations Re and Im and the Laplace transform is taken into account. The use of the symbols $\widetilde{\mathrm{Re}}$ and $\widetilde{\mathrm{Im}}$ underlines that the imaginary and real parts are taken conditionally (i.e., assuming that s is the real number). The following proceeds immediately from the above-said

$$P(s) = [\dot{N}(s) + \dot{N}^*(s)]/2\,,\ R(s) = [\dot{N}(s) - \dot{N}^*(s)]/2j\,,$$

where the symbol (*) determines a complex conjugate function.

Let's represent the forced and free components of the transients in the form

$$\dot{y}_{forced}(t) = Y_{m.stat}e^{j(\omega_c t+\beta)}1(t) = \dot{y}_{stat}(t)1(t)\,,\ \dot{y}_{free}(t) = \dot{Y}_{m.free}(t)e^{(-\alpha+j\omega_0)t}\,,\ \dot{Y}_{m.free}(t) = Y_{m.free}(t)e^{j\gamma(t)}\,.$$

Then proceeding from (4)

$$\dot{N}(t) = \dot{y}(t)\dot{y}_{stat}^{-1}(t) = \dot{y}(t)[Y_{m.stat}e^{j(\omega_c t+\beta)}]^{-1}\,. \tag{7}$$

In accordance with (7), proceeding from the shift theorem

$$\dot{N}(s) = \dot{Y}(s + j\omega_c)Y_{m.stat}^{-1}e^{-j\beta}\,.$$

According to (2), we will have the following for the function image $\dot{N}(t)$

$$\dot{N}(s) = [\dot{Y}_{forced}(s + j\omega_c) + \dot{Y}_{free}(s + j\omega_c)]Y_{m.stat}^{-1}e^{-j\beta}\,. \tag{8}$$

Let's demonstrate the application of the orthogonal components method by the help of the example of the PSK sequence passing through the telecommunication system selective path (SSP). Let's assume that SSP is realized by a series connection of the n identical unidirectional oscillating sections for which the system function has the form $k(s) = K_0[(s+a)/(s^2 + 2\alpha s + \omega_r^2)]^n$, where α - the damping constant of each bandpass filter which is equal to a half of its bandpass, ω_r -a resonance frequency.

In general case, the signal with a two-position PSK may be written down in the following form

$$x(t) = A_0 \sum_{\mu=0}^{M} \sin(\omega_c t + \psi + \varphi_\mu)\{1(t - \mu\tau) - 1[t - (\mu+1)\tau]\}\,, \tag{9}$$

where τ - the sequence element width, $(M+1)\tau$ - the information sequence duration, $\varphi_\mu = q_\mu \pi$, q_μ assumes the value of 0 or 1 depending on the specific form of the selected code sequence.

There is a good reason to start the research of the phase shift keying with the steady-state mode of the SSP operation making a supposition that there was a continuous sequence of the zeros (or ones) before the moment of switching. On the other hand, the transient proves to be the most evident at switching the phase on the adjacent elements. Let's choose a section of the PSK sequence with four switchings on the adjacent elements. The initial energy reservoirs have been taken into account by the moment of the phase shift keying by the free component occurring after switching off the preceding element of the sequence. According to the calculation, such a quantity of the frequency switchings is sufficient for predicting the selective path response to the PSK sequence of the radio signal within any of its intervals. Let's accept the following conformity of the phases with the number of the μ element for the sequence fragment.

μ	0	1	2	3	4	5	6
φ_μ	0	0	π	0	π	0	0

We model the phase-shift keying for the next element of the sequence by switching on the radio step of the polarity that is opposite to the previous discrete but of a double amplitude. Then the input signal (9) will be rewritten in the following form

$$x(t) = x_{stat}(t) + \sum_{\mu=2}^{5} x_\mu(t)\,,\ t > 0 \tag{10}$$

where $x_{stat}(t) = A_0\sin(\omega_c t + \psi)$, $x_\mu(t) = 2(-1)^{\mu-1}x_{stat}(t)1(t - \mu\tau) = 2(-1)^{\mu-1}A_0\sin[\omega_c(t - \mu\tau) + \psi_\mu]1(t - \mu\tau)$, $\psi_\mu = \psi + \mu\omega_c\tau$.

Let's change the variable $t_\mu = t - \mu\tau$ that will permit us to get the identical expressions for each element of the sequence $x_\mu(t_\mu) = 2(-1)^{\mu-1}A_0\sin(\omega_c t_\mu + \psi_\mu)1(t_\mu)$.

According to expression (1), the image of the SSP response to the exciting signal $x_\mu(t)$ will be determined by the formula

$$Y_{\mu}(s)=2(-1)^{\mu-1}A_0\frac{s\sin\psi_{\mu}+\omega_c\cos\psi_{\mu}}{s^2+\omega_c^2}K_0\left[\frac{s+a}{s^2+2\alpha s+\omega_r^2}\right]^n, \quad (11)$$

for which we have the complex-conjugate "forced" poles $s_{1,2}=\pm j\omega_c$ and the "free" ones $s_{3,4}=-\alpha\pm j\omega_0$, where $\omega_0=\sqrt{\omega_r^2-\alpha^2}$ - the natural frequency of the bandpass filter SSP.

Proceeding from (11) and according to the method FILT [5], we will get the expressions for the imaging function of the forced and free components of the transient

$$\dot{Y}_{\mu.\,forced}(s)=2(-1)^{\mu-1}\frac{A_0\dot{k}(j\omega_c)e^{j\psi_{\mu}}}{s-j\omega_c}e^{-s\mu\tau},\ \dot{Y}_{\mu.\,free}(s)=4(-1)^{\mu-1}\sum_{l=0}^{n-1}\frac{(n-l-1)!\dot{C}_{l\mu}}{(s+\alpha-j\omega_0)^{n-l}}e^{-s\mu\tau}, \quad (12)$$

$$\dot{C}_{l\mu}=A_0K_0\sum_{h=0}^{l}\frac{(-1)^h C_{n+h-1}^h}{(n-l-1)!(l-h)!}\frac{1}{(2j\omega_0)^{n+h}}\frac{d^{l-h}}{ds^{l-h}}\left[\frac{s\sin\psi_{\mu}+\omega_c\cos\psi_{\mu}}{s^2+\omega_c^2}(s+a)^n\right]_{s=-\alpha+j\omega_0},$$

where multiplier $e^{-s\mu\tau}$ is introduced for the transition to the initial time reference system t instead of t_{μ}.

Then proceeding from (8), we have the following image of the transient function (TF)

$$\dot{N}_{\mu}(s)=[\dot{Y}_{\mu.\,forced}(s+j\omega_c)+\dot{Y}_{\mu.\,free}(s+j\omega_c)][A_0\dot{k}(j\omega_c)e^{j\psi_{\mu}}]^{-1}.$$

Or after substitutions (12), assuming $\Omega=\omega_0-\omega_c$, we will get

$$\dot{N}_{\mu}(s)=2(-1)^{\mu-1}\left[\frac{1}{s}+\frac{2}{A_0\dot{k}(j\omega_c)e^{j\psi_{\mu}}}\sum_{l=0}^{n-1}\frac{(n-l-1)!\dot{C}_{l\mu}}{[s+\alpha-j\Omega]^{n-l}}\right]e^{-s\mu\tau}, \quad (13)$$

Then taking into account (10), the image for the final TF can be found in a compact form

$$\dot{N}(s)=\frac{1}{s}+\sum_{\mu=2}^{5}\dot{N}_{\mu}(t).$$

Taking into account the expressions (6), we will find the image for the orthogonal components $P(s)=\widetilde{\mathrm{Re}}\{\dot{N}(s)\}$, $R(s)=\widetilde{\mathrm{Im}}\{\dot{N}(s)\}$.

For the transient function, the transition from the image space into the originals space will result in

$$\dot{N}(t)=1(t)+\sum_{\mu=2}^{5}2(-1)^{\mu-1}\left[1(t-\mu\tau)+\frac{2}{A_0\dot{k}(j\omega_c)e^{j\psi_{\mu}}}\sum_{l=0}^{n-1}\dot{C}_{l\mu}(t-\mu\tau)^{n-l-1}e^{[-\alpha+j\Omega](t-\mu\tau)}1(t-\mu\tau)\right], \quad (14)$$

from which the orthogonal components can be found as $P(t)=\mathrm{Re}\,\dot{N}(t)$, $R(t)=\mathrm{Im}\,\dot{N}(t)$.

As it proceeds from (13) and (14), TF describes the complex envelope curve of the transient.

As $P(t)$ is given by the real part of the function $\dot{N}(t)$ and $R(t)$ - by its imaginary part, then proceeding from the expression (14), $R(t)$ characterizing the phase deviation at the transient is determined by the free component of the transient. Hence, when using the method of the orthogonal components for the SSP synthesis according to the criteria of the minimal phase overshoot on the sequence element, attention should be paid to the application of the minimax criteria for the quadrature component $R(t)$ [7].

Fig.1 shows the calculated charts in nondimensional time αt for TF and the orthogonal components at $n=3$ for the cases of the tuned bandpass filters and their symmetrical detuning. The assumed width of the elements of the sequence is $\alpha\tau=4$ which corresponds to $\tau=4/\pi\Delta F$ in the real time, ΔF - the filters bandwidth in Hertz.

The charts show that in case with the tuned bandpass filters we have rather reliable phase behavior also when acquiring the phase information in the dynamic mode. However, detuning results in smoothing the phase behavior and thus impairs the phase discriminator operation conditions. Hence, we can find the allowable value of detuning for the specific realization of the SSP bandpass filters. For the SSP containing the identical bandpass filters, we have a delay of the filter response relatively the moment of the phase shift keying of the input signal (at $n=3$ this value constitutes approximately $\alpha\tau_{delay}=1.5$). This should be taken into account when realizing the correlation devices for the PSK sequences processing.

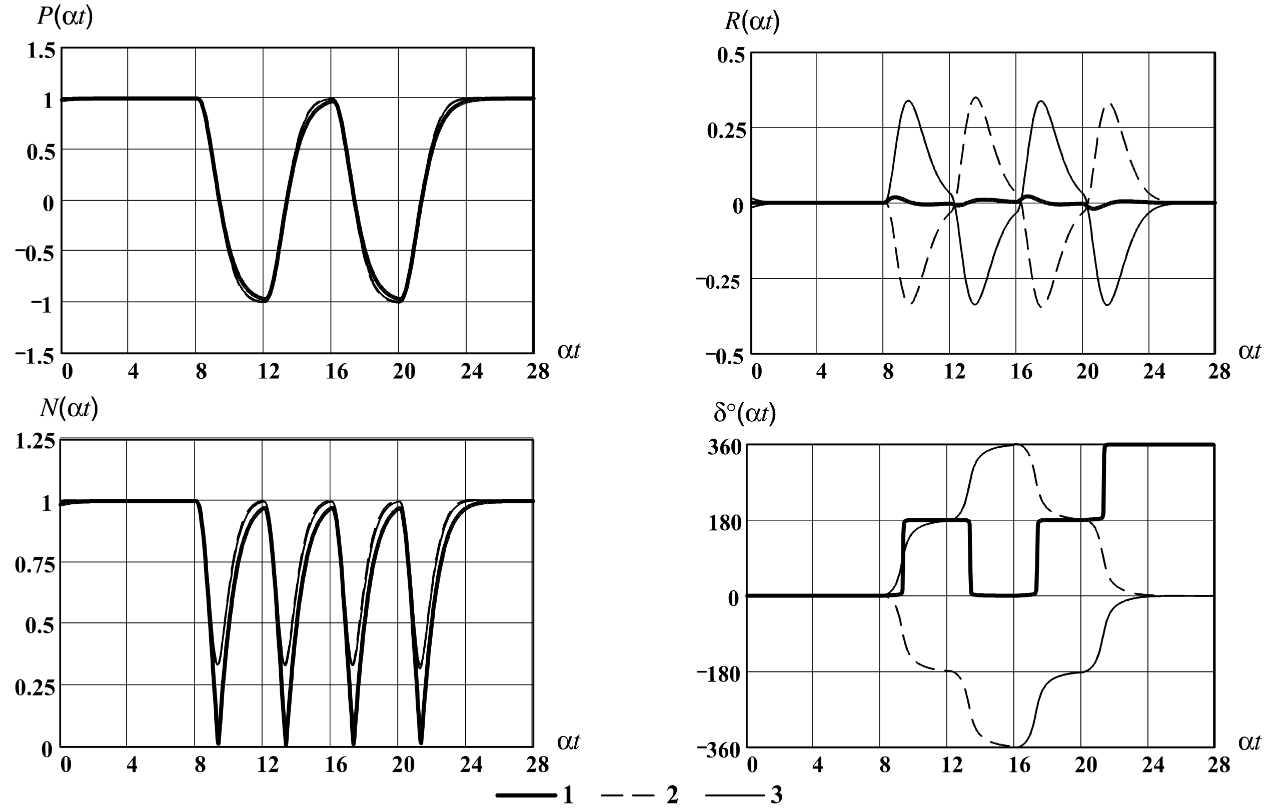

Fig. 1. The charts of behavior of the orthogonal components and TF of the SSP response to the PSK signal at an identical implementation of the unidirectional bandpass filters of the path for the following parameters of the bandpass filters and signal: the Q-factor of the bandpass filters $Q = 25$, $n = 3$, the width of the element of sequence $\alpha\tau = 4$, $\psi = 0$, $\xi = (\omega_r - \omega_c)/\alpha$, 1 — $\xi = 0$; 2 — $\xi = 0.5$; 3 — $\xi = -0.5$

There has been developed the orthogonal components concept concerning the telecommunication phase systems using the radio signal fine microstructure. The given way is connected with the transient function introduced in the report. The solution is found on the base of combining the orthogonal components method with the one of the fast Laplace transform. The methodology that is under discussion in the given work may be used for a wide range of the systems using the radio signal phase as an informative attribute. The given approach makes it possible to solve an important problem of the unambiguous determination of the envelope curve and the radio signal phase independently from the level of its broadbandness (the Amplitude-phase-frequency problem, the APF problem in radio electronics).

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