

DOA ESTIMATION BASED ON THE RELATION BETWEEN DOAS AND NON-DIAGONAL CORRELATION MATRIX ELEMENTS

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Abstract

In this paper, we propose a novel DOA estimation algorithm based on the fact that the correlation between the received signals at different antenna elements can be written as a function of DOAs. The computational cost of the proposed method is much smaller than that by MUSIC method. Moreover, the present method does not require the space averaging technique even though the case of correlated incident waves although MUSIC requires it which reduces the practical number of antenna elements. The estimation accuracy and the computational cost are evaluated through computer simulation in comparison of those by MUSIC and Beamformer methods.

1. Introduction

Mobile communication technology has remarkably developed in the last decade. It is important to accurately detect radio wave propagation environment around the base station. Therefore, we should estimate the Direction of Arrival (DOA) of highly coherent waves like desired wave and the delay waves. The high resolution DOA estimation algorithm using array antenna is attracted attention to achieve these demands.

Many DOA estimation algorithms have been already proposed. Beamformer method which scans the main beam of array antenna is the most fundamental technique. The other techniques based on eigenvalue decomposition of array input correlation matrix are the Min-Norm method, MUSIC (Multiple Signal Classification) [1], and ESPRIT (Estimation of Signal Parameters via Rotational Invariance Techniques) [2], are attracted attention in the mobile communication system. However, these algorithms employ eigenvalue decomposition and angle search that require very large computational cost. Moreover, in the case of correlated incident waves, they require space averaging technique which reduces the practical number of antenna elements.

In this paper, we propose a simple DOA estimation algorithm based on the fact that the correlation between the received signals at different antenna elements can be written as a function of DOAs. Although the proposed method needs more array elements in comparison with MUSIC method, the computational cost of the proposed method is much smaller than that by MUSIC method. Moreover, the present method does not require the space averaging technique even though the case of correlated incident waves. The estimation accuracy and the computational cost are evaluated through computer simulation in comparison of those by MUSIC and Beamformer methods.

2. Preliminaries

In this section, we formulate signals received at array antenna and the correlation between received signals at different antenna elements.

As Fig.1, assume that L incident waves $s_i (i = 1, 2, \dots, L)$ arrive at K -elements linear array antenna of half-wave length. The received signal x_k of the k -th antenna element can be written as

$$x_k(n) = \sum_{i=1}^L s_i(n) e^{-j\pi(k-1)\sin\theta_i} + u_k(n), \quad k = 1, 2, \dots, K \quad (1)$$

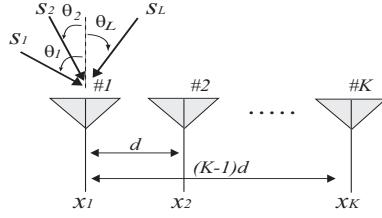


Figure 1: composed array antenna

where $d = \lambda/2$ denotes the element interval. The angle θ_i is DOA of the i -th incident wave which changes within $(-90^\circ, 90^\circ)$. Let denote u_k the receiver noise of the k -th element where its average is 0.

Using (1), the correlation R_{1k} between the received signal x_1 at the first element and x_k at the k -th element can be written as

$$\begin{aligned} R_{1k} &= E[x_1(n)x_k^*(n)] = E\left[\left(\sum_{i=1}^L s_i(n) + u_1(n)\right)\left(\sum_{i=1}^L s_i(n)e^{-j\pi(k-1)\sin\theta_i} + u_k(n)\right)^*\right] \\ &= \sum_{i=1}^L C_i \Theta_i^{k-1}, \quad k = 2, 3, \dots, K. \end{aligned} \quad (2)$$

where $N, E[\cdot]$ denote the number of snapshots and the ensemble average, respectively. Besides, C_i and Θ_i are given by

$$C_i = \frac{1}{N} \sum_{n=1}^N \left\{ s_i^*(n) \sum_{i'=1}^L s_{i'}(n) \right\}, \quad (3)$$

$$\Theta_i = e^{j\pi \sin\theta_i}. \quad (4)$$

3. Proposed DOA estimation method

In this section, we show that the DOA estimation problem can be replaced by the problem of solving nonlinear simultaneous equation.

In (2), the correlations $\{R_{1k}\}_{k=1}^K$ and the number of snapshots N are given. On the contrary, the coefficients $\{C_i\}_{i=1}^L$ and the angles $\{\Theta_i\}_{i=1}^L$ (or the unknown signals s_i and the DOAs θ_i) are unknown. Therefore, we need the $2L$ number of independent equations to obtain their solutions.

We focus on that k in eq.(2) corresponds to the k -th antenna element. When the number of antenna elements is $(2L+1)$, we can prepare the $2L$ number of equations by changing $k = 2, 3, \dots, 2L+1$. The $2L$ number of DOA θ_i can be solved by considering to be simultaneous equations. We describe the DOA estimation formulae in the case of one wave and two waves in the following section. The proposed algorithm can reduce the computational cost much lower than that of MUSIC which need eigendecomposition and Beamformer which needs the angle search. But the proposed algorithm ($2L+1$ elements) needs more array elements in comparison with MUSIC algorithm ($L+1$ elements) in case of L incident waves of arrival.

3.1 The case of one wave

Let us describe the case of one incident wave (DOA θ_1) and three array elements. Equation (2) can be reduced into

$$R_{1k} = C_1 \Theta_1^{k-1}, \quad k = 2, 3. \quad (5)$$

Considering (5) to be simultaneous equation, Θ_1 and θ_1 are obtained as

$$\theta_1 = \sin^{-1} \left(-\frac{j \ln \Theta_1}{\pi} \right), \quad \Theta_1 = \frac{R_{13}}{R_{12}} \quad (6)$$

3.2 The case of two waves

In the same way, let us describe the case of two wave (DOA θ_i) and five array elements. Equation (2) can be reduced into

$$R_{1k} = \sum_{i=1}^2 C_i \Theta_i^{k-1}, \quad k = 2, 3, 4, 5 \quad (7)$$

Considering (7) to be simultaneous equation, Θ_1 and Θ_2 are obtained by solving quadratic equation. Based on this fact, DOA θ_1, θ_2 ($\theta_1 < \theta_2$) can be written as

$$\theta_i = \sin^{-1} \left(-\frac{j \ln \Theta_i}{\pi} \right), \quad \Theta_i = \frac{-a_2 + (-1)^i \sqrt{a_2^2 - 4a_1 a_3}}{2a_1}, \quad i = 1, 2 \quad (8)$$

$$a_1 = R_{12}R_{14} - R_{13}^2, \quad a_2 = R_{13}R_{14} - R_{12}R_{15}, \quad a_3 = R_{13}R_{15} - R_{14}^2. \quad (9)$$

Similarly the case of three or four DOAs, the DOA estimation problem can be replaced by the problem of solving L -order nonlinear simultaneous equation of solution derivation. We can algebraically derive DOAs of up to 4 incident waves based on the fact that the polynomials of up to 4-th order can be algebraically solved [3,4]. However, since it is not possible to algebraically solve the case of more than four waves, we can estimate DOAs by deriving a numerical solution by Newton-Rapson method.

4. Simulation Results

In this section, the estimation accuracy and the computational cost of the proposed method are evaluated through the computer simulation in comparison of those by MUSIC and Beamformer methods. Specifications of simulation are shown in Table 1 and 2, respectively. The estimation accuracy is evaluated by RMSE (Root Mean Square Error). The estimated DOAs are evaluated by the average of 100 simulation results because the influence of the noise is not uniform in one estimation.

Table 1: Simulation data (common items)

Array antenna	Half-wave length uniform linear array
Carrier frequency	2.0 GHz
Modulation method	QPSK
Number of snapshots	200

Table 2: Simulation data (individual items)

	Fig.2	Fig.3	Fig.4	Fig.5
Number of elements	3		5	
Number of arrival waves		1		2
DOA	30°	change	0°, 30°	0°, change
Kind of waves	—		Correlated	
SNR	change	10[dB]	change	10[dB]

4.1 The case of one incident wave

Let us describe the case of one wave and three array elements in Figs.2–3. Although the accuracy doesn't reach the level by MUSIC and the beamformer methods, the proposed method can estimate DOAs with good accuracy. And the proposed method accuracy reaches the level by MUSIC with two elements.

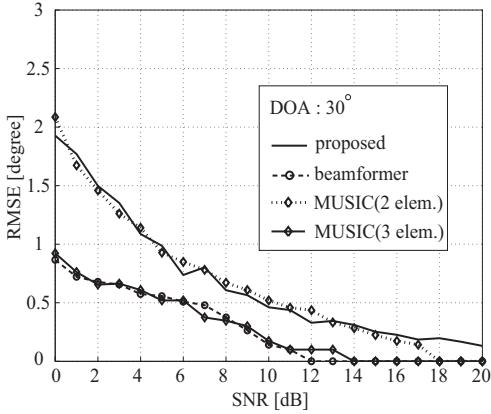


Figure 2: RMSE in DOA of one wave (the DOA = 30°)

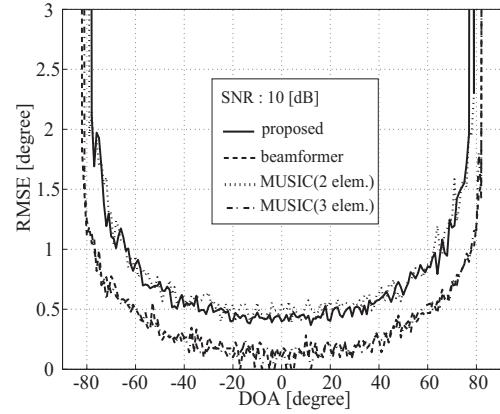


Figure 3: RMSE in DOA of one wave (SNR = 10 dB)

4.2 The case of two correlated waves

Let us describe the case of two complete correlated wave (the delayed 10 symbols) and five linear array elements in Fig.4–5. MUSIC method employs the space averaging technique to estimate DOAs of correlated waves. In Fig.4, The

accuracy of the Beamformer is remarkably worse. Also in this case, the accuracy of the present method doesn't reach the level by MUSIC method but with good accuracy.

In Fig.5, the accuracy by any method becomes worse as two DOAs become closer. Especially, the accuracy of Beamformer is remarkably bad. The accuracy of proposed method doesn't achieve but can follow the level by MUSIC method, in fact the angular difference is almost the same as MUSIC. The proposed method is effective considering the computational cost discussed in the next section.

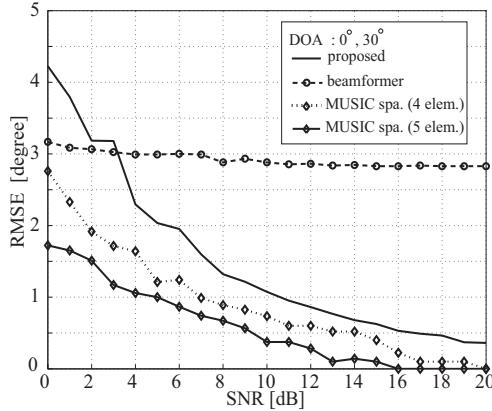


Figure 4: RMSE in DOA of two correlated waves (the DOA = 0° , 30°)

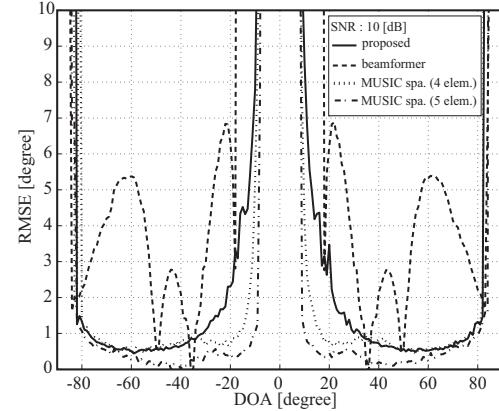


Figure 5: RMSE in DOA of two correlated waves (SNR = 10 dB)

4.3 Comparison of the computational cost

The computational cost of the proposed method is evaluated through the computer simulation in comparison it by MUSIC and Beamformer. Under the same calculation condition, the time required to estimate DOAs of 10000 times is indicated in Table 3. The angular search of MUSIC and Beamformer was done with 1° interval. From Table 3, the computational cost of the proposed method is much smaller than that by MUSIC and Beamformer methods. Although the DOA estimation accuracy of the proposed method is inferior to these methods, the computational cost can be greatly shortened. Therefore the proposed algorithm can be suitable to be used in high-speed mobile telecommunications as DOA estimation method.

Table 3: Comparison with the computational cost

	Beamformer	MUSIC	Proposed method
3 elements, one wave	24.63 (145)	47.90 (281)	0.17 (1)
5 elements, two waves	65.22 (33)	83.59 (42)	1.98 (1)

Note: Unit of computation time is [sec], and (·) indicates ratio

5. Conclusion

In this paper, we proposed a simple DOA estimation algorithm based on the fact that the correlation between the received signals at different antenna elements can be written as a function of DOAs. Although the proposed method needs more array elements in comparison with MUSIC to estimate the same number of DOAs, the computational cost of the proposed method is greatly reduced in comparison with MUSIC and Beamformer methods. Moreover, the proposed method does not require the space averaging technique even though the case of correlated incident waves.

Reference

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