# A RIGOROUS ANALYSIS OF WIDE BEAM SCALAR FEED HORN

Bahattin TÜRETKEN
*TUBITAK-UEKAE EMC Laboratory PO.Box:74 41470 Gebze Kocaeli, TURKEY

Different types wide beam scalar feed horns are commonly used as feeds in reflector antenna systems which are used in acoustics and microwave communications because due to their well-known symmetric pattern and zero or low cross-polarization. To analyze the performance of such feeds, one needs to know accurately their near- and far-field patterns.

## INTRODUCTION

The radiation characteristics of circular waveguides and horns have been the subject of numerous past investigations. The first rigorous analytical solution of the radiation from a semi-infinite, infinitely thin unflanged circular rigid pipe, has been obtained by Levine and Schwinger [1]. Recently, Turetken et al [2] have considered the radiation of the scalar waves in a circular waveguide horn formed by flaring out a circular waveguide. The aim of the present work is to produce an analysis of the case where the aperture's inner surface is treated as impedance material ($\eta_1$) (Figure 1.). The impedance of the aperture's inner surface presents a corrugated structure. In the electromagnetic case, circular waveguide fed, the corrugation depth changes from $\lambda/2$ , where the corrugation act like a conducting surface, to $\lambda/4$ where corrugations present a high impedance [3]. The analysis of the problem yields the radiation pattern of the wide-beam scalar feed horn. To this end, by introducing the Fourier transform for the scattered field and applying the boundary conditions in the transform domain, the problem is reduced into a modified Wiener–Hopf equation. Using the mode matching method in conjunction with the Wiener-Hopf technique treats the radiation of plane harmonic scalar waves from a wide beam scalar feed horn. The solution is exact but formal since infinite series of unknowns and some branch-cut integrals with unknown integrands are involved. Approximation procedures based on rigorous asymptotic are used and the approximate solution to the Wiener-Hopf equations are derived in terms of infinite series of unknowns, which are determined from infinite systems of linear algebraic equations. Numerical solution of these systems is obtained for various values of the parameters of the problem and their effects on the directivity of the scalar feed horn is presented.

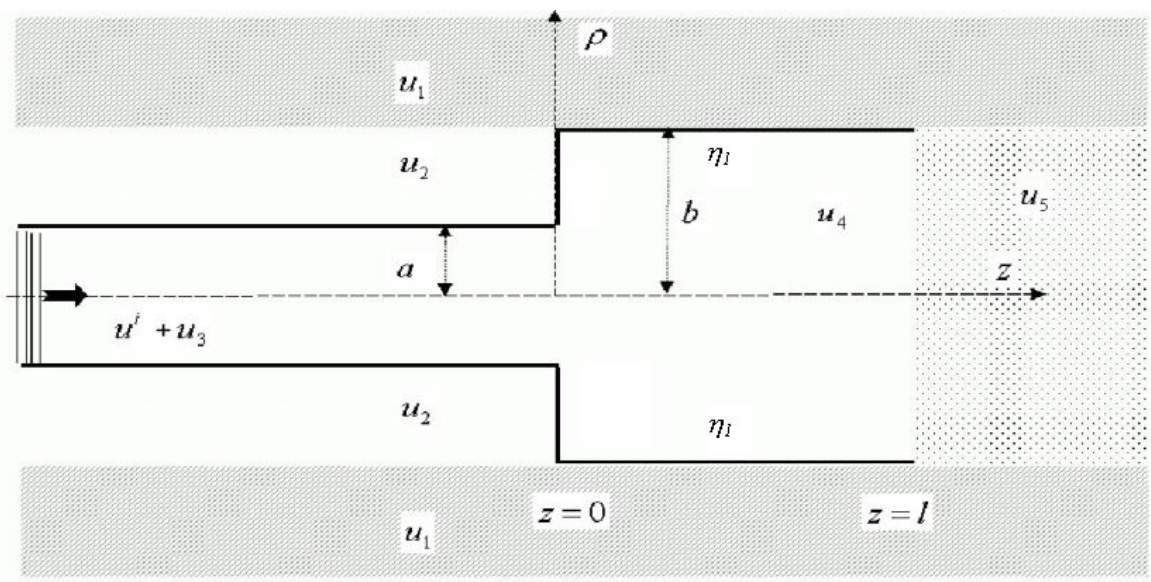


**Figure 1.** Geometry of the problem

## ANALYSIS and RESULTS

For analysis purposes, it is convenient to express the field constituents in the various sub domains explicitly as follows:

$$u^T(\rho,z)=\begin{cases} u_1(\rho,z) & ;\rho>b \quad z\in(-\infty,\infty) \\ u_2(\rho,z) & ;\ \rho\in(a,b),\quad z<0 \\ u_3(\rho,z)+u^i(\rho,z) & ;\ \rho\in(0,a),\quad z<0 \\ u_4(\rho,z) & ;\rho\in(0,b),\quad z\in(0,l) \\ u_5(\rho,z) & ;\rho\in(0,b),\quad z>l \end{cases} \tag{1}$$

Here, $u^i$ is the incident field given by

$$u^i(\rho,z)=e^{ikz} \tag{2}$$

In (2) k is the free space wave number, which is assumed to have small positive imaginary part. $u_j(\rho,z),\ j=1-5$ which satisfy the Helmholtz equation in their corresponding regions, are to be determined with the aid of boundary and continuity conditions related to this problem. By introducing the Fourier transform in z for the scattered field and applying the boundary conditions in the transform domain $\alpha$ , after some mathematical manipulations, the problem is reduced into a scalar modified Wiener-Hopf equation of the third kind valid in the strip $\mathrm{Im}(-k)<\mathrm{Im}(\alpha)<\mathrm{Im}(k)$

$$-\frac{b}{2}F_1(b,\alpha)+\frac{\dot{F}^-(b,\alpha)}{K^2(\alpha)}\ L(\alpha)+\frac{e^{i\alpha l}\dot{F}^+(b,\alpha)}{K^2(\alpha)N(\alpha)}=\frac{i\alpha}{\pi}\sum_{m0}^{\infty}\frac{J_1(Z_m a)}{J_1(Z_m b)}\frac{f_m}{Z_m}\frac{1}{\delta_m^2-\alpha^2}$$
$$+e^{i\alpha l}\frac{b}{2}\sum_{m=0}^{\infty}\frac{J_0(\xi_m b)}{\alpha_m^2-\alpha^2}\left[g_m-i\alpha h_m\right] \tag{3a}$$

where

$$N(\alpha)=\pi i J_1(Kb)H_1^{(1)}(Kb) \tag{3b}$$

$$L(\alpha)=\frac{H_1^{(1)}(Ka)}{\pi H_1^{(1)}(Kb)\left[\left[J_1(Ka)Y_1(Kb)-J_1(Kb)Y_1(Ka)\right]\right]} \tag{3c}$$

$\delta_m$ and $\alpha_m$ are the simple poles $G^-\ (\rho,\alpha)$ and $G^+(\rho,\alpha)$ lying at the lower and upper half planes, $Z_m=K(\delta_m)$, m=0,1,2,... are related to integrable functions satisfying Dini condition . By using the factorization and the decomposition procedures together with the Liouville theorem, the modified Wiener-Hopf equation in (3a) can be reduced to the system of Fredholm integral equations of the second kind. For kl>>1, the coupled system of Fredholm integral equations of the second kind , are susceptible to a treatment by iterations.

$$\dot{F}^-(b,\alpha)=\dot{F}^{-(1)}(b,\alpha)+\dot{F}^{-(2)}(b,\alpha)+\ldots\ldots\ldots \tag{4a}$$

$$\dot{F}^+(b,\alpha)=\dot{F}^{+(1)}(b,\alpha)+\dot{F}^{+(2)}(b,\alpha)+\ldots\ldots\ldots \tag{4b}$$

The formal solution of $\dot{F}^+(b,\alpha)$ ve $\dot{F}^-(b,\alpha)$ can easily be obtained through the classical Wiener-Hopf technique. The total scattered field in the region $y>b$ (Figure 2.) can be obtained by taking the inverse Fourier transform of the expression $F(\rho,\alpha)$ .

$$u_1(\rho,z)=-\frac{1}{2\pi}\int_L\frac{H_0^{(1)}(K\rho)}{K(\alpha)H_1^{(1)}(Kb)}\left[\dot{F}_-\ (b,\alpha)+e^{i\alpha l}\dot{F}_+(b,\alpha)\right]e^{-i\alpha z}d\alpha \tag{5}$$

where $\boldsymbol{L}$ is a straight line parallel to the real axis lying in the strip $\mathrm{Im}(-k)<\mathrm{Im}(\alpha)<\mathrm{Im}(k)$ . Utilizing the asymptotic expansion of $H_0^{(1)}(K\rho)$ as $k\rho\to\infty$

$$H_0^{(1)}(K\rho) = \sqrt{\frac{2}{\pi K\rho}} e^{i(K\rho - \pi/4)} \tag{6}$$

The asymptotic evaluation of the integral in (5) can be performed via the saddle point technique; the diffracted field is obtained as:

$$u_1(\rho, z) = \frac{i}{\pi}\left\{\frac{e^{ikr}}{kr_1}\frac{\dot{F}^+(b, -k\cos\theta_1)}{\sin\theta_1 H_1^{(1)}(kb\sin\theta_1)} + \frac{e^{ikr}}{kr_2}\frac{\dot{F}^+(b, -k\cos\theta_1)}{\sin\theta_2 H_1^{(1)}(kb\sin\theta_2)}\right\} \tag{7a}$$

where $r_1, \theta_1$, and $r_2, \theta_2$ are the spherical coordinates defined by

$$\rho = r_1 \sin\theta_1 \qquad z = r_1 \cos\theta_1 \tag{7b}$$

and

$$\rho = r_2 \sin\theta_2 \qquad z - l = r_2 \cos\theta_2 \tag{7c}$$

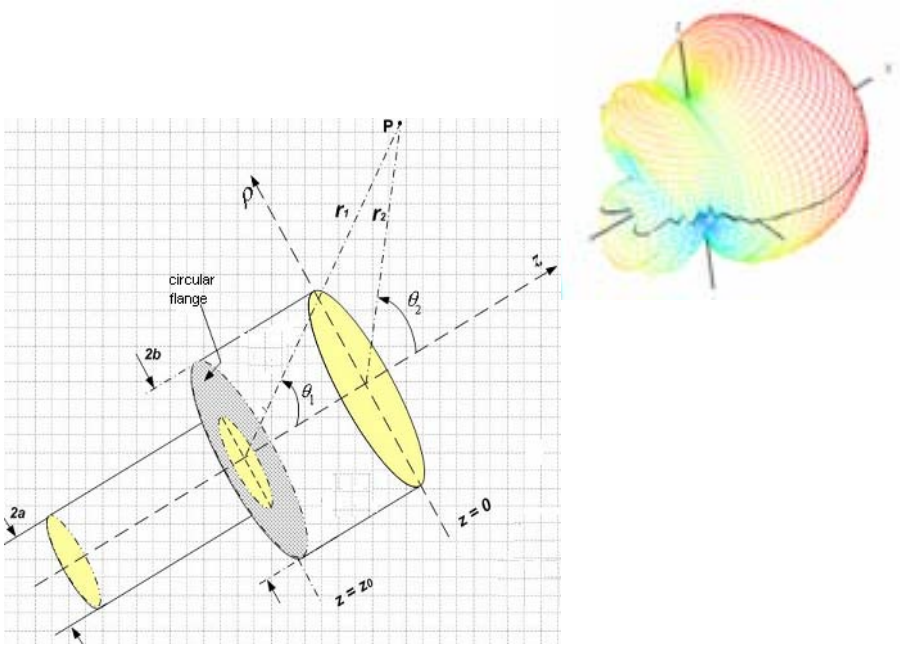


**Figure 2.** Wide beam scalar feed horn

In order to show the influence of the values of the aperture's inner surface on the radiation the numerical results showing the observation angle are presented.

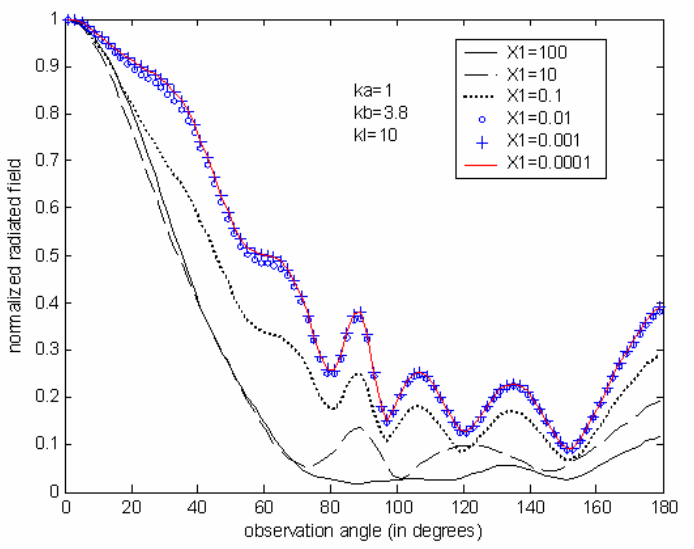


**Figure 3.** Normalized radiated field versus the observation angle for different values of the $X_1$

**($\eta_1$=i$X_1$ )**

When X1>0 the directivity of the wide beam scalar horn increases with the increasing values of $X_1$ (Figure 3) and we should also note that the variation of the radiated field is not sensitive to the variation of the impedance of the aperture's inner surface $X_1$ for $X_1$<0.001.

## REFERENCES


[1] H. Levine and J. Schwinger, "On the sound radiation of sound from an unflanged circular pipe J. Phys.Rev, 73, 383 1948

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[3] John D. Kraus , Antennas, Mc.Graw-Hill Book Company, 1988