The Challenge of Metamaterials Homogeneization: New Architectures and Technologies for Real - Artificial Materials

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1 Introduction

Two categories of metamaterials (MTMs) have been explored so far: resonant structures made of thin wires (TWs) and/or split-ring resonators (SRRs) [1, 2, 3], and nonresonant transmission line (TL) structures made of lumped inductors and capacitors [4, 5, 6, 7]. Both can in principle exhibit novel properties, such as left-handedness [8] in one, two or three dimensions. The MTMs of the first category are narrow-band or high-loss structures due to their resonant nature, whereas the MTMs of the second category have low loss over a broad bandwidth under appropriate matching conditions. However, the two types of MTMs share a common property: they are effective structures, i.e. they exhibit a macroscopic medium behavior because they are operated in a frequency range where the average lattice feature p is much smaller than the guided wavelength λ_g, $p \ll \lambda_g$. This is why they can be characterized in terms of the constitutive parameters ε and μ, in contrast to structures operated in the Bragg regime such as typically photonic crystals. MTMs are essentially transparent media where refractive phenomena take place, whereas photonic crystals are bandgap structures where diffraction dominates.

Due to their unique phase properties, left-handed (LH) MTMs represent a new paradigm in microwave and optics engineering and, when paired with conventional right-handed (RH) materials, may lead to a vast range of novel devices with unique functionalities.

However, such a brilliant future will be possible for MTMs only on condition that high-quality structures can be fabricated and integrated into modern circuit and systems. The present paper describes one of the major technological challenges to meet in this direction: homogenization, and suggests a few potential solutions. The discussion is mostly based on TL LH MTMs, and more specifically on composite right/left-handed (CRLH) MTMs [9], which represent a broadband generalization of TL LH MTMs. However, the considerations and results presented also largely pertain to other types of LH structures.

2 CRLH TL MTMs

The fundamental relations for a CRLH TL structure can be derived from standard TL theory extended to 2D (or 3D). Assuming that the electrical size of the unit cell is small enough ($p \ll \lambda_g$), one obtains the dispersion (β) and attenuation (α) relations

$$\beta(\omega) = \frac{1}{p}\left(\omega\sqrt{L_R C_R} - \frac{1}{\omega\sqrt{L_L C_L}}\right), \quad \alpha = 0, \quad Z_c = \sqrt{\frac{L_R}{C_R}} = \sqrt{\frac{L_L}{C_L}}, \tag{1}$$

where the *balanced-design* condition $\omega_{se} = \omega_{sh}$, with $\omega_{se} = 1/\sqrt{L_R C_L}$ and $\omega_{sh} = 1/\sqrt{L_L C_R}$, has been implicitly assumed. Fig. 1 shows a typical 2D CRLH TL MTM structure, its circuit model, dispersion diagram $[\beta(\omega)]$ and refractive index $[n = c\beta/\omega]$. The last two graphs also show the case of an unbalanced design with a midgap due to the non-canceling resonances ω_{se}/ω_{sh} [9].

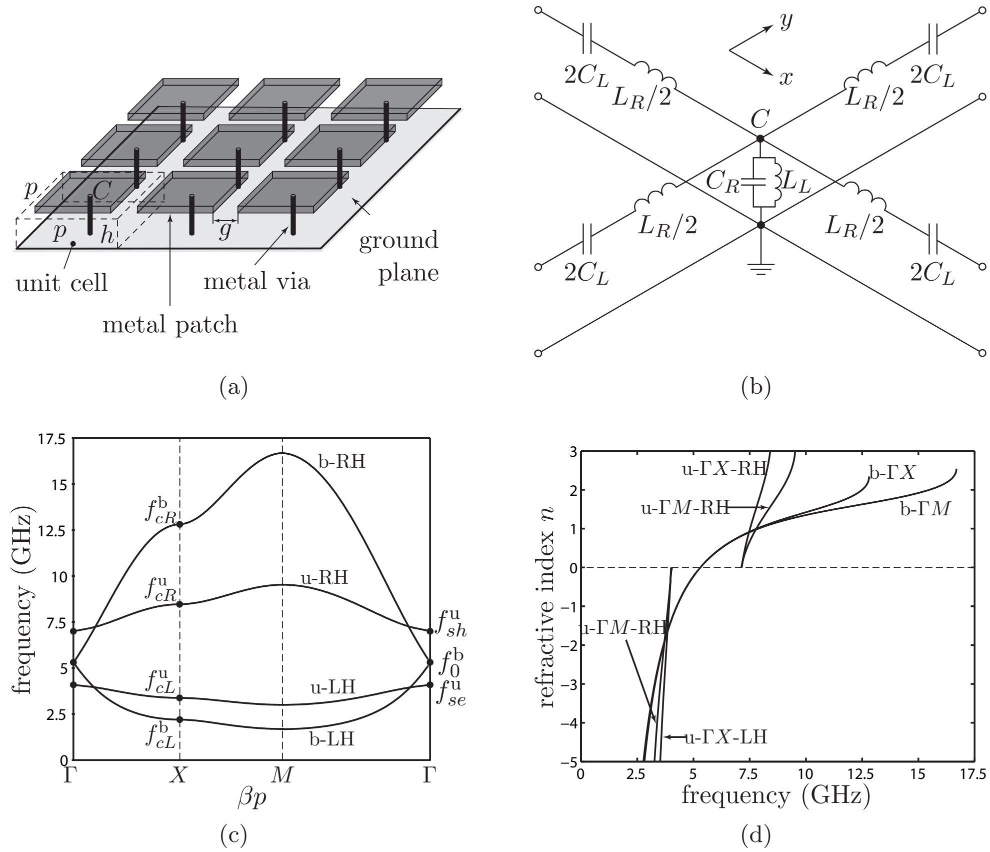


Figure 1: Planar (2D) CRLH TL MM. (a) Mushroom structure. (b) Equivalent circuit unit cell, where the node C corresponds the top of a metal via. (c) Dispersion curves, for an unbalanced (label u) design with $L_R = 3.03$ nH, $C_R = 2.00$ pF, $L_L = 0.77$ nH, $C_L = 0.17$ pF corresponding to $(f_{cL}, f_{se}, f_{sh}, f_{cR}) = (3.38, 4.05, 7.06, 8.46)$ GHz, and for a balanced (label b) design with $L_R = L_L = 1.50$ nH, $C_R = C_L = 0.60$ pF, corresponding to $(f_{cL}, f_{se} = f_0 = f_{sh}, f_{cR}) = (2.20, 5.31, 12.81)$ GHz. (d) Refractive index obtained from (c) by $n = \beta/k_0$ or from $n = c\sqrt{\mu\varepsilon}$ with (**??**).

The CRLH TL structure is in essence a band-pass filter, the cutoff frequencies of which limit the long-wavelength MTM regime located around the transition frequency $\omega_0 = 1/\sqrt{LC}$ ($LC = L_R C_L = L_L C_R$) between the LH and RH ranges. These cutoff frequencies are given as [9] (balanced design)

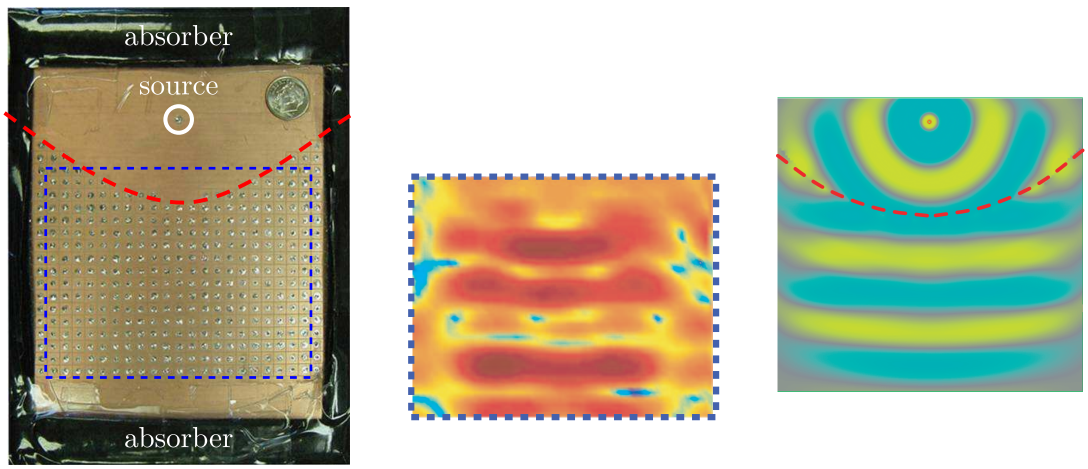
$$\omega_{cL} = \omega_R \left| 1 - \sqrt{1 + \frac{\omega_L}{\omega_R}} \right| \overset{\omega_L \ll \omega_R}{\approx} \frac{\omega_L}{2}, \tag{2}$$

$$\omega_{cR} = \omega_R \left(1 + \sqrt{1 + \frac{\omega_L}{\omega_R}} \right) \overset{\omega_L \ll \omega_R}{\approx} 2\omega_R + \frac{\omega_L}{2}, \tag{3}$$

where $\omega_R = 1/\sqrt{L_R C_R}$ and $\omega_L = 1/\sqrt{L_L C_L}$.

3 Homogenization

For the sake of illustration, let us consider the perfect reflection less RH/LH parabolic interface shown in Fig. 2. The near-field measured field [Fig. 2(b)] shows that, although the principle of circular to plane wave transformation is verified, the quality of this transformation is far poorer than that which would be provided by a perfectly homogeneous medium with same constitutive parameters [Fig. 2(c)]. The reason for this fact is that in the experimental prototype p/λ is not small enough to suppress all diffraction-scattering effects. These effects not only perturb the refraction patterns expected for an homogeneous medium, but also induce power loss losses.



(a) (b) (c)

Figure 2: Focusing/collimating parabolic interface between a RH parallel-plate structure and a LH mushroom structure with design parameters $p = 5$ mm, gaps between adjacent patches $g = 0.2$ mm, substrate (Rogers RT 6010) permittivity $\varepsilon_r = 10.2$, substrate and via height $h = 1.27$ mm. (a) Prototype. (b) Measured E-field at 3.35 GHz ($p \approx \lambda_r/8$) in the mushroom region. (c) E-field at 3.35 GHz in the corresponding homogenous medium.

If we homogenize the MTM by decreasing p/λ_g by a factor ζ ($\zeta > 0$), we observe from Eq. (1) that the dispersion relation is transformed to

$$\beta(\omega) = \frac{\zeta}{p}\left(\omega\sqrt{L_{R\zeta}C_{R\zeta}} - \frac{1}{\omega\sqrt{L_{L\zeta}C_{L\zeta^2}}}\right). \tag{4}$$

In order to preserve the refractive features (such as focal length), we need to keep the refractive index $n(\omega) = \beta(\omega)/k_0$ of the material and therefore $\beta(\omega)$ unchanged. This implies, also taking into account that matching must be also preserved, that the LC quantities need to be adjusted to the new values

$$L_{R\zeta} = \frac{L_R}{\zeta}, \quad C_{R\zeta} = \frac{C_R}{\zeta}, \quad L_{L\zeta} = L_L \cdot \zeta. \quad C_{L\zeta} = C_L \cdot \zeta, \tag{5}$$

which reveals the fundamental challenge in metamaterial homogenization: Whereas the RH parameters need to be reduced, which is a natural effect obtained as size is reduced, the LH parameters need to be *increased*, which means that more reactive energy need to be stored over a smaller volume!

Reduction of even a factor 5, from the typical current $p \approx \lambda/10$ to $p \approx \lambda/50$, would yield dramatic improvement in the production of very pure quasi-optical refractive effect of immediate interest for a vast array of novel component and antenna applications.

A direct consequence of RH parameters reduction and LH parameters reduction may be inferred from Eqs. (2) and (2): as $L_{R\zeta}, C_{R\zeta} \to 0$ and $L_{L\zeta}, C_{L\zeta} \to \infty$, $\omega_{cL\zeta} \to 0$ and $\omega_{cR\zeta} \to \infty$, which means that bandwidth become infinite, just like in an ideal TL (except that the phase center is at ω_0 instead of at DC as in a PRH TL).

4 Conclusions

The spurious effects of insufficiently small electrical length of the structural features of MTMs have been described, and the challenge of homogenization, which consist in the necessity of squeezing more reactive energy within smaller volumes, has been pointed out. It has also been shown that this process results in bandwidth enhancement. Meeting the challenge of homogenization will require resort of low loss high-permittivity ferroelectrics or ceramic for generation of high series capacitors and of low loss high-permeability ferrimagnetic/ferromagnetic nanostructured materials for the generation of high shunt inductors.

References

[1] D. R. Smith, W. J. Padilla, D. C. Vier, S. C. Nemat-Nasser, and S. Schultz, "Composite medium with simultaneously negative permeability and permittivity," *Phys. Rev. Lett.,* vol. 84, no. 18, pp. 4184–4187, May 2000.

[2] J. B. Pendry, A. J. Holden, D. J. Robbins, and W. J. Stewart, "Low frequency plasmons in thin-wire structures," *J. Phys. Condens. Matter,* vol. 10, pp. 4785–4809, 1998.

[3] J. B. Pendry, A. J. Holden, D. J. Robbins, and W. J. Stewart, "Magnetism from conductors and enhanced nonlinear phenomena," *IEEE Trans. Micr. Theory. Tech.,* vol. 47, no. 11, pp. 2075–1084, Nov. 1999.

[4] C. Caloz and T. Itoh, "Application of the transmission line theory of left-handed (LH) materials to the realization of a microstrip LH transmission line," in Proc. *IEEE-AP-S USNC/URSI National Radio Science Meeting,* vol. 2, pp. 412–415, San Antonio, TX, USA, June 2002.

[5] A. K. Iyer and G. V. Eleftheriades, "Negative refractive index metamaterials supporting 2-D waves," in *IEEE-MTT Int'l Symp.,* vol. 2, Seattle, WA, USA, June 2002, pp. 412–415.

[6] G. V. Eleftheriades, A. K. Iyer, and P. C. Kremer, "Planar Negative Refractive Index Media Using Periodically L-C Loaded Transmission Lines," *IEEE Trans. Microwave Theory Tech.,* vol. 50, no. 12, pp. 2702–2712, Dec. 2002.

[7] A. Sanada, C. Caloz, and T. Itoh, "Planar distributed structures with negative refractive index," *IEEE Trans. Microwave Theory Tech.,* vol. 52, no. 4, pp. 1252–1263, April 2004.

[8] V. Veselago, "The electrodynamics of substances with simultaneously negative values of ε and μ," *Soviet Physics Uspekhi,* vol. 10, no. 4, pp. 509–514, Jan.-Feb. 1968.

[9] C. Caloz and T. Itoh. *Electromagnetic Metamaterials: Transmission Line Theory and Microwave Applications,* Wiley-Interscience, 2005, *to be published*.