PLANAR SLOTTED LINES WITH FINITE WIDTH OF THE SUBSTRATE

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ABSTRACT

This paper uses the spectral domain method to analyse a flat rectangular waveguide with a slotted partition in the H plane and a waveguide with the longitudinal slot in its wider wall. Even and odd modes have been identified on both guides and their suitability for practical utilization in circuits is shown. Our calculated propagation constants compare well with data published elsewhere. The analysis provides a new view on the field propagation in the slotted waveguide that is in accord with its hitherto known behavior.

INTRODUCTION

Transmission lines are the basic building blocks of all HF and MW circuits. Intended utilization, frequency band and available technology influence the selection of the line and its cross-section. Planar technology is nowadays the most widespread technology, but it provides open transmission lines predisposed to producing leaky waves at higher frequencies. Leakage needs to be suppressed in circuits and supported in antennas. A practically realizable line must have conductors and substrates of finite proportions. In this paper we investigate the propagation of waves along two lines with small height and finite width of the substrate, Fig. 1 and 2.

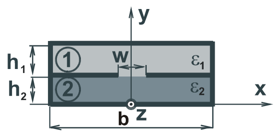

Fig. 1 Cross-section of a finned waveguide

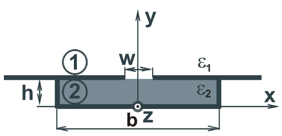

Fig. 2 Cross-section of a flat waveguide

Planar transmission lines with metallic walls on the top, bottom and sidewalls have the advantages of both planar technology and waveguide technology. They do not suffer from losses of energy by radiation or crosstalk, and they have great electromagnetic resistance. A slotline located in the E-plane of the rectangular waveguide is a typical representative [1,2], usually known as a unilateral finline.

Prevention of leakage from the slotline and an inquiry into its behavior in a wide frequency range was the motivation for investigation of a completely shielded conductor-backed slotline, shown in Fig. 1. The line is, in fact, a rectangular waveguide with a conductive partition in the H plane with a longitudinal slot. Due to its proportions we design it as a finned waveguide (FNW), regardless of the same or different permittivities in the upper and lower portion of the cross-section. A rectangular waveguide with a longitudinal slot in the wider wall has been used since the beginning of microwave technique development as the measuring line with a probe movable along the slot. It has also been studied for radiation purposes as a traveling-wave antenna [3], or as a leaky-wave antenna [4]. Our investigation of this line from the leakage point of view has resulted in a new view on, and an alternate explanation, of its characteristics. For brevity we call it a flat waveguide (FW). Our approach has more general validity since it removes the restriction set on the ratio of the cross-sectional proportions in [4].

METHOD OF ANALYSIS

A substantial feature of the two explored lines in Figs. 1 and 2 is the finite width of their substrates metallized on the sidewalls. This fact determines a method for analyzing them. When the concept of transversal waves is adopted, and all the field components have a dependence of the form $e^{-j\gamma_z z}$, where the propagation constant in the z direction $\gamma_z = \beta - j\alpha$ and β is the phase constant while α is the attenuation constant, the solution of the wave equation in 2D for the electric and magnetic Hertz potential Φ^e and Φ^m

$$\left[\Delta + \left(k_j^2 - \gamma_z^2\right)\right] \Phi(x, y) = 0 \,, \tag{1}$$

is transferred to 1D solution by the Fourier transform

$$\tilde{\Phi}(\xi_n, y) = \frac{2}{b} \int_{-b/2}^{b/2} \Phi(x, y) e^{-j\xi_n x} \, dx \,, \tag{2}$$

where the spectral variable $\xi_n = 2n\pi / b$ for even modes, $\xi_n = (2n-1)\pi / b$ for odd modes, n=0, ±1, ±2, … ∞ , and k_j is the wave number in the corresponding medium j=1,2. The transversal electric component of the electric field E_x

within the slot can have an even or odd symmetry with regard to the axis of the line. Consequently, we distinguish even and odd modes. Applying (2) in (1) we get

$$\left[\frac{\partial^2}{\partial y^2} - \gamma_j^2\right]\tilde{\Phi}(\xi_n, y) = 0 \tag{3}$$

$$\gamma_j^2 = \xi_n^2 + \gamma_z^2 - \varepsilon_{rj} k_0^2 \ . \tag{4}$$

The solution of (3) is known. When the boundary conditions on the planes y=0, h_2 are imposed in the spectral domain and the current densities on the partition of the FNW are eliminated by the Parseval theorem, and the tangential components of the electric field in the slot are expressed in terms of the basis functions, then the Galerkin method provides a set of linear equations for the amplitudes of the basis functions. For nontrivial solutions, the determinant of the matrix must be zero. From this condition, known as the dispersion equation, the propagation constant γ_z is determined at a chosen frequency [5,6]. The field components are determined by the backward Fourier transform

$$\Phi(x, y) = \frac{1}{2}\sum_{n=-\infty}^{\infty}\tilde{\Phi}(\xi_n, y)e^{j\xi_n x} \ . \tag{5}$$

In the case of the FW in the upper unbounded area, the Fourier transform

$$\tilde{\Phi}(\xi, y) = \int_{-\infty}^{\infty}\Phi(x, y)e^{-j\xi x}dx \tag{6}$$

is applied and the field is accessible by the backward transform

$$\Phi(x, y) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\tilde{\Phi}(\xi, y)e^{j\xi x}d\xi \ , \tag{7}$$

while $\gamma_1^2 = \xi^2 + \gamma_z^2 - \varepsilon_{r1}k_0^2$ and $\gamma_2^2 = \xi_n^2 + \gamma_z^2 - \varepsilon_{r2}k_0^2$.

The boundary conditions record on the y=h plane in the space domain is more complex. Equation (7) represents the field in the first area while (5) gives it in the second area. Consequently, in the spectral domain two different images are related. The introduction of the finite function and its periodic extension, on y=h in the second area, provides the relation between these images

$$\tilde{\Phi}_1(\xi, h) = \frac{b}{2}\sum_{n=-\infty}^{\infty}\tilde{\Phi}_2(\xi_n, h)\cdot Sa\left[\frac{b}{2}(\xi - \xi_n)\right] \tag{8}$$

where the sample function

$$\mathrm{Sa}\left[\frac{b}{2}(\xi - \xi_n)\right] = \frac{\sin\left[\frac{b}{2}(\xi - \xi_n)\right]}{\frac{b}{2}(\xi - \xi_n)} \ . \tag{9}$$

The characteristic impedance of the dominant mode is essential for the design of circuits. Because it is a hybrid mode, the definition of the characteristic impedance is not unique. For the slotted lines we use the power-voltage definition, where V is the voltage across the slot and P is the time averaged power flow along the line.

$$Z = \frac{V^2}{2P} . \tag{10}$$

RESULTS AND DISCUSSION

We checked the correctness of our code produced for analysis of the FNW by computing the dominant mode phase constant. Due to the symmetry, FNW represents one half of the fin line given in [2]. Published data agrees well with ours, as is shown in Fig. 3. The second test is shown in Tab. 1 where the normalized cut-off frequency b/λ_c of FNW calculated in [7] by the transmission line matrix method agrees well with the cut-off frequency calculated by our code. Regardless of the $E_x(x)$ symmetry there are two sets of modes. The first set contains modes that we term slot-type modes. Their field distribution in the neighborhood of the slot resembles the field of modes propagating along the conductor-backed slotline, influenced now by the shielding. The modes that we term waveguide-type modes constitute the second set, since their field resembles the field of the TE modes in the rectangular waveguide perturbed now by the

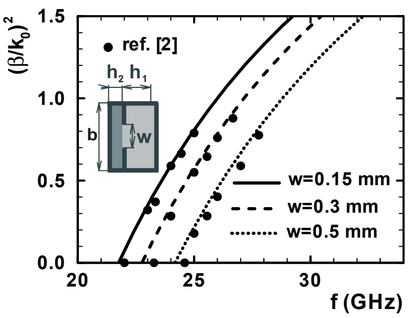

Fig. 3 The calculated dispersion characteristics of the dominant even mode on the FNW with ε_{r1}=1, ε_{r2}=3.75, h_1= 3.4875 mm, h_2=0.0625 mm, b=3.55 mm and three different w, and data taken from [2].

conductive partition in the H plane. The difference between the slot-type modes and the waveguide-type modes is most distinct when ε_1=ε_2.

We checked the phase constant of the even and also the odd dominant modes on the FNW by measuring their standing waves. The nodes of the standing wave were detected by a movable probe over a narrow longitudinal slot made in the shielding and placed where the longitudinal component of the magnetic field $H_z(x)$ was zero. The measured phase constant of the even and odd modes, providing a good trace of the calculated characteristics, is shown in Figs. 4 and 5. Glass was used in the lower portion and plexiglass in the upper portion of the FNW. The phase constants of both dominant modes, calculated by the CST Microwave Studio, also fit well with our data.

Tab. 1 The normalized cutoff frequency of the FNW [7] with ε_{r1}= ε_{r2}=1, h_1=h_2=16 mm, b=16 mm.

	b/λ_c	
w (mm)	[7]	this work
12	0.243	0.241
10	0.235	0.234
8	0.224	0.227
6	0.210	0.213
4	0.192	0.192

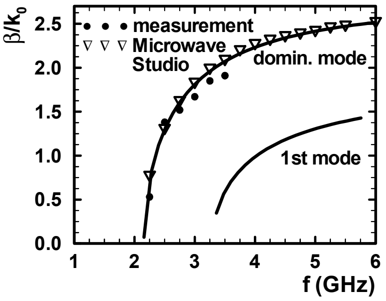

Fig. 4 Calculated and measured phase constant of the even dominant mode on the FNW with h_1=5 mm, h_2=3 mm, w=10 mm, b=30 mm, ε_{r1}=2.6, ε_{r2}=8.

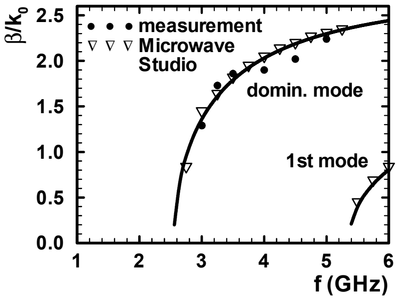

Fig. 5 Calculated and measured phase constant of the odd dominant mode on the FNW with h_1=5 mm, h_2=3 mm, w=10 mm, b=30 mm, ε_{r1}=2.6, ε_{r2}=8.

Calculation of the cutoff frequency f_c of the modes is based on an auxiliary capacitance or extension of the line in the transversal resonance model. The phase constant of the modes in the FNW follows the function β_w/k_0=Sqr[$1-(f_c/f)^2$] , as do the phase constant of the TE modes in the rectangular waveguide drawn in Fig. 6. The diamonds are an approximation of the dispersion characteristic by the squared root function.

From the point of view of circuit applications, the waveguide-type modes of both the even and the odd symmetry on the FNW are of low practical importance. Their field in the slot area is directed dominantly perpendicular to the slot and the voltage across the slot is low. Applications of the FNW in microwave and millimeter wave circuits are possible when the even dominant slot-type mode is excited. The line is usable as a standard slotline, possessing the advantages of the shielded structure.

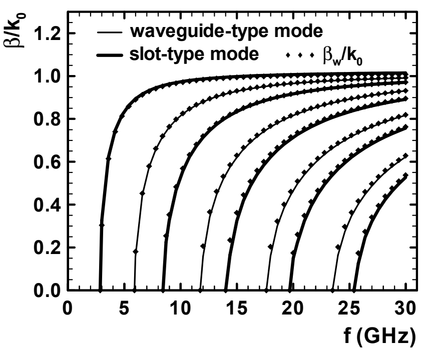

Fig. 6 Dispersion characteristics of the even modes in the FNW with b=50 mm, h_1=h_2=2 mm, w=1 mm ε_{r1}=ε_{r2}=1.

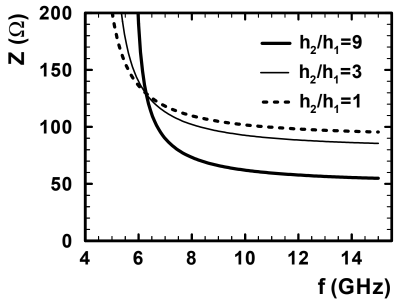

Fig. 7 Characteristic impedance of the dominant even mode of the FNW when h_1+h_2=10 mm, w=1 mm and b=23 mm.

The complex propagation constant on a transmission line with a slot in the wider wall of an air-filled waveguide with TM_{11} excitation was measured and calculated in [3]. It was also calculated by a transverse resonant procedure employing a transverse equivalent network and the perturbation solution of the resonance equation [4]. We have tested a code computing the complex propagation constant by the method presented in this paper. Acceptable agreement of the data from [3] and [4] with our data for the squared slotted wave-guide is shown in Figs. 8 and 9, where h=b.

We have identified even and odd modes on the guide. The dispersion characteristics of the first five modes are plotted in Fig. 10. These are the space leaky modes, and correspond to the modes of the waveguide perturbed by the longitudinal slot. The first space leaky mode is related to the dominant mode of the waveguide TE_{10}. The second mode corresponds to the TM_{11} mode, and the field of the third mode resembles the field of the TE_{11} mode. The fourth space

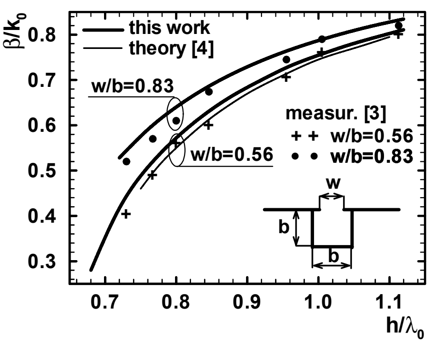

Fig. 8 Normalized phase constant of the squared slotted waveguide.

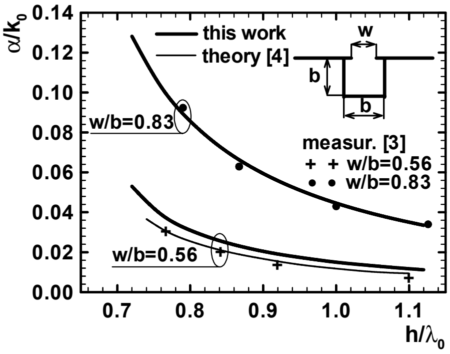

Fig. 9 Normalized leakage constant of the squared slotted waveguide

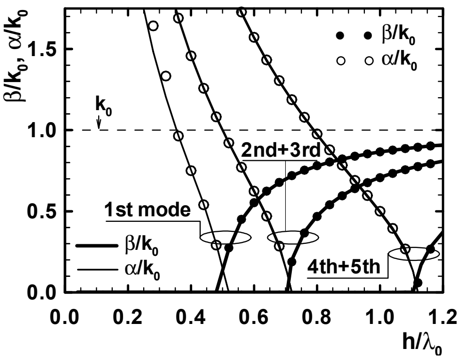

Fig. 10 Dispersion characteristics of five first odd space leaky modes on the squared slotted waveguide with w=1 mm, h=b=10 mm, $\varepsilon_{r1}=\varepsilon_{r2}=1$.

leaky mode is a modification of the TM_{12} mode, while the fifth has a link to the TE_{12} mode. Their phase constants trace the propagation constants of the modes in an unperturbed waveguide, dots in Fig. 10. Similarly, the attenuation constant, typical for evanescent modes below the cutoffs, coincides with the leakage constant, circles in Fig. 10. Accordingly, a waveguide with a narrow slot does not radiate since the leakage constant is very great below the cutoff. It also does not radiate above the cutoff, since the leakage constant is almost zero. The narrower the slot width, the closer together are the frequencies at which β and α approach zero. The space leaky mode converts into a bound mode when $\beta=k_0$. When a dielectric slab with permittivity $\varepsilon_r>1$ is placed on the top of the guide with $\varepsilon_{r1}=\varepsilon_{r2}=1$ the initial space leaky mode converts into the bound mode. The bound mode changes into the mode leaking into the slab above the spectral gap.

CONCLUSION

The FNW and FW were analyzed by the spectral domain method. The method is easier asserted on the FNW compared to the mode matching method. The even dominant mode is usable in circuits and possesses the characteristic impedance adjustable by the height of either area of the cross-section. Closed-form formula of the cutoff is available when homogeneous dielectric fills the guide. An alternative analysis of the slotted waveguide has been presented. Even and odd modes have been identified on the FW. Space leaky modes are dominant modes. At higher frequencies they convert into the bound modes when the permittivity of the substrate in the FW is greater then permittivity outside. The propagation constant of the first odd space leaky mode agrees well with the propagation constant of the TE_{10} waveguide mode. Our results are in good accord with data published in the past. The low profile of the FNW and FW allows integration with circuits based on other planar transmission lines.

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