

# FUNDAMENTAL PROPERTIES OF BROADSIDE RADIATION FROM UNIFORM LEAKY-WAVE ANTENNAS

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**Abstract:** In this work, radiation at broadside is studied for a general class of one- and two-dimensional uniform leaky-wave antennas. The analysis, based on a simple transverse equivalent network model of the antenna, reveals that a fundamental condition for optimum antenna operation at broadside is the equality between the phase and attenuation constants of the leaky mode responsible for radiation. This corresponds to having the maximum level of radiation at broadside and the beam being on the verge of splitting into two distinct peaks off broadside. Simple design formulas based on the optimized conditions are provided for both lossless and lossy structures. In addition, the antenna bandwidth for operation at broadside is studied and a useful approximate expression for the fractional bandwidth is given in terms of the physical parameters of the antenna.

## INTRODUCTION AND BACKGROUND

In this work, a study of one- and two-dimensional (1D and 2D) uniform leaky-wave antennas (LWAs) is presented, with the aim of characterizing their fundamental radiation properties when the beam points at broadside. In particular, it is shown how to optimize radiation at broadside, and the general properties of the broadside radiation, including pattern bandwidth, are examined. Although directed at antennas, this work also has important applications in other areas such as the enhanced transmission through subwavelength apertures, where the phenomenon is due to leaky-wave radiation at broadside [1].

The class of LWAs considered here consists of a grounded dielectric slab excited by either a horizontal electric line source (1D LWA) or a horizontal electric dipole source (2D LWA) placed inside the slab, oriented in the  $x$  direction. At the air-slab interface a partially-reflecting surface (PRS) is assumed, which may take the form of a high permittivity layer, or a metallic plate with a periodic arrangement of holes, or some other structure that partially shields the slab, allowing radiation to occur [2]. The PRS creates a leaky parallel-plate waveguide (PPW) region, which is excited by the source. The source launches the first higher-order ( $n = 1$ ) TE and/or TM PPW modes (depending on the source), which are leaky modes due to the PRS. In this class of structures radiation occurs from the fast-wave nature of the PPW modes, and not from a space harmonic, and thus the structure is classified as a *uniform* type of LWA structure.

The analysis of both the radiated far field and the modal properties of the structure is carried out by means of a transverse equivalent network in which the PRS is modelled as a constant shunt susceptance  $B_s$  (this has been shown to accurately model a 2D periodic array of metal patches or slots in a metallic plane, as well as superstrates with high permittivity [2], [3]). It is then possible to obtain in a simple way a closed-form expression for the power density radiated in the far-field region.

## THEORY

The uniform leaky-wave antennas considered here consist of a grounded dielectric slab of thickness  $h$ , with relative permeability and permittivity  $\mu_r$  and  $\epsilon_r' (1 - j \tan \delta_e)$ , respectively, covered with a partially-reflecting surface (PRS) and excited by either an electric line (1D LWA) or an electric dipole source (2D LWA), placed at a distance  $h_s$  from the interface. A sketch of the latter is reported in Fig. 1(a), while in Fig. 1(b) examples of practical realizations of PRSs are illustrated. In Fig. 1(c), the transverse equivalent network (TEN) used for the study of the antenna is reported.

Considering for simplicity the 1D case, it is well known that the electric far field  $E_x = E_{ff}$  radiated by a leaky wave longitudinally propagating along the  $y$  axis with complex propagation constant  $k_{yLW} = \beta - j\alpha$  is given by [4]

$$E_{ff}(\theta) = E_0(k_{yLW}) \frac{2jk_{yLW}}{k_0^2 \sin^2 \theta - k_{yLW}^2} \cos \theta \quad (1)$$

where  $k_0$  is the free-space wavenumber,  $\theta$  is the angle measured from broadside, and  $E_0(k_{yLW})$  is the amplitude of the leaky wave, which is proportional to the spectral Green's function (SGF) of the problem. A simple analysis of the radiation pattern when the beam points close to broadside (i.e.,  $\theta \ll 1$ ) allows one to establish that the condition  $\beta = \alpha$  implies the splitting of a single beam pointing exactly at broadside (when  $\beta < \alpha$ ) into a beam with two peaks close to,

but off broadside (when  $\beta > \alpha$ ), regardless of the amount loss in the dielectric.

To derive a condition that maximizes the broadside radiated power density, a knowledge of the dependence of the leaky-wave amplitude  $E_0(k_{yLW})$  on  $k_{yLW}$  is necessary. This can be obtained by means of an asymptotic study of the SGF in the limit of large values of  $B_S$  with respect to the free-space admittance  $\eta_0$  (this implies a quasi-closed structure) and small values of the leaky-wave propagation constant  $k_{yLW}$  with respect to  $k_0$  (this implies a highly directive beam close to broadside). It turns out that the broadside radiated power density  $P(0)$  is inversely proportional to a  $(\hat{\beta}^2 + \hat{\alpha}^2)^2$  term, where the hat ^ indicates normalization with respect to  $k_0$ . Next, the dependence of  $\hat{k}_{yLW}$  with frequency can be obtained in an approximate way by modeling the structure as a closed PPW filled with an equivalent lossy dielectric, which takes into account the radiation losses; it turns out that the trajectory of the leaky-wave pole in the complex  $\hat{k}_y$  plane by varying frequency is a hyperbola of the form  $\hat{\beta}\hat{\alpha} = \text{constant}$ , symmetric with respect to the  $\hat{\beta} = \hat{\alpha}$  line. Therefore, the condition for having the maximum radiated power density at broadside is  $\hat{\beta} = \hat{\alpha}$ , both in the lossless and in the lossy case. Furthermore, in the same limit as above, it can easily be proved that the  $\hat{\beta} = \hat{\alpha}$  condition is equivalent to the following equality:

$$\cot(k_0 h \sqrt{\mu_t \epsilon'_r}) = \frac{B_S}{\eta_0} \sqrt{\frac{\epsilon'_r}{\mu_t}} \quad (2)$$

which expresses the optimum condition in terms of the physical and geometrical parameters of the structure, and coincides with the expression found in [2] in the lossless case. Note that the optimum frequency predicted by (2) is independent of substrate loss. Vice versa, it can be shown that by maximizing the total radiated power density at broadside one obtains (2), which in turn can be shown to imply the presence of a leaky wave with  $\hat{\beta} = \hat{\alpha}$ . An asymptotic expression valid for large values of  $B_S$  can be derived for  $\hat{\beta}$  and  $\hat{\alpha}$  when the optimum condition (2) is satisfied:

$$\hat{\beta} \simeq \hat{\alpha} \sim \sqrt{\frac{(\mu_t \epsilon'_r)^{3/2} \eta_0^2}{\mu_t \pi B_S^2} + \frac{1}{2} \mu_t \epsilon'_r \tan \delta_\epsilon} \quad (3)$$

In the lossless case, the broadside power density from an optimized structure increases indefinitely as  $B_S$  tends to infinity since, while the beamwidth of the radiated beam tends to zero, the directivity at broadside tends to infinity. However, in the lossy case, this is not to be expected on physical grounds and, since the structure tends to become a closed PPW when  $B_S$  tends to infinity, the radiation efficiency of the antenna (the ratio between the radiated power and the power dissipated into the substrate) tends to zero. Therefore, in the lossy case, an optimum value for the normalized shunt susceptance  $B_S/\eta_0$  can be shown to exist, which gives rise to a maximum level of power density radiated at broadside. In the limit of small loss tangent  $\tan \delta_\epsilon$ , it can be shown that the optimum value for  $B_S$  is given by

$$B_S^{\text{opt}} \simeq \eta_0 \sqrt{\frac{2}{\pi \tan \delta_\epsilon}} \sqrt{\frac{\epsilon'_r}{\mu_t}} \quad (4)$$

Based on an equivalent lossy-filled PPW model for the antenna (where an equivalent loss tangent accounts for both radiation and the actual substrate loss tangent), the exact positions of the leaky pole on the hyperbola of the  $\hat{k}_y$  plane corresponding to a broadside power density 3 dB lower than its maximum value can be determined, and it turns out that a 3-dB region can be defined: in particular, when the leaky pole lies inside this 3-dB region, the level of the broadside power density is within 3 dB of its maximum. It can be shown that such 3-dB region is an angular region of the  $\hat{k}_y$  plane symmetric with respect to the  $\hat{\beta} = \hat{\alpha}$  line, with internal angle  $\phi = \pi/4$ .

Finally, an approximate expression for the fractional bandwidth of the antenna for broadside radiation (defined as the frequency range over which the broadside power density level is within 3 dB of its maximum) can be derived, which is given by

$$\text{FBW} \simeq \frac{2\eta_0^2}{\pi B_S^2} \sqrt{\frac{\epsilon'_r}{\mu_t}} + \tan \delta_\epsilon \quad (5)$$

The case of a 2D-LWA, for which a horizontal dipole source is considered, can be treated by showing that the radiation patterns in the principal (E and H) planes are equal for observation angles  $\theta$  close to broadside and for frequencies close

to the optimum frequency defined by (2). Since the radiation pattern of a horizontal dipole in the plane orthogonal to its direction is the same of that produced by the corresponding horizontal line source in the same direction, all the results obtained in the 1D case can be directly extended to the 2D case. In the 2D case, the dipole excites both a  $TM_1$  and a  $TE_1$  cylindrical leaky-wave, varying as  $\cos\phi$  and  $\sin\phi$ , respectively. These leaky waves determine the E- and H-plane patterns, respectively. Under the approximations made previously, both leaky waves have the same propagation constant, given by (3).

## NUMERICAL RESULTS

To verify the validity of the above analysis, a structure with parameters  $\epsilon'_r = 2.2$ ,  $\mu_r = 1$ ,  $h = 5$  mm,  $h_s = h/2$ , and  $B_s = 20 \eta_0$  excited by a unit-amplitude electric line source is considered. In Fig. 2(a) the dispersion diagram for the  $TE_1$  leaky wave is reported together with the behavior of the broadside power density  $P(0)$  with frequency. It can be seen that at the optimum frequency  $f_{opt} = 20.687$  GHz (for which  $\beta = \alpha$ ) the broadside power density  $P$  has a maximum, both in the lossless and in the lossy case (with  $\tan \delta_e = 10^{-3}$ ). Moreover, in Fig. 2(b) it can also be verified that at the frequency  $f_{opt}$  the beam is on the verge of splitting into two distinct peaks off broadside: in particular, at  $f = 20.684$  GHz (i.e., when  $\beta < \alpha$ ) only one peak is present, pointing at broadside, but with a peak value lower than the one achieved at  $f_{opt} = 20.687$  GHz (where  $\beta = \alpha$  and only one peak is still present). As expected, at  $f = 20.69$  GHz, (i.e., when  $\beta > \alpha$ ), two separate peaks exist at  $\theta \approx \pm 1.45^\circ$ .

In Fig. (3), the  $TE_1$  pole loci in the complex  $\hat{k}_y$  plane corresponding to the dispersion diagrams of Fig. 2(a) are reported for both the lossless and lossy cases. It can be observed that, by varying frequency, the pole trajectories are very well approximated by hyperbolas; the squares indicate the maximum points, i.e., the values of  $\hat{\beta}$  and  $\hat{\alpha}$  for which the broadside radiated power density is maximum; as expected from the above analysis, they lie along the  $\hat{\beta} = \hat{\alpha}$  line. The circles indicate the 3 dB points, i.e., the values of  $\hat{\beta}$  and  $\hat{\alpha}$  for which the broadside radiated power density is 3 dB below its maximum (the relevant frequencies are also indicated in Fig. (3)). As stated in the previous section, the 3 dB points are located exactly on the edges of the 3 dB region (indicated with the straight gray lines in Fig. (3)) in both the lossless and lossy cases.

Finally, in Fig. (4), the angular power density  $P(\theta)$  in both the H plane (*black solid lines*) and the E plane (*gray dashed lines*) is reported for a structure with parameters as in Fig. (2) excited by a horizontal electric dipole placed at  $z = -h_s = -h/2$ , at  $f_{opt}$ ,  $f_{3dB-}$ , and  $f_{3dB+}$ . Here  $f_{3dB-}$  and  $f_{3dB+}$  are defined as the lower and upper frequencies at which the broadside power density level is 3 dB lower than its maximum (reached at  $f_{opt}$ ). It can be seen that for each frequency the E- and H-plane patterns are in excellent agreement throughout the entire angular range shown, which extends significantly beyond the angle(s) of maximum radiation.

## CONCLUSION

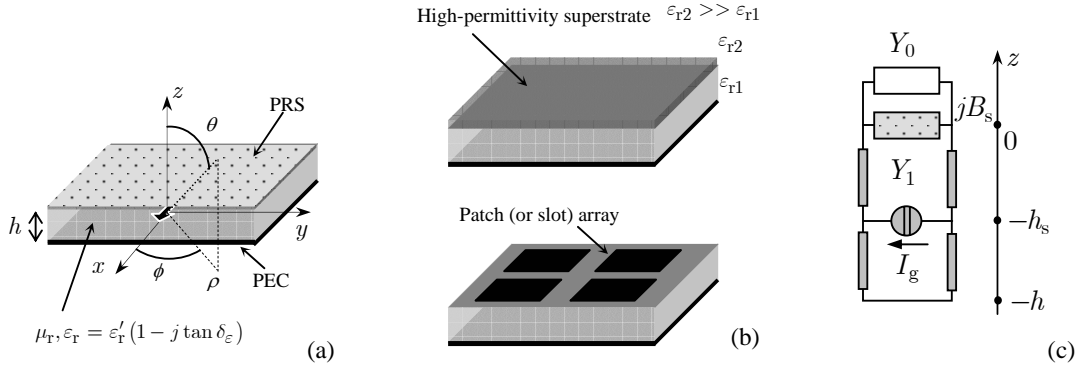
In this work, the fundamental characteristics of radiation at broadside from one- and two-dimensional uniform leaky-wave antennas have been illustrated, with reference to a class of antennas based on a grounded dielectric slab covered with a partially-reflecting surface. The analysis reveals that a fundamental condition for maximum antenna radiation at broadside is the equality between the phase and attenuation constants of the leaky mode. This condition corresponds to the beam being on the verge of splitting into two separate peaks.

For both lossless and lossy structures, design formulas have been obtained based on an approximate asymptotic expression for the leaky-mode propagation constant in the limit of large  $B_s$  (i.e., small beamwidth). These formulas show that in the lossless case the maximum level of power density radiated at broadside increases indefinitely as  $B_s$  increases, whereas in the lossy case an optimum value for the shunt susceptance  $B_s$  exists.

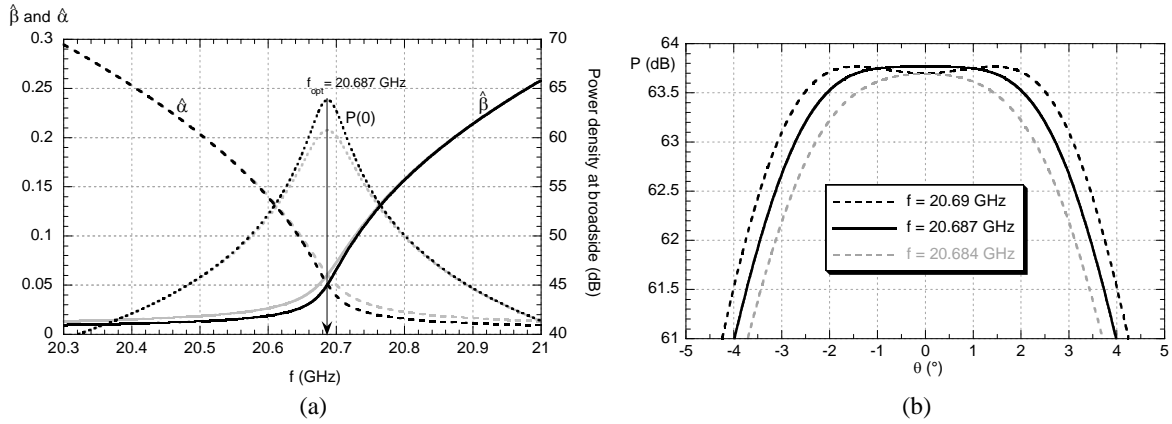
In addition, the antenna bandwidth for operation at broadside (defined as the frequency range over which the power density radiated at broadside is within 3 dB of the maximum) has been characterized in terms of the location of the leaky poles in the complex plane, and a simple and useful approximate expression for the fractional bandwidth has been given in terms of the physical parameters of the antenna.

## REFERENCES

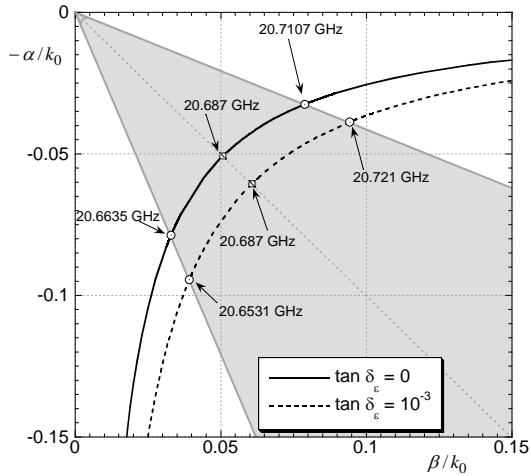
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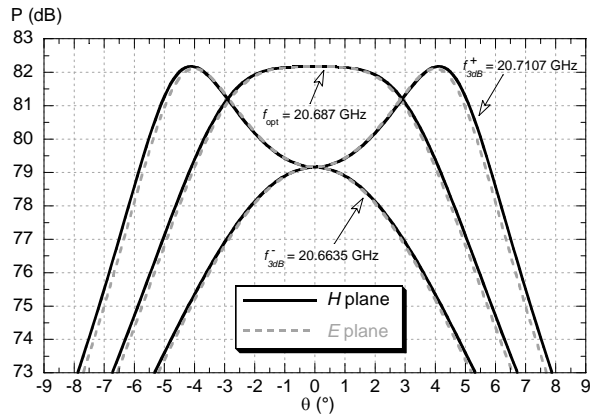
**Fig. 1** – (a) Three-dimensional view of the structure considered here, with the relevant physical and geometrical parameters, excited by a horizontal electric dipole (2D LWA); (b) examples of practical realizations of PRSs; (c) transverse equivalent network (TEN) for the structure in (a).



**Fig. 2** – (a) Dispersion diagram for the  $TE_1$  mode and broadside power density  $P(0)$  (in dB, relative to one W/m-rad) for a uniform structure as in Fig. 1 with  $\mu_r = 1$ ,  $\epsilon_r' = 2.2$ ,  $h = 5$  mm,  $h_s = h/2$ ,  $B_s = 20 \eta_0$ , excited by a unit-amplitude electric line source; *black curves*:  $\tan \delta_e = 0$ ; *gray curves*:  $\tan \delta_e = 10^{-3}$ ; (b) radiated power density in dB as a function of the angle  $\theta$  in a neighborhood of the optimum frequency  $f_{opt} = 20.687$  GHz for the structure considered in (a).



**Fig. 3** – Leaky-pole trajectories (*black curves*) and 3-dB region (*shaded area*) for a structure as in Fig. 2; *squares*: maximum points; *circles*: 3-dB points.



**Fig. 4** – Radiated power density in dB relative to one W/rad<sup>2</sup> as a function of the angle  $\theta$  in the H plane (*black solid lines*) and the E plane (*gray dashed lines*) for a structure with parameters as in Fig. 2 that is excited by an electric dipole source placed at  $z = -h_s = -h/2$ .