

# HIGH-FREQUENCY EXCITATION OF MICROSTRIP LINES BY GAP SOURCES AND PROBES

Francisco Mesa<sup>(1)</sup>, Raúl Rodríguez Berral<sup>(1)</sup>, and David R. Jackson<sup>(2)</sup>

<sup>(1)</sup>*University of Seville, Dept. of Applied Physics I, 41012 Seville, Spain  
Email: mesa@us.es*

<sup>(2)</sup>*University of Houston, Dept. of Electrical and Computer Engr., Houston, Texas, 77204-4005, USA  
Email: djackson@uh.edu*

## ABSTRACT

The nature of the current excited on an infinite microstrip line by a gap voltage source or a vertical probe feed at high frequency is examined. At high frequency the continuous-spectrum current excited on the line may become quite significant, due to both leaky-mode excitation as well as direct radiation from the source. However, even at arbitrarily high frequency the amplitude of the bound mode that is excited may be calculated from simple formulas for limiting cases. The effects of loss are also explored, and it is seen that conductor and dielectric loss affects the bound mode significantly more than the continuous-spectrum current.

## INTRODUCTION

Microstrip lines are one of the most commonly used transmission lines at microwave frequencies. They can be excited in a variety of ways, including end-launch connectors and vertical probe feeds. An end-launch connector can be modeled approximately as a gap voltage source. A coaxial feed is approximately modeled by a vertical probe excitation. One of the goals of this presentation is to investigate the high-frequency excitation of the current on a microstrip line by both gap voltage sources and vertical probe sources.

At low frequency, the excitation of a microstrip line by a gap voltage source or a vertical probe source is accurately described by simple transmission-line theory. As the frequency increases, however, transmission-line theory becomes increasingly less accurate as the continuous-spectrum current excited by the source becomes increasingly important. The continuous-spectrum current is that part of the total current that accounts for radiation effects as well as leaky-mode excitation, and it cannot be predicted from simple transmission-line analysis.

Semi-analytical formulations for both the gap voltage excitation and the probe excitation are used here, which are accurate at high frequency. The current on the line is obtained by solving the EFIE using a moment method in the spectral domain. The result is that the Fourier transform of the current in the longitudinal ( $z$ ) direction can be obtained in closed form (or by solving a small matrix equation). A numerical inverse Fourier transform is then used to recover the actual current on the line as a function of the distance  $z$  from the source.

It is shown that surprisingly simple approximate formulas exist for determining the amplitude of the bound mode that is excited by either source at arbitrarily high frequency. High-frequency effects due to the continuous-spectrum current are also examined for both types of excitations. Finally, the effects of dielectric and conductor loss are also examined, and it is shown how loss affects the bound-mode and continuous-spectrum currents on the line differently.

## CALCULATION OF THE STRIP CURRENT

An infinite microstrip line excited by a gap voltage source or a vertical probe current is shown in Fig. 1a or 1b, respectively. In either case, a semi-analytical moment-method approach in the spectral domain is used to calculate the Fourier transform of the strip current  $\tilde{I}(k_z)$ . The Fourier transform of the strip current is known in closed form when a single basis function is used to describe the transverse variation of the strip current. A small matrix equation is used to obtain this function when multiple basis functions are used to describe the transverse current variation. The details of the calculation procedure may be found in [1] for the gap excitation and in [2] for the probe excitation. The strip current then has the form

$$I(z) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} \tilde{I}(k_z) e^{-jk_z z} dk_z . \quad (1)$$

As discussed in [3,4], branch points appear in the complex  $k_z$  plane at  $k_z = \pm k_0$  and at  $k_z = \pm k_{TM0}$ , where  $k_{TM0}$  is the wavenumber of the fundamental  $TM_0$  surface wave on the grounded dielectric slab (it is assumed here that only this surface wave is above cutoff). The branch points and corresponding Sommerfeld (hyperbolic) branch cuts are shown in Fig. 2. The path of integration  $C_z$  in (1) is along a Sommerfeld path, staying above the singularities in the right-half plane and below the singularities in the left-half plane. For lossless structures, the branch points are on the real  $k_z$  axis. Also appearing on the real axis in the lossless case are bound-mode poles at  $k_z = \pm k_z^{BM}$ . The wavenumber  $k_z^{BM}$  is that of the bound (non-radiating) mode of propagation on the microstrip line.

The path of integration may be deformed into a path  $C_{BM}$  around the bound-mode pole together with a path  $C_b$  around the branch cuts in the fourth quadrant (see Fig. 2). In this way the total strip current excited by the source is decomposed into a bound-mode (BM) current and a continuous-spectrum (CS) current. The residue contribution at the bound-mode pole defines the BM current amplitude that is launched by the source, while the branch-cut integration defines the CS current. The BM current is the current of the desired transmission-line mode, while the CS current is a radiating type of current that is responsible for spurious effects that generally increase with frequency. In the lossless case the BM current will always dominate at large distances from the source (since there is no attenuation), although the CS current may be strong for distances close to the source, especially at high frequency.

### APPROXIMATE FORMULAS FOR THE BOUND-MODE EXCITATION

The reciprocity theorem may be used to derive simple formulas for the bound mode that get launched by either the gap voltage source or the vertical probe source, in the limiting case of a narrow gap or probe. A summary of these formulas is given below.

#### Gap Voltage Excitation

For a narrow gap source, the BM current for the mode launched in the  $+z$  direction for  $z > 0$  is

$$I_{BM}(z) = \left( \frac{V_g}{2Z_0^{PI}} \right) e^{-jk_z^{BM} z}, \quad (2)$$

where  $Z_0^{PI}$  is the characteristic impedance of the line defined from a power-current definition. That is, using the gap voltage  $V_g$  along with  $Z_0^{PI}$  predicts the correct BM current at any frequency, regardless of the amplitude of the CS current. This is in agreement with the conclusion from [5]. If the gap source has a finite width  $\Delta$  and it is assumed that the impressed field  $E_z$  within the gap region has the form

$$E_z(z) = \frac{-V_g}{\pi \sqrt{(\Delta/2)^2 - z^2}}, \quad (3)$$

then the result in (2) is modified to become (with  $J_0$  being the Bessel function of order zero)

$$I_{BM}(z) = \left( \frac{V_g}{2Z_0^{PI}} \right) J_0(k_z^{BM} \Delta/2) e^{-jk_z^{BM} z}. \quad (4)$$

#### Probe Excitation

The probe is centered at  $(x, z) = (x_0, 0)$  and has a radius  $a$  ( $x_0$  is zero in Fig. 1b). If we assume that the probe radius is very small so that there is no variation in the surface current density azimuthally around the perimeter, and also that the substrate thickness  $h$  is small so that the probe current  $I_p(y) = I_0$  is constant in the vertical direction, the reciprocity method shows that the BM current launched by the probe is

$$I_{BM}(z) = I_0 \left( \frac{Z_0^{VI}}{2Z_0^{PI}} \right) e^{-jk_z^{BM} z}, \quad (5)$$

where  $Z_0^{VI}$  is the characteristic impedance defined using a voltage-current definition, with the voltage being calculated at  $(x_0, 0)$ . Other models for the vertical current variation may be used to improve the accuracy for thick substrates. A cosine model assumes that the probe current has the form

$$I_p(y) = I_0 \cos[k_1(y-h)], \quad (6)$$

where  $k_1$  is the wavenumber inside the substrate and  $I_0$  is the probe current at the attachment point with the strip.

A more sophisticated approximation for the probe current is to assume that the vertical variation of the current on the probe is that corresponding to the same probe inside of an infinite parallel-plate waveguide, excited by a coaxial magnetic frill, where the magnetic surface current of the frill has the same form as the radial electric field of the TEM mode of the corresponding coaxial cable. In this case the probe current is given by

$$I_p(y) = \frac{I_0}{S_A} \sum_{n=0}^{\infty} A_n \cos(k_{yn}y) \quad (7)$$

with

$$S_A = \sum_{n=0}^{\infty} A_n (-1)^n \quad (8)$$

and

$$A_n = \frac{H_0^{(2)}(k_{\rho n}b) - H_0^{(2)}(k_{\rho n}a)}{(1 + \delta_{n0})k_{\rho n}^2 H_0^{(2)}(k_{\rho n}a)}, \quad (9)$$

where  $a$  and  $b$  are the inner and outer radii of the frill,  $k_{\rho n} = \sqrt{k_1^2 - k_{yn}^2}$ ,  $k_{yn} = n\pi/h$ ,  $\delta_{n0}$  is the Kronecker delta function (0 for  $n \neq 0$ , 1 for  $n = 0$ ), and  $H_0^{(2)}$  is the zero-order Hankel function of the second kind.

For either model, the BM current for  $z > 0$  excited by a narrow probe is then given by

$$I_{BM}(z) = \left( \frac{Z_0^{VI}}{2Z_0^{PI}} \right) \left( \frac{\int_0^h I_p(y) E_z^{BM}(x_0, y, 0) dy}{\int_0^h E_z^{BM}(x_0, y, 0) dy} \right) e^{-jk_z^{BM}z}, \quad (10)$$

where  $E_z^{BM}(x_0, y, 0)$  is the electric field of the bound mode as a function of  $y$ , evaluated at  $x = x_0$  and  $z = 0$ .

Numerical results (omitted here) for a narrow probe verify the accuracy of the above formulas. Additional numerical results (omitted here) are also obtained for the case of a finite-radius probe, whose radius is not necessarily small compared to the line width.

## EFFECTS OF THE CONTINUOUS-SPECTRUM CURRENT

Results (omitted here) show how the CS current becomes increasingly important as the frequency increases. This results in considerable interference between the CS and BM currents at high frequency, which results in significant oscillations that are observed in the total current on the microstrip line when plotted versus the distance  $z$  from the source.

## EFFECTS OF LOSS

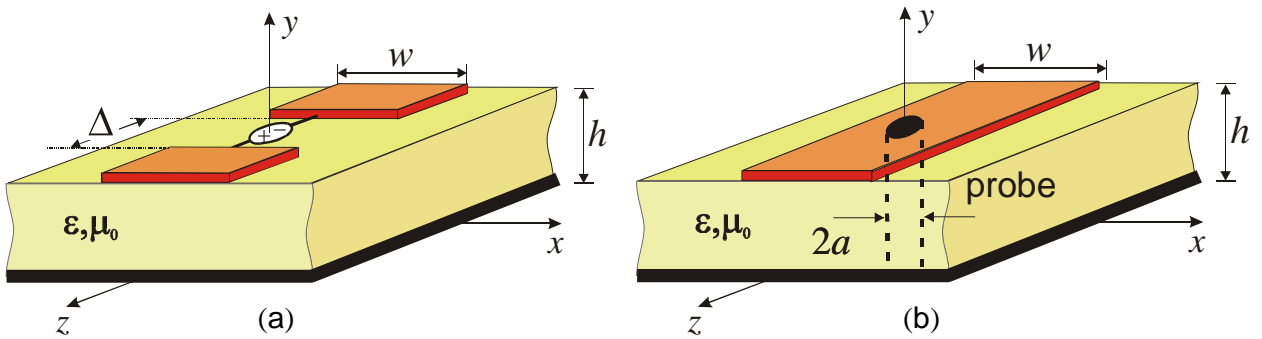
Numerical results (omitted here) show that when loss is added, either to the substrate or to the conductors, the BM current attenuates exponentially, as expected. The CS current does not attenuate exponentially, however, but algebraically. Therefore, at large distance from the source, the CS current will always be the dominant one in the lossy case, in contrast to the lossless case where the BM current is always the dominant one far away from the source.

## CONCLUSIONS

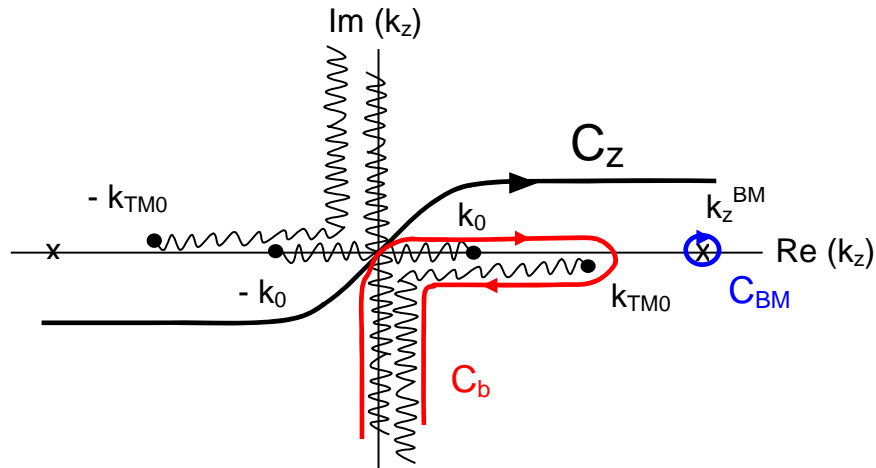
The high-frequency excitation of a microstrip line by a gap voltage source or a vertical probe has been studied. Approximate formulas for the bound-mode current excited on the line, which are accurate at any frequency, have been given for simple limiting cases. Results verify these formulas, and also show the importance of the continuous-spectrum current at high frequency. Results also show how loss on the microstrip line affects the bound-mode and continuous-spectrum currents differently.

## REFERENCES

- [1] C. DiNallo, F. Mesa, and D. R. Jackson, "Excitation of leaky modes on multilayer stripline structure," *IEEE Trans. Microwave Theory Tech.*, vol. 46, no. 8, pp. 1062-1071, Aug. 1998.
- [2] R. Rodriguez-Berral, F. Mesa, and D. R. Jackson, "High-frequency excitation of a microstrip line by a probe," *IEEE Intl. Microwave Symp.*, Long Beach, CA, June 12-17, 2005.
- [3] F. Mesa, C. Di Nallo, and D. R. Jackson, "The theory of surface-wave and space-wave leaky mode excitation on microstrip lines," *IEEE Trans. Microwave Theory and Tech.*, vol. 47, pp. 207-215, Feb. 1999.
- [4] D. C. Chang and E. F. Kuester, "An analytic theory for narrow open microstrip," *AEU*, Band 33, Heft 5, pp. 199-206, 1979.
- [5] E. F. Kuester, D. C. Chang, and L. Lewin, "Frequency-dependent definitions of microstrip characteristic impedance," *URSI EM Theory Symp.*, pp. 335 B/1 – 335 B/3, Munich, March 26-29, 1980.



**Fig. 1.** (a) An infinite microstrip line excited by a gap voltage source of length  $\Delta$ . (b) An infinite microstrip line excited by a vertical probe current of radius  $a$ .



**Fig. 2.** The complex  $k_z$  plane for an open microstrip line. Branch points are shown at  $\pm k_0$  and  $\pm k_{TM0}$ , as well as bound-mode poles at  $\pm k_z^{BM}$ . The original path of integration  $C_z$  is shown along with the path  $C_{BM}$  that defines the bound-mode current and the path  $C_b$  around the branch cuts that defines the continuous-spectrum current.