# Network Time Domain Transmission Line Representation for Short-Pulse Radiation by Periodic Phased Arrays

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**Abstract**

We are in the process of performing an effective wide-band analysis for characterizing the electrodynamic behavior of phased array antennas, infinite periodic structures, frequency selective surfaces and related applications, with emphasis on gaining physical insight into the phenomenology of short-pulse radiation. The present contribution shows the current status of our network-oriented *dyadic* TD GF for a planar array of sequentially excited dipoles that constitutes a prototype study of sequentially short-pulsed radiation by infinite periodic arrays. In order to mitigate the dispersive effect of the TD-Floquet waves we will also consider alternative eigenvector formulations for the transverse field within a periodic cell of the array that yield a more favorably-computable TD-Transmission Line -Green's function.

## I. Introduction

Wide-band radiation by periodic arrays of sequentially pulse-excited antennas, or pulse-induced radiation by passive scatterers, suggests parameterization in terms of wave constituents in the time domain (TD), which is better matched to the relevant phenomenologies than its frequency domain (FD) counterpart. The purpose of this paper is to lay the foundation for TD field representations and their physical interpretations. In particular, we aim at a TD network formulation that may lead to efficient representations of the vector electromagnetic field radiated by phased arrays. We have previously investigated canonical TD dipole-excited Green's functions (GF) for infinite [1] and truncated [2] periodic line arrays, and for infinite [3] and semi-infinite [4] periodic planar arrays. The radiated field there has been expressed and parameterized in terms of TD Floquet waves (FW). The resulting *scalar* TD GF has already been used advantageously to construct a fast TD method of moments algorithm for wide-band analysis of infinite periodic structures [5]. The current status of a network-oriented *dyadic* TD GF is presented here for a planar array of sequentially excited dipoles that constitutes a prototype study of sequentially short-pulsed radiation by infinite periodic arrays.
A principal feature of the network-oriented approach is that $E$ $(TM)$ and $H$ $(TE)$-type TD-FW modes can be separated and treated individually. The field is expressed in terms of TD transmission line (TL) Green's functions that obey standard network theory. Therefore, possible infinite planar vertically inhomogeneous media may readily be incorporated into the formalism. Interesting causality issues accompany such $E$ and $H$ mode decompositions: it is found that individually, each $E$ and $H$ mode is *noncausal*, and that it can be expressed in closed form in terms of a convolution between characteristic noncausal functions and the causal TL GFs. Causality on the total TD-FW vector mode field is recovered by summing the $E$ and $H$ mode contributions. In order to parameterize the TD-FW behavior, we begin with the solution in the FD, with subsequent inversion to the TD. Asymptotic inversion from the FD yields the instantaneous frequencies which parameterize the constituent TD-FWs. The localization of the synthesizing wide-band frequency spectrum around instantaneous frequencies is due to the periodicity-induced dispersive FW behavior. The formal aspects of the analysis follow the traditional lines in [6], to which we refer frequently. Thus, the transverse $p, q$th vector FW mode fields are expressed in terms of transverse FW-mode scalar eigenfunctions. The longitudinal fields are described by voltage and current TL GFs, $Z_{pq}$ and $T^{I}_{pq}$; in the TD, these TL-GFs can be evaluated in terms of Bessel functions and incomplete Lipschitz-Hankel integrals [7]. Numerical examples of radiation from infinite planar arrays of dipoles with short-pulse band-limited excitation demonstrate the accuracy of the TD-FW algorithm, and illustrate the rapid convergence of the (TD-FW)-based field representation since only a few terms are required for describing the off-surface field radiated by the planar array. Results for the nonphased case have already been used in a combined (TD-FW)-FDTD algorithm, shown in [8], for the analysis of periodic arrays of complex scatterers.

## II. Statement of the Problem

We consider the generic infinite periodic array geometry shown in Fig.1a, with periodicities $d_x$ and $d_y$ along the $x$ and $y$ directions, respectively; the corresponding TD transmission line (TL) representations for the FW-based modal fields and Green's functions are schematized in Figs.1b and 1c, respectively. Concerning

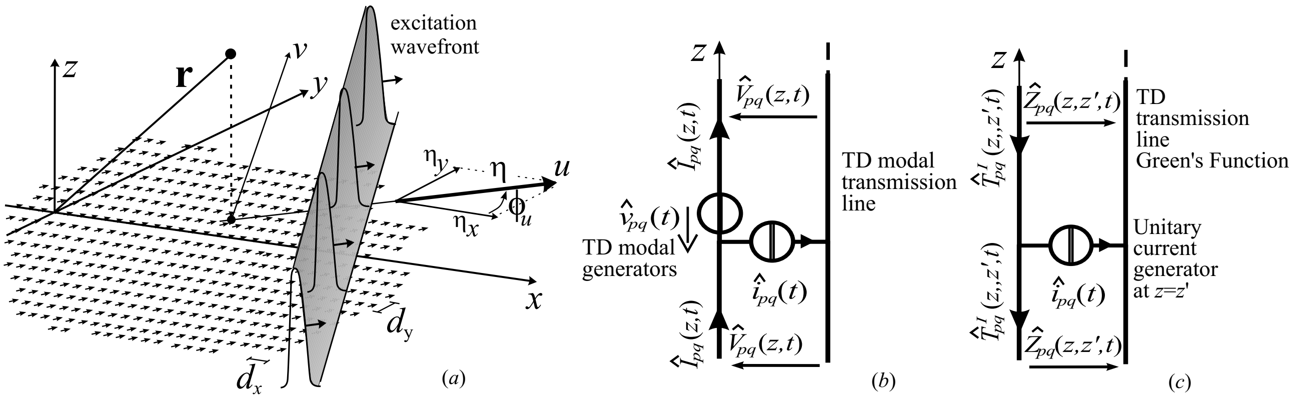


Fig. 1. Generic planar periodic array geometry of elementary radiators, and TD-TL schematizations of the FW-based modal fields and Green's functions. (a) Array geometry. $d_x$, $d_y$: interelement spacing along $x$ and $y$, respectively; $\eta\omega/c = k\eta$: phase gradient of the excitation (i.e., the wavefront) along the direction $\mathbf{1}_u$(see [3]); $v_u^{(p)} = c/\eta$: phase speed along $\mathbf{1}_u$. (b) Equivalent TD transmission line for $pq$th FW, with modal voltage $\hat{V}_{pq}$ and current $\hat{I}_{pq}$, excited by modal voltage and current sources $\hat{v}_{pq}$ and $\hat{i}_{pq}$. (c) TD transmission line voltage and current Green's functions $\hat{Z}(z, z', t)$ and $T^I(z, z', t)$ excited by a unit *current* generator at $z = z\prime$.

notation, a caret ˆ tags time-dependent quantities; bold face symbols define vector quantities; $\mathbf{1}_x$, $\mathbf{1}_y$ and $\mathbf{1}_z$ denote unit vectors along $x$, $y$, and $z$, respectively; the observation point is denoted by $\mathbf{r} = \boldsymbol{\rho} + z\mathbf{1}_z$, with $\boldsymbol{\rho} = x\mathbf{1}_x + y\mathbf{1}_y$. The FW-based modal FD and TD fields due to the array are related by the Fourier transform pair $f(\mathbf{r}, \omega) = \int_{-\infty}^{\infty} \hat{f}(\mathbf{r}, t)e^{-j\omega t}dt$, $\hat{f}(\mathbf{r}, t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} f(\mathbf{r}, \omega)e^{j\omega t}d\omega$, in which $f$ can be either a scalar or a vector quantity. These Fourier -transform-related fields satisfy the respective FD and TD periodicity conditions

$$\mathbf{E}(\mathbf{r} + \mathbf{d}, \omega) = \mathbf{E}(\mathbf{r}, \omega)e^{-j\eta(\omega/c)(\mathbf{1}_u \cdot \mathbf{d})}, \qquad \hat{\mathbf{E}}(\mathbf{r} + \mathbf{d}, t) = \hat{\mathbf{E}}[\mathbf{r}, t - \eta(\mathbf{1}_u \cdot \mathbf{d})/c] \tag{1}$$

where $\mathbf{d} = d_x\mathbf{1}_x + d_y\mathbf{1}_y$, $k = \omega/c$ denotes the ambient wavenumber and $c$ denotes the ambient wavespeed. In the FD, the composite linear phasing on the array is along the direction $\mathbf{1}_u$, perpendicular to $\mathbf{1}_v = \mathbf{1}_z \times \mathbf{1}_u$ (see Fig.1a), with *projected* phasings $\omega\eta_x/c$ and $\omega\eta_y/c$ along the $x$ and $y$ directions, respectively. In the TD, this translates into intercell excitation delayed by $\eta(\mathbf{1}_u \cdot \mathbf{d})/c$. The important nondimensional *single* parameter $\eta = \sqrt{\eta_x^2 + \eta_y^2} = c/v_u^{(p)}$, which is tied to the rotated coordinate system defined by $u$ (see Fig.1), combines both phasings $\eta_x$ and $\eta_y$, and $v_u^{(p)} = c/\eta$ is the impressed phase speed along $u$. The TD cutoff condition $\eta = 1$ ($v_u^{(p)} = c$) separates two distinct wave dynamics. Here, we treat the case $\eta < 1$ which implies excitation phase speeds $v_u^{(p)} = c/\eta$ (and corresponding *projected* phase speeds $c/\eta_x$ and $c/\eta_y$) larger than the ambient wavespeed $c$.

## III. Time-Domain Modal Representation of the FW-based Fields and Their Sources: TD Modal Transmission Line Fields and Green's Functions

The TD-TL representations are here obtained by Fourier-transforming their FD counterparts such as $\mathbf{E}_t(\mathbf{r}, \omega) = \sum_{p,q} V_{pq}(z, \omega)\mathbf{e}_{pq}(\boldsymbol{\rho}, \omega)$, where $V_{pq}(z, \omega)$ is the associated TL voltage, and $\mathbf{e}_{pq}(\boldsymbol{\rho}, \omega)$ is the transverse eigenvector. Thus, the transverse (to $z$) TD field is expressed in terms of a time-dependent complete orthogonal eigenvector set comprising both $E$ (TM) and $H$ (TE) mode functions $\hat{\mathbf{e}}_{pq}(\boldsymbol{\rho}, t)$ and $\hat{\mathbf{h}}_{pq}(\boldsymbol{\rho}, t)$,

$$\hat{\mathbf{E}}_t(\mathbf{r}, t) = \sum_{p,q} \hat{V}_{pq}(z, t) \otimes \hat{\mathbf{e}}_{pq}(\boldsymbol{\rho}, t), \qquad \hat{\mathbf{H}}_t(\mathbf{r}, t) = \sum_{p,q} \hat{I}_{pq}(z, t) \otimes \hat{\mathbf{h}}_{pq}(\boldsymbol{\rho}, t), \tag{2}$$

where $\otimes$ denotes time convolution and the summation extends over both $E$ and $H$ modes. As in the FD case, excitations by TD modal sources $\hat{\mathbf{J}}_{te}(\mathbf{r}, t)$ and $\hat{\mathbf{M}}_{te}(\mathbf{r}, t)$ are likewise represented in terms of this eigenbasis, with amplitudes given by the strengths $\hat{i}_{pq}$ and $\hat{v}_{pq}$ of current and voltage generators: $\hat{\mathbf{J}}_{te}(\mathbf{r}, t) = \sum_{p,q} \hat{i}_{pq}(z, t) \otimes$

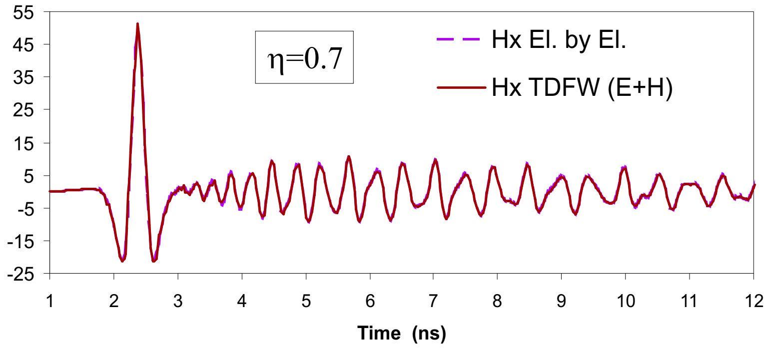


Fig. 2. $\hat{H}_x(\mathbf{r},t)$ component of the magnetic field radiated by an infinite sequentially pulsed planar array of $y$-directed dipoles with $\eta = 0.7$, observed at $\mathbf{r} = (x, y, z) = (0, 0, 10d_x)$ as a function of time $t$. Parameters: $d_x = d_y = 0.1m$; $\lambda_M = 2d_x$. The TD-FW expansion consists of all FWs with $|p|, |q| \le 2$, and agrees well with the element-by-element reference solution.

$\hat{\mathbf{e}}_{pq}(\boldsymbol{\rho}, t)$, $\hat{\mathbf{M}}_{te}(\mathbf{r}, t) = \sum_{p,q} \hat{v}_{pq}(z,t) \otimes \hat{\mathbf{h}}_{pq}(\boldsymbol{\rho}, t)$. The eigenfunctions used in the FD are transformed into the TD with a proper normalization, leading to the closed form

$$\hat{\mathbf{e}}^E_{pq}(\boldsymbol{\rho}, t) = \frac{e^{-j\boldsymbol{\alpha}_{pq}\cdot\boldsymbol{\rho}}}{\sqrt{d_x d_y}} \left\{ \frac{\eta}{c} \mathbf{1}_u \delta'(\tau) + j\boldsymbol{\alpha}_{pq}\delta(\tau) \right\}, \qquad \tau = t - \eta \mathbf{1}_u \cdot \boldsymbol{\rho}/c, \tag{3}$$

together with $\hat{\mathbf{e}}^H_{pq}(\boldsymbol{\rho}, t) = \hat{\mathbf{e}}^E_{pq}(\boldsymbol{\rho}, t) \times \mathbf{1}_z$, where $\boldsymbol{\alpha}_{pq} = \alpha_{x,p}\mathbf{1}_x + \alpha_{y,q}\mathbf{1}_y$, and $\alpha_{x,p} = 2\pi p/d_x$, $\alpha_{y,q} = 2\pi q/d_y$. Thus, the eigenfunctions are pulsed FW-modulated slant-stacked plane wavefronts. The magnetic mode functions are given by $\hat{\mathbf{h}}_{pq} = \mathbf{1}_z \times \hat{\mathbf{e}}_{pq}$. Expression (3) is particularly convenient since the Dirac delta functions reduce the convolutions in (2) to closed-form sampling of the voltages and currents (and their derivatives) at the retarded time $\tau = t - \eta \mathbf{1}_u \cdot \boldsymbol{\rho}/c$ (see [1], [3] for a physical interpretation in terms of a moving coordinate system). Other normalizations, in particular the standard *orthonormal* choice which has a factor $k_{t,pq}$ in the denominator of (3), do not simplify the convolution and therefore require numerical techniques for evaluation.
For both $E$ and $H$ modes, the TD-TL voltages $\hat{V}_{pq}(z,t)$ in (2) are obtained by superposing contributions from appropriate *point* voltage and current generators distributed along $z'$: $\hat{V}_{pq}(z,t) = -\int dz' \hat{T}^V_{pq}(z,z',t) \otimes \hat{v}_{pq}(z',t) - \int dz' \hat{Z}_{pq}(z,z',t) \otimes \hat{i}_{pq}(z',t)$. Solutions for the TD-TL Green's functions $\hat{Z}_{pq}(z,z',t)$ and $\hat{T}^V_{pq}(z,z',t)$, excited by delta function current and voltage generators at $z'$ (see Figs.1b,c), are found via Fourier inversion from the FD solutions in [6, pp.207], which can be evaluated in closed forms in terms of Hankel functions and Incomplete Lipschitz-Hankel Integrals (see [7], and [9],[10],[11] for use of Incomplete Lipschitz-Hankel Integrals ). The TD modal voltage and current generator strengths $\hat{v}_{pq}(t)$ and $\hat{i}_{pq}(t)$ are found as before by projecting the total equivalent electric and magnetic TD currents onto each FW mode: $\hat{v}_{pq}(z,t) =< \hat{\mathbf{M}}_{te}(\mathbf{r},t); \hat{\mathbf{e}}^\dagger_{pq}(\boldsymbol{\rho},t) >_t$, $\hat{i}_{pq}(z,t) =< \hat{\mathbf{J}}_{te}(\mathbf{r},t); \hat{\mathbf{e}}^\dagger_{pq}(\boldsymbol{\rho},t) >_t$, in which the subscript $_t$ denotes time convolution in the inner product. The eigenfunction $\hat{\mathbf{e}}^\dagger_{pq}(\boldsymbol{\rho},t)$ is defined as the Fourier transform of $k^{-2}_{t,pq}(\omega)\mathbf{e}_{pq}(\boldsymbol{\rho},\omega)$ which can also be evaluated in simple closed form.

For the particular but interesting case of an array of sequentially pulsed dipoles, all oriented along $\mathbf{J}_t$, it may be convenient to group the transverse eigenvector together with the current generator. This procedure leads to (as an example) the electric field

$$\hat{\mathbf{E}}^{E,H}_{t,pq}(\mathbf{r},\mathbf{r}',t) = \hat{\underline{\mathbf{D}}}^{E,H}_E(t) \cdot \mathbf{J}_t \frac{e^{-j\boldsymbol{\alpha}_{pq}\cdot(\boldsymbol{\rho}-\boldsymbol{\rho}')}}{d_x d_y} \otimes \hat{Z}^{E,H}_{pq}(z,z',\tau) \tag{4}$$

which is usefully expressed in terms of the noncausal dyad $\hat{\underline{\mathbf{D}}}^E_E(t) = \hat{a}_{pq}(t)\mathbf{1}_u\mathbf{1}_u + \hat{b}_{pq}(t)(\mathbf{1}_u\mathbf{1}_v + \mathbf{1}_v\mathbf{1}_u) + \hat{c}_{pq}(t)\mathbf{1}_v\mathbf{1}_v$ where $\hat{a}_{pq}(t) = \delta(t) - \hat{c}_{pq}(t)$, $\hat{b}_{pq}(t) = j\,\mathrm{sgn}(t)\,\hat{c}_{pq}(t)$, and $\hat{c}_{pq}(t)$ is a short pulse TD noncausal function that is time-spread around $t = 0$. However, as demonstrated in [7], causality is recovered by summing the $E$ and $H$ constituents.

## IV. **Illustrative Examples**

To check the accuracy of the TD-FW-based network formalism for the pulse-excited planar phased array of dipoles, we have implemented the magnetic field solution analogous to (4), convolved with a BL excitation. The solution is compared here with a reference solution obtained via element-by-element summation over the pulsed radiations from all dipoles, i.e., $\hat{\mathbf{H}}(\mathbf{r},t) = \sum_{m,n=-\infty}^{\infty} \left[\hat{P}'(t-t_{mn})/(4\pi R_{mn}^2 c) + \hat{P}(t-t_{mn})/(4\pi R_{mn}^3)\right](\mathbf{J}_t \times \mathbf{R}_{mn})$, with $\mathbf{R}_{mn} = \mathbf{r} - (\mathbf{r}' + \boldsymbol{\rho}_{mn})$, $t_{mn} = \eta \mathbf{1}_u \cdot \boldsymbol{\rho}_{mn}/c + R_{mn}/c$, and $m,n = 0, \pm 1, \pm 2, .....$ The $mn$-series has been truncated when contributions from far-away elements are negligible, i.e., retaining elements with $|m|,|n| < 80$. The selected BL excitation is a normalized Rayleigh pulse $\widehat{P}(t) = \Re e[j/(j+\omega_M t/4)^5]$ (i.e., $\widehat{P}(0) = 1$ ) [12], with FD spectrum $P(\omega) = \pi(6\omega_M)^{-1}(j4\omega/\omega_M)^4 \exp(-4|\omega|/\omega_M)$ and central radian frequency $\omega_M$, which corresponds to a central wavelength $\lambda_M = 2\pi c/\omega_M$. We present here only the magnetic field results because the electric field evaluation would require the more involved TL-GF in $\hat{Z}_{pq}^E(z,z',\tau)$. Consider an array of dipoles oriented along $\mathbf{1}_y$, with periods $d_x = d_y = 0.1m$. The exciting waveform is chosen such that $\lambda_M = 2d_x$, so that the the average length of the pulse is twice the period of the array (see [3] for more details). Results for a phased case ($\eta = 0.7$ along the direction $\mathbf{1}_u \equiv \mathbf{1}_x$) are displayed in Fig.2. The magnetic field $\hat{H}_x$ is observed at the location $(x,y,z) = (0,0,10d_x)$, versus time $t$. It is remarkable that TD-FWs with only $|p|,|q| \leq 2$ are adequate to represent the TD radiated field at any time $t$ in the plotted time interval. The agreement with the element-by-element reference solution is excellent (as a further check on the numerics, both the TD-FW expansion and the element-by-element reference solution yield a negligible $\hat{H}_y$ component). The total magnetic field is obtained by numerically summing its $E$ and $H$-mode constituents; for the phased case $\eta = 0.7$, it is seen that this sum cancels the small noncausal components, and renders the total signal causal.

## V. **Conclusions**

A Network-Oriented Dyadic Green's function has here been investigated for a planar infinite periodic array of sequentially BL-pulse-excited dipoles. Via the network-oriented approach, the $E$ ($TM$) and $H$ ($TE$) mode contributions can be separated and treated individually in a systematic fashion. The thus reduced modal field is expressed in terms of transmission line (TL) Green's functions that behave according to standard network theory. Therefore, possible infinite planar transversely homogeneous layers with longitudinal inhomegeneities can be readily incorporated within the formalism. It has been found that individually, each TD-FW $E$ and $H$ mode is noncausal and can be obtained in closed form in terms of a convolution between characteristic noncausal dyadic functions and the causal TL Green's function. Causality of the total mode field is recovered in the $E$ and $H$ mode sum. The total radiated field can be constructed at any location and at any time within our numerical experiments by retaining only a few TD-FWs. It should also be noted that "physically observable" $pq$th TD-FWs are synthesized by $(+p,+q)$, $(-p,-q)$ superposition.

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