EFFICIENT COMPUTATION OF SURFACE FIELDS EXCITED ON AN ELECTRICALLY LARGE CIRCULAR CYLINDER WITH AN IMPEDANCE BOUNDARY CONDITION

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ABSTRACT

An alternative numerical approach is presented for the evaluation of the Fock type integrals which exist in the Uniform Geometrical Theory of Diffraction (UTD) based asymptotic solution for the surface fields excited by a magnetic or an electric source located on the surface of an electrically large circular cylinder with an impedance boundary condition (IBC). This alternative approach is based on numerical integration for the Fock type integrals on a deformed path on which the integrands are non oscillatory and rapidly decaying. Comparing this approach with the previously developed one presented in [1] reveals that the alternative approach requires less computational time and is easier to implement.

INTRODUCTION

Many military and commercial applications (e.g. missiles, mobile base stations, transreceivers of Multi Input Multi Output (MIMO) systems, etc.) have stringent aerodynamic constraints that require the use of antennas that conform to their host platforms. This requires the development of efficient and accurate design and analysis tools for this class of antennas. Therefore, the study of surface fields, created by a current distribution on the surface of a thin material coated (lossy or lossless) perfect electric conducting (PEC) circular cylinder using an impedance boundary condition (IBC) has been a subject of interest due to its applications in the analysis of slot/aperture antennas as well as antennas on partially coated host platforms for many years [1]-[5]. Furthermore, it acts as a canonical problem useful toward the development of asymptotic solutions valid for arbitrary smooth convex thin material coated/partially material coated surfaces.

In this paper, we concentrate on the evaluation of the Uniform Theory of Diffraction (UTD) based asymptotic representation of the appropriate Green's function pertaining to the surface fields excited by a magnetic/electric current source located on the surface of an electrically large circular impedance cylinder, which is assumed to be infinitely long along its axial direction. The UTD-based solution for the surface fields of such a geometry involves some Fock type integrals which have to be evaluated numerically. Since the numerical evaluation of these Fock type integrals (and their derivatives) strongly affects the efficiency and accuracy of the solution, an alternative numerical approach to the previously developed one presented in [1] is offered. In [1], these integrals are evaluated by invoking the Cauchy's residue theorem via finding the pole singularities numerically. In this study, a numerical integration is performed along a deformed path on which the integrands are non oscillatory and rapidly decaying. During the path deformation, certain parameters are determined carefully to guarantee that all pole singularities are captured. Furthermore, this alternative approach is easier to implement and requires less computational time compared to the approach presented in [1].

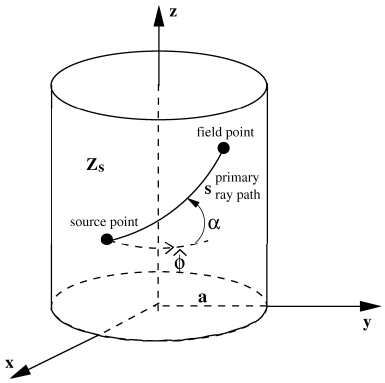

Figure 1: Geometry of a circular cylinder with a radius a

UTD SOLUTION

Consider an electrically large circular cylinder with an IBC as shown in Fig.1. The cylinder has a radius a, a uniform surface impedance Z_s, and is assumed to be infinitely long along its axial direction. For such a cylinder, the tangential surface field excited by a tangential magnetic source

$$\vec{P}_m = P_m^z \hat{z} + P_m^\phi \hat{\phi} \tag{1}$$

located on the surface is expressed in [1] as

$$\vec{H}_{t_\ell}^{\pm} = \vec{P}_m \cdot (\hat{z}'\hat{z}G_{zz}^{\ell\pm} + \hat{\phi}'\hat{z}G_{z\phi}^{\ell\pm} + \hat{z}'\hat{\phi}G_{\phi z}^{\ell\pm} + \hat{\phi}'\hat{\phi}G_{\phi\phi}^{\ell\pm}) \tag{2}$$

where $G_{pq}^{\ell\pm}$ is a UTD based Green's function for a $\hat{p}$ oriented surface magnetic field due to a $\hat{q}$ directed magnetic current. Note that $G_{pq}^{\ell+}$ pertain to the Green's function which is responsible from the surface waves propagating around the cylinder in the positive $\hat{\phi}$ direction, whereas $G_{pq}^{\ell-}$ corresponds to those propagating in the negative $\hat{\phi}$ direction. The generic form of the Green's function representations for various source and field orientations given in [1] can be written as

$$G_{pq}^{\ell\pm} \sim G_0 f(k, \alpha, s, \Upsilon_r). \tag{3}$$

In (3), $G_0 = -\frac{jke^{-jks}}{2\pi Z_0 s}$, k is the free space wave number, Z_0 is the free space impedance, s is the geodesic ray path, α is the angle between s and the positive $\hat{\phi}$ direction as shown in Fig.1. Finally,

$$\Upsilon_r = \int_{-\infty}^{\infty} d\tau e^{-j\xi\tau} \frac{(R_w)^r}{D_w} \qquad r = 0, 1, 2 \tag{4}$$

are the Fock type integrals.

Using a procedure similar to the one presented in [1], the tangential surface field excited by a tangential electric source can be derived for the same geometry.

NUMERICAL EVALUATION OF SURFACE FIELDS

Numerical Approach 1

This approach is based on performing a residual series expansion to each Fock type integral and invoking the Cauchy's residue theorem, which requires numerically finding the corresponding pole singularities. The poles are located in the fourth quadrant of the complex τ-plane. In general, the first 20 terms of this series expansion are included to obtain accurate results as explained in [1].

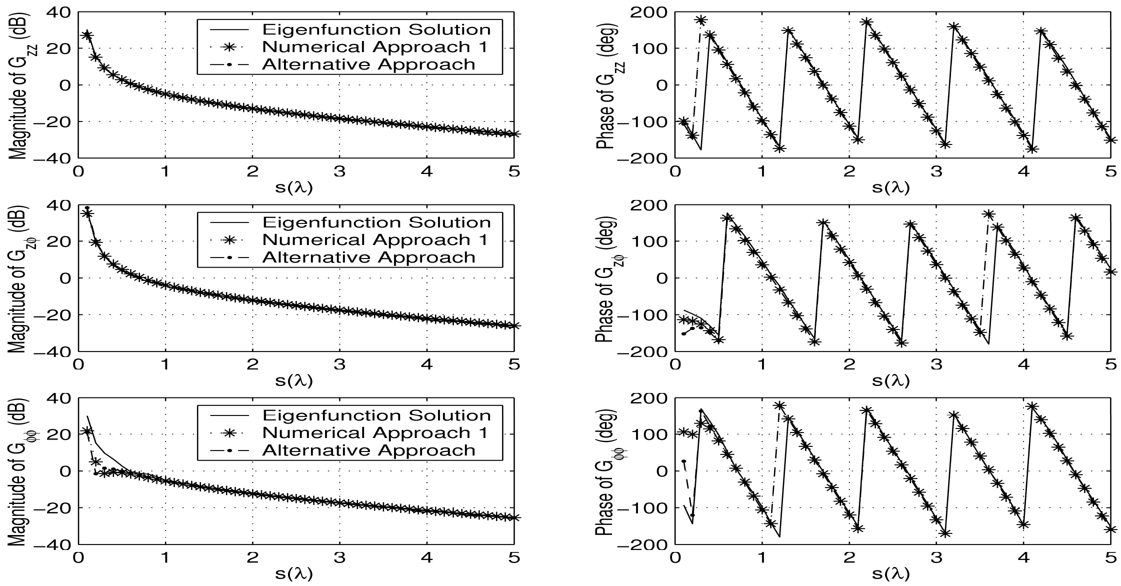

Figure 2: Comparison of the magnitude (in dB) and phase of the various components of Green's function representations versus separation, s, for a magnetic source obtained by the eigenfunction solution and the numerical approaches for $f = 7GHz$, $a = 5\lambda$, $\alpha = 45°$ and $\Lambda = 0.1$.

The Alternative Approach

This approach is based on performing a numerical integration for the Fock type integrals. The original integration path for the Fock type integrals given by (4) ranges from $-\infty$ to ∞ on the complex τ-plane. Unfortunately, the integrals may not converge rapidly when this path is used since the integrands have a highly oscillatory and slowly convergent behavior on this path. Therefore, these integrals are evaluated on a deformed path similar to the one in [6]-[7]. Note that, similar deformed paths have been previously used for the coated cylinder case in [8]-[10]. Each integral is split into three integrals ranging from $(-\infty, 0)$, $(0, k_{big})$, and (k_{big}, ∞), respectively, where k_{big} is chosen approximately $1.5k$. Then, the integration path for the first and third integrals are deformed to $(\infty e^{-j2\pi/3}, 0)$, and $(k_{big}, \infty e^{-j\pi/3})$, respectively. The choice of $k_{big} = 1.5k$ guarantees that all pole singularities are captured. Therefore, in principle, this approach is more accurate than the numerical approach 1. Finally, the integration variable τ is changed to $\tau e^{j2\pi/3}$ for the first integral, and to $(\tau - k_{big})e^{j\pi/3}$ for the third integral resulting the Airy function and its derivative to be non oscillatory and decay most rapidly (an exponential decay is achieved) as $|\tau| \to \infty$ along the path where $arg(\tau) = 0$ [10]. Consequently, the first and the third integrals now range from 0 to ∞, they are fast decaying, non oscillatory and hence, they can be integrated efficiently using a simple Gaussian quadrature algorithm.

NUMERICAL RESULTS AND DISCUSSIONS

Numerical results for the surface fields are obtained using the aforementioned approaches and compared with the eigenfunction solution to assess and compare their accuracy and efficiency. On a circular cylinder with a radius 5λ, and a normalized surface impedance $\Lambda = Z_s/Z_0 = 0.1$, for a fixed azimuthal angle($\alpha = 45°$) various components of Green's function representations are computed and plotted in Fig.2 for the geodesic path length varying from 0.1λ to 5λ at $f = 7GHz$.

Both approaches are very accurate and much faster than the eigenfunction solution as expected. It should be noted, however, the alternative approach has several advantages over numerical approach 1. Firstly, performing a numerical integration along the deformed contour is significantly faster than the residual series expansion approach as it is seen from Table I. Secondly, locating the poles requires a difficult and complex procedure. One can easily miss a pole, or pole search algorithm needs to be modified for some geometries and physical parameters. Finally, a finite number of poles are taken into consideration in numerical approach 1 whereas all poles are included in this approach.

TABLE I. COMPUTATIONAL TIME

	Numerical Approach 1		Alternative Approach	
	Magnetic source	Electric source	Magnetic source	Electric source
G_{zz}	92.4 sec.	92.5 sec.	38.7 sec.	38.5 sec.
$G_{z\phi}$	92.5 sec.	93.1 sec.	38.6 sec.	56.6 sec.
$G_{\phi\phi}$	92.7 sec.	92.6 sec.	56.9 sec.	38.4 sec.

CONCLUSION

An alternative numerical approach is presented for the evaluation of UTD based asymptotic solution for the surface fields excited by a magnetic or an electric source located on the surface of an electrically large circular cylinder with an IBC. This alternative approach is based on performing a numerical integration for the Fock type integrals on a deformed path, which is the major burden in the evaluation of the UTD solution. The accuracy and efficiency of this approach is compared with the previously developed one presented in [1], which is based on invoking the Cauchy's residue theorem by finding the pole singularities numerically. They both yield accurate results as they are compared with the eigenfunction solution in [1]. However, the alternative approach has several advantages over the first approach such as having an easier formulation and less computational time. Having these advantages makes this approach more appealing than the numerical approach 1 for the evaluation of the UTD based asymptotic solution for the surface fields excited on an electrically large circular cylinder with an IBC.

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