EIGENFUNCTION EXPANSIONS OF SOURCE-EXCITED ELECTROMAGNETIC FIELDS IN UNBOUNDED GYROTROPIC ANISOTROPIC MEDIA

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ABSTRACT

We consider electromagnetic fields excited by spatially bounded, arbitrary given sources in the presence of a uniform gyrotropic cylinder surrounded by an infinitely extended homogeneous gyrotropic medium. The axis of symmetry of the considered cylindrical structure is assumed to coincide with the gyrotropic axis. The total field is sought in terms of the vector modal solutions of source-free Maxwell's equations. We determine the content of the modal spectrum and obtain an eigenfunction expansion of the source-excited field in terms of discrete- and continuous-spectrum modes. We also discuss the case of sources in an unbounded homogeneous gyrotropic medium.

INTRODUCTION

Electromagnetic fields excited by sources in anisotropic media are usually sought using Green's dyadics or Fourier transforms. In the case of gyrotropic anisotropic media described by the permittivity and/or permeability tensors with nonzero off-diagonal elements, neither of these approaches has substantial advantages over each other, since the dyadic Green's functions in such media cannot be expressed in closed forms and are represented in terms of Fourier-type integrals. Moreover, in resonant gyrotropic anisotropic media in which the refractive index surfaces may have unbounded branches, the Green's functions are singular not only at the source point but also on the surfaces of the so-called resonance cones. Because of these difficulties, an alternative approach based on using eigenfunction expansions of source-excited fields turns out to be more suitable for field evaluations in the aforementioned media. Although this approach was proposed for isotropic and anisotropic media long ago (see [1, 2] and references therein), eigenfunction expansions are rarely used for analysis of fields excited by arbitrary sources in unbounded gyrotropic media, which is apparently explained by difficulties of constructing the desired eigenfunctions in closed from. Recent studies of source-excited fields on open magnetized (gyrotropic) plasma waveguides [3] have shown that the systematic use of eigenfunction expansions provides a convenient and easily calculated method of finding the total field of a given source located in a gyrotropic medium.

In this paper, we present a rigorous and concise formulation of the complete eigenfunction expansion of the source-excited field on a uniform cylindrical guiding structure immersed in a gyrotropic anisotropic background medium. The limiting transition to the case of sources located in an unbounded homogeneous gyrotropic anisotropic medium will also be discussed.

FORMULATION OF THE PROBLEM AND BASIC EQUATIONS

Consider time-harmonic ($\sim \exp(i\omega t)$) given electric and magnetic currents with densities $\boldsymbol{j}^{\mathrm{e}}(\rho,\phi,z)$ and $\boldsymbol{j}^{\mathrm{m}}(\rho,\phi,z)$, respectively, in a medium described by the permittivity tensor $\boldsymbol{\varepsilon} = \epsilon_0(\varepsilon_1\hat{\boldsymbol{\rho}}_0\hat{\boldsymbol{\rho}}_0 + i\varepsilon_2\hat{\boldsymbol{\rho}}_0\hat{\boldsymbol{\phi}}_0 - i\varepsilon_2\hat{\boldsymbol{\phi}}_0\hat{\boldsymbol{\rho}}_0 + \varepsilon_1\hat{\boldsymbol{\phi}}_0\hat{\boldsymbol{\phi}}_0 + \varepsilon_3\hat{\boldsymbol{z}}_0\hat{\boldsymbol{z}}_0)$ and the permeability tensor $\boldsymbol{\mu} = \mu_0(\mu_1\hat{\boldsymbol{\rho}}_0\hat{\boldsymbol{\rho}}_0 + i\mu_2\hat{\boldsymbol{\rho}}_0\hat{\boldsymbol{\phi}}_0 - i\mu_2\hat{\boldsymbol{\phi}}_0\hat{\boldsymbol{\rho}}_0 + \mu_1\hat{\boldsymbol{\phi}}_0\hat{\boldsymbol{\phi}}_0 + \mu_3\hat{\boldsymbol{z}}_0\hat{\boldsymbol{z}}_0)$, where ρ, ϕ, and z are cylindrical coordinates and ϵ_0 and μ_0 are the electric and magnetic constants, respectively. Such tensors are typical of a general gyrotropic anisotropic medium with the gyrotropic axis parallel to the z axis. Let the elements of the tensors $\boldsymbol{\varepsilon}$ and $\boldsymbol{\mu}$ vary with ρ in such a way that they undergo a jump at $\rho = a$ and are constant for $\rho < a$ and $\rho > a$, where a is the radius of the cylindrical guiding structure considered. In what follows we denote the elements of the tensors $\boldsymbol{\varepsilon}$ and $\boldsymbol{\mu}$ in an inner region ($\rho < a$) and in an outer medium ($\rho > a$) by $\tilde{\varepsilon}_{1,2,3}$, $\tilde{\mu}_{1,2,3}$ and $\varepsilon_{1,2,3}$, $\mu_{1,2,3}$, respectively. We will consider only the case where $\varepsilon_1^{-1}\varepsilon_2 + \mu_1^{-1}\mu_2 \neq 0$. A peculiar case $\varepsilon_1^{-1}\varepsilon_2 = -\mu_1^{-1}\mu_2$ when the medium has the property of uniaxial anisotropy needs separate consideration and will not be discussed here.

Solutions of source-free Maxwell's equations in such a cylindrically stratified medium can be sought in terms of the vector functions

$$\begin{bmatrix} \boldsymbol{E}_{m,s,\alpha}(\boldsymbol{r},q) \\ \boldsymbol{H}_{m,s,\alpha}(\boldsymbol{r},q) \end{bmatrix} = \begin{bmatrix} \boldsymbol{E}_{m,s,\alpha}(\rho,q) \\ \boldsymbol{H}_{m,s,\alpha}(\rho,q) \end{bmatrix} \exp[-im\phi - ik_0 p_{s,\alpha}(q) z], \tag{1}$$

where k_0 is the wave number in free space, q is the normalized (to k_0) transverse wave number in the outer medium, m is the azimuthal index ($m = 0, \pm 1, \pm 2, \ldots$), the functions $p_{s,\alpha}(q)$ describe the dependence of p, the axial wave number normalized to k_0, on the transverse wave number q for the "ordinary" ($\alpha = \mathrm{o}$) and "extraordinary" ($\alpha = \mathrm{x}$) normal waves of the outer medium, the subscript s denotes the wave propagation direction ($s = -$ and $s = +$ denote waves propagating in the negative and positive directions of the z axis, respectively), and $\boldsymbol{E}_{m,s,\alpha}(\rho,q)$ and $\boldsymbol{H}_{m,s,\alpha}(\rho,q)$ are the vector functions describing the radial distribution of the field of an eigenwave corresponding to the transverse wave number q and the indices m, s, and α. The functions $p_{s,\alpha}(q)$ obey the relation $p_{+,\alpha}(q) \equiv p_\alpha(q) = -p_{-,\alpha}(q)$, where

$$\begin{aligned} p_\alpha(q) &= 2^{-1/2}\left[\tau_{\mathrm{e}}\kappa_{\mathrm{e}}^2 + \tau_{\mathrm{m}}\kappa_{\mathrm{m}}^2 + \tau_{\mathrm{g}}^2\kappa^2 - (\tau_{\mathrm{e}}+\tau_{\mathrm{m}})q^2 + \chi_\alpha R_p(q)\right]^{1/2}, \qquad \chi_{\mathrm{o}} = -\chi_{\mathrm{x}} = -1, \\ R_p(q) &= \left\{(\tau_{\mathrm{e}}-\tau_{\mathrm{m}})^2 q^4 - 2\left[(\tau_{\mathrm{e}}+\tau_{\mathrm{m}})(\tau_{\mathrm{e}}\kappa_{\mathrm{e}}^2 + \tau_{\mathrm{m}}\kappa_{\mathrm{m}}^2 + \tau_{\mathrm{g}}^2\kappa^2) - 2\tau_{\mathrm{e}}\tau_{\mathrm{m}}(\kappa_{\mathrm{e}}^2+\kappa_{\mathrm{m}}^2)\right]q^2 \right. \\ &\quad \left. + (\tau_{\mathrm{e}}\kappa_{\mathrm{e}}^2 + \tau_{\mathrm{m}}\kappa_{\mathrm{m}}^2 + \tau_{\mathrm{g}}^2\kappa^2)^2 - 4\tau_{\mathrm{e}}\tau_{\mathrm{m}}\kappa_{\mathrm{e}}^2\kappa_{\mathrm{m}}^2\right\}^{1/2}. \end{aligned} \tag{2}$$

Here,

$$\tau_{\mathrm{e}} = \frac{\varepsilon_1}{\varepsilon_3}, \quad \tau_{\mathrm{m}} = \frac{\mu_1}{\mu_3}, \quad \tau_{\mathrm{g}} = \frac{\varepsilon_2}{\varepsilon_1} + \frac{\mu_2}{\mu_1}, \quad \kappa_{\mathrm{e}} = \left(\varepsilon_3\frac{\mu_1^2-\mu_2^2}{\mu_1}\right)^{1/2}, \quad \kappa_{\mathrm{m}} = \left(\mu_3\frac{\varepsilon_1^2-\varepsilon_2^2}{\varepsilon_1}\right)^{1/2}, \quad \kappa = (\varepsilon_1\mu_1)^{1/2} \tag{3}$$

(see [4]). It is assumed that $\mathrm{Re}\,R_p(q) > 0$ and $\mathrm{Im}\,p_\alpha(q) < 0$. To find the eigenvalues q, it is required that the functions $\boldsymbol{E}_{m,s,\alpha}(\rho,q)$ and $\boldsymbol{H}_{m,s,\alpha}(\rho,q)$ obtained as a result of solution of Maxwell's equations be regular on the z axis and satisfy both the boundary conditions, which consist in the continuity of the components $E_{\phi;m,s,\alpha}(\rho,q)$, $E_{z;m,s,\alpha}(\rho,q)$, $H_{\phi;m,s,\alpha}(\rho,q)$, and $H_{z;m,s,\alpha}(\rho,q)$ at $\rho = a$, and the following boundedness conditions at $\rho \to \infty$ [3]:

$$\rho^{1/2}\left|\boldsymbol{E}_{m,s,\alpha}(\rho,q)\right| < R_{m,\alpha}^{(1)}, \qquad \rho^{1/2}\left|\boldsymbol{H}_{m,s,\alpha}(\rho,q)\right| < R_{m,\alpha}^{(2)}, \tag{4}$$

where $R_{m,\alpha}^{(1)}$ and $R_{m,\alpha}^{(2)}$ are certain constants. It can be shown that the total field yielded by summing (integrating) eigenwaves over the found values of q satisfies the radiation condition at infinity ($r = (\rho^2+z^2)^{1/2} \to \infty$).

The transverse components of the vector functions $\boldsymbol{E}_{m,s,\alpha}(\rho,q)$ and $\boldsymbol{H}_{m,s,\alpha}(\rho,q)$ are readily expressed in terms of their axial components $E_{z;m,s,\alpha}(\rho,q)$ and $H_{z;m,s,\alpha}(\rho,q)$. In the outer homogeneous medium ($\rho > a$), these components are written as follows:

$$\begin{aligned} E_{z;m,s,\alpha}(\rho,q) &= \frac{i}{\varepsilon_3}\Big[\sum_{k=1}^{2} C_{m,s,\alpha}^{(k)}(q) n_{s,\alpha}^{(1)} q H_m^{(k)}(k_0 q\rho) + C_{m,s,\alpha}(q) n_{s,\alpha}^{(2)} q_\alpha H_m^{(2)}(k_0 q_\alpha\rho)\Big], \\ H_{z;m,s,\alpha}(\rho,q) &= -\frac{1}{Z_0\mu_3}\Big[\sum_{k=1}^{2} C_{m,s,\alpha}^{(k)}(q) q H_m^{(k)}(k_0 q\rho) + C_{m,s,\alpha}(q) q_\alpha H_m^{(2)}(k_0 q_\alpha\rho)\Big]. \end{aligned} \tag{5}$$

Here, $C_{m,s,\alpha}^{(1)}$, $C_{m,s,\alpha}^{(2)}$, and $C_{m,s,\alpha}$ are coefficients to be determined, $H_m^{(1)}$ and $H_m^{(2)}$ are Hankel functions of the first and second kinds, respectively, of order m, and Z_0 is the impedance of free space. Other quantities in (5) are given by the formulas

$$\begin{aligned} n_{s,\alpha}^{(1,2)}(q) &= \Big[\big(q_\alpha^{(1,2)}\big)^2 + \mu_1^{-1}\mu_3\, p_\alpha^2(q) + \varepsilon_1^{-1}(\varepsilon_2^2-\varepsilon_1^2)\mu_3\Big]\Big[\mu_3\tau_{\mathrm{g}} p_{s,\alpha}(q)\Big]^{-1}, \\ q_\alpha^{(1)} = q, \quad q_\alpha^{(2)} &= q_\alpha(q) = \Big[\kappa_{\mathrm{m}}^2 - \tau_{\mathrm{m}}^{-1} p_\alpha^2(q) - \varepsilon_3\tau_{\mathrm{g}} p_{s,\alpha}(q)\big(n_{s,\alpha}^{(1)}(q)\big)^{-1}\Big]^{1/2}. \end{aligned} \tag{6}$$

In the above expressions, we put $\mathrm{Im}\,q_\alpha(q) < 0$. Because of such a choice of the branches of $q_\alpha(q)$, we rejected the solution comprising $H_m^{(1)}(k_0 q_\alpha\rho)$ to ensure that the functions $\boldsymbol{E}_{m,s,\alpha}(\rho,q)$ and $\boldsymbol{H}_{m,s,\alpha}(\rho,q)$ do not contradict the boundedness conditions (4).

In the uniform inner region ($\rho < a$), the axial field components can be written as

$$E_{z;m,s,\alpha}(\rho,q) = \frac{i}{\tilde{\varepsilon}_3}\sum_{k=1}^{2} B_{m,s,\alpha}^{(k)}(q)\,\tilde{n}_{s,\alpha}^{(k)}\,\tilde{q}_\alpha^{(k)} J_m(k_0\tilde{q}_\alpha^{(k)}\rho),$$

$$H_{z;m,s,\alpha}(\rho,q)=-\frac{1}{Z_0\tilde{\mu}_3}\sum_{k=1}^{2}B^{(k)}_{m,s,\alpha}(q)\,\tilde{q}^{(k)}_{\alpha}J_m(k_0\tilde{q}^{(k)}_{\alpha}\rho). \tag{7}$$

Here, J_m are Bessel functions of the first kind of order m, $B^{(1)}_{m,s,\alpha}$ and $B^{(2)}_{m,s,\alpha}$ are coefficients to be determined, and $\tilde{q}^{(1)}_{\alpha}$ and $\tilde{q}^{(2)}_{\alpha}$ are the transverse wave numbers in the inner region, which correspond to the axial wave number $p_\alpha(q)$:

$$\tilde{q}^{(1,2)}_{\alpha}(q)=2^{-1/2}\left[\tilde{\kappa}^2_{\rm e}+\tilde{\kappa}^2_{\rm m}-\left(\tilde{\tau}^{-1}_{\rm e}+\tilde{\tau}^{-1}_{\rm m}\right)p^2_\alpha(q)-\chi^{(1,2)}R_q\big(p_\alpha(q)\big)\right]^{1/2},\qquad \chi^{(1)}=-\chi^{(2)}=1,$$

$$R_q(p)=\left\{\left(\frac{1}{\tilde{\tau}_{\rm e}}-\frac{1}{\tilde{\tau}_{\rm m}}\right)^2p^4-2\left[\left(\frac{1}{\tilde{\tau}_{\rm e}}+\frac{1}{\tilde{\tau}_{\rm m}}\right)(\tilde{\kappa}^2_{\rm e}+\tilde{\kappa}^2_{\rm m})-2\left(\frac{\tilde{\kappa}^2_{\rm e}}{\tilde{\tau}_{\rm m}}+\frac{\tilde{\kappa}^2_{\rm m}}{\tilde{\tau}_{\rm e}}+\tilde{\varepsilon}_3\tilde{\mu}_3\tilde{\tau}^2_{\rm g}\right)\right]p^2+\left(\tilde{\kappa}^2_{\rm e}-\tilde{\kappa}^2_{\rm m}\right)^2\right\}^{1/2}. \tag{8}$$

The quantities $\tilde{\tau}_{\rm e}$, $\tilde{\tau}_{\rm m}$, $\tilde{\tau}_{\rm g}$, $\tilde{\kappa}_{\rm e}$, $\tilde{\kappa}_{\rm m}$, and $\tilde{n}^{(1,2)}_{s,\alpha}$ are obtained from the formulas for $\tau_{\rm e}$, $\tau_{\rm m}$, $\tau_{\rm g}$, $\kappa_{\rm e}$, $\kappa_{\rm m}$, and $n^{(1,2)}_{s,\alpha}$ in (3) and (6) if we replace $\varepsilon_{1,2,3}$, $\mu_{1,2,3}$, and $q^{(1,2)}_{\alpha}$ by $\tilde{\varepsilon}_{1,2,3}$, $\tilde{\mu}_{1,2,3}$, and $\tilde{q}^{(1,2)}_{\alpha}$, respectively.

Calculating the transverse components of the field and satisfying the continuity conditions for tangential field components on the boundary $\rho=a$, we arrive at the system of linear equations for unknown coefficients $B^{(1,2)}_{m,s,\alpha}$, $C^{(1,2)}_{m,s,\alpha}$, and $C_{m,s,\alpha}$. This system can thus be represented in matrix form:

$$\mathbf{S}\cdot\boldsymbol{G}=C^{(1)}_{m,s,\alpha}\boldsymbol{F}, \tag{9}$$

where the components of the column vector $\boldsymbol{G}$ are given by the expressions $G_{1,2}=B^{(1,2)}_{m,s,\alpha}$, $G_3=C^{(2)}_{m,s,\alpha}$, and $G_4=C_{m,s,\alpha}$. The elements of the matrix $\mathbf{S}$ and the components of the column vector $\boldsymbol{F}$, which are not written here, are expressible in terms of cylindrical functions involved in the expressions for tangential fields at $\rho=a$.

The coefficients $B^{(1,2)}_{m,s,\alpha}$, $C^{(1,2)}_{m,s,\alpha}$, and $C_{m,s,\alpha}$ are determined up to a factor independent of spatial coordinates. It is convenient to put $C^{(1)}_{m,s,\alpha}=\det|\mathbf{S}|$. Then the other coefficients are easily calculated. Their expressions turn out to be very cumbersome and are not presented here for brevity.

FIELD EXPANSIONS IN TERMS OF DISCRETE- AND CONTINUOUS-SPECTRUM MODES

The obtained field representation allows us to find the spectrum of eigenvalues q and the corresponding eigenfunctions of the guiding structure. First, it is easy to verify that the field (5) satisfies the boundedness conditions (4) for all real transverse wave numbers q. Next, based on the approach developed in [3], it can be shown that $\boldsymbol{E}_{m,s,\alpha}(\rho,q\exp(\pm i\pi))=\boldsymbol{E}_{m,s,\alpha}(\rho,q)$ and $\boldsymbol{H}_{m,s,\alpha}(\rho,q\exp(\pm i\pi))=\boldsymbol{H}_{m,s,\alpha}(\rho,q)$, whence it follows that the negative values of q can be excluded from the analysis. Thus, all positive values of q constitute the continuous eigenvalue spectrum.

Along with the continuous spectrum of the values of q, conditions (4) can also be satisfied for certain discrete complex values $q=q_{m,n}$ $(n=1,2,\ldots)$ which are roots of the equation $C^{(1)}_{m,s,\alpha}(q_{m,n})=0$ for $\operatorname{Im}q_{m,n}<0$ or the equation $C^{(2)}_{m,s,\alpha}(q_{m,n})=0$ for $\operatorname{Im}q_{m,n}>0$. With allowance for the properties of Hankel functions, it can easily be verified that roots of the latter equation do not yield new solutions for the field and can therefore be rejected. It is evident that the waves corresponding to the discrete values $q_{m,n}$ are localized eigenmodes (discrete-spectrum modes) of the considered guiding structure. The eigenmode fields are obtained by putting $q=q_{m,n}$ in (1) and will further be denoted as $\boldsymbol{E}_{m,n_s}(\boldsymbol{r})$ and $\boldsymbol{H}_{m,n_s}(\boldsymbol{r})$, where the indices $n_+=n>0$ and $n_-=-n<0$ mark discrete-spectrum modes propagating in the positive and negative directions of the z axis, respectively.

Since the set of discrete- and continuous-spectrum modes is complete, the total field outside the source region can be expanded in the form

$$\begin{bmatrix}\boldsymbol{E}(\boldsymbol{r})\\ \boldsymbol{H}(\boldsymbol{r})\end{bmatrix}=\sum_{m=-\infty}^{\infty}\left(\sum_{n_s}a_{m,n_s}\begin{bmatrix}\boldsymbol{E}_{m,n_s}(\boldsymbol{r})\\ \boldsymbol{H}_{m,n_s}(\boldsymbol{r})\end{bmatrix}+\sum_{\alpha={\rm o,x}}\int_0^\infty a_{m,s,\alpha}(q)\begin{bmatrix}\boldsymbol{E}_{m,s,\alpha}(\boldsymbol{r},q)\\ \boldsymbol{H}_{m,s,\alpha}(\boldsymbol{r},q)\end{bmatrix}{\rm d}q\right), \tag{10}$$

where a_{m,n_s} and $a_{m,s,\alpha}$ are the expansion coefficients of the discrete- and continuous-spectrum modes, respectively. In (10), one should put $n_s=n>0$ and $s=+$ for positive z and $n_s=-n$ and $s=-$ for negative z outside the source region. The field expansion inside the source region is not given here for the sake of brevity.

The discrete- and continuous-spectrum modes entering expansion (10) satisfy some general conditions under which orthogonality relations for these modes can be established. In the considered case, the orthogonality relations have a form similar to that obtained in [3] for channels in a gyroelectric background medium. Using the orthogonality relations and the well-known method developed for finding the expansion coefficients of modes of closed and open waveguides and based on Lorentz's theorem (see, e.g., [3]), we obtain the formulas

$$a_{m,\pm n} = \frac{1}{N_{m,n}} \int \left[\boldsymbol{j}^{\mathrm{e}}(\boldsymbol{r}) \cdot \boldsymbol{E}^{(\mathrm{T})}_{-m,\mp n}(\boldsymbol{r}) - \boldsymbol{j}^{\mathrm{m}}(\boldsymbol{r}) \cdot \boldsymbol{H}^{(\mathrm{T})}_{-m,\mp n}(\boldsymbol{r}) \right] \mathrm{d}\boldsymbol{r}, \tag{11}$$

$$a_{m,\pm,\alpha}(q) = \frac{1}{N_{m,\alpha}(q)} \int \left[\boldsymbol{j}^{\mathrm{e}}(\boldsymbol{r}) \cdot \boldsymbol{E}^{(\mathrm{T})}_{-m,\mp,\alpha}(\boldsymbol{r},q) - \boldsymbol{j}^{\mathrm{m}}(\boldsymbol{r}) \cdot \boldsymbol{H}^{(\mathrm{T})}_{-m,\mp,\alpha}(\boldsymbol{r},q) \right] \mathrm{d}\boldsymbol{r}. \tag{12}$$

Here, integration is performed over the region occupied by currents, the superscript (T) denotes fields taken in an auxiliary ("transposed") medium described by the transposed tensors $\boldsymbol{\varepsilon}^{\mathrm{T}}$ and $\boldsymbol{\mu}^{\mathrm{T}}$, and the normalization quantities for the corresponding modes are given by the formulas

$$N_{m,n} = 2\pi \int_0^{\infty} \left[\boldsymbol{E}_{m,n}(\boldsymbol{r}) \times \boldsymbol{H}^{(\mathrm{T})}_{-m,-n}(\boldsymbol{r}) - \boldsymbol{E}^{(\mathrm{T})}_{-m,-n}(\boldsymbol{r}) \times \boldsymbol{H}_{m,n}(\boldsymbol{r}) \right] \cdot \hat{\boldsymbol{z}}_0 \, \rho \, \mathrm{d}\rho, \tag{13}$$

$$N_{m,\alpha}(q) = -\frac{16\pi}{Z_0 k_0^2} \left(\frac{\mathrm{d}p_\alpha(q)}{\mathrm{d}q} \right)^{-1} \left[\mu_3^{-1} + \varepsilon_3^{-1} \left(n^{(1)}_{s,\alpha}\right)^2 \right] C^{(1)}_{m,s,\alpha}(q) C^{(2)}_{m,s,\alpha}(q). \tag{14}$$

If the medium is homogeneous in the entire space, i.e., for $0 < \rho < \infty$, then the eigenvalue spectrum comprises only the continuous part constituted by positive real transverse wave numbers q. The fields of the corresponding eigenmodes and their norms $N_{m,\alpha}(q)$ are yielded by putting $C^{(1,2)}_{m,s,\alpha} = 1$ and $C_{m,s,\alpha} = 0$ in (5) and (14). In this case, the expansion for the source-excited field is obtained from (10) by retaining only the continuous-spectrum modes whose expansion coefficients are given by previous formula (12).

CONCLUSIONS

In this paper, we presented the complete eigenfunction expansion of the total electromagnetic field excited by spatially bounded given sources in a cylindrically stratified gyrotropic medium. The field has been expanded in terms of modes whose spectrum comprises both the discrete and continuous parts, and the expansion coefficients of discrete- and continuous-spectrum modes have been calculated. Our analysis generalizes the theory of excitation of open waveguides in an infinitely extended gyroelectric medium [3] to the case of open guiding structures located in a general gyrotropic anisotropic medium. Although the problem of excitation of guiding structures in such media can be solved using the dyadic Green's functions (see, e.g., [5]), the approach developed herein makes it possible to immediately obtain the source-excited field without preliminary calculation of the dyadic Green's functions.

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REFERENCES

[1] L. B. Felsen and N. Marcuvitz, *Radiation and Scattering of Waves*. Englewood Cliffs: Prentice-Hall, 1973.

[2] V. V. Shevchenko, *Continuous Transitions in Open Waveguides*. Boulder: Golem Press, 1971.

[3] I. G. Kondrat'ev, A. V. Kudrin, and T. M. Zaboronkova, *Electrodynamics of Density Ducts in Magnetized Plasmas*. Amsterdam: Gordon and Breach, 1999.

[4] S. Przeździecki and R. A. Hurd, "Diffraction by a half plane perpendicular to the distinguished axis of a gyrotropic medium (oblique incidence)," *Can. J. Phys.*, vol. 59, pp. 403–424, 1981.

[5] E. L. Tan and S. Y. Tan, "Cylindrical vector wave function representations of the dyadic Green's functions for cylindrical multilayered gyrotropic bianisotropic media," *Progress In Electromagnetics Research*, vol. 26, pp. 199–222, 2000.