# MULTIPLE DIFFRACTION OF ELECTROMAGNETIC WAVES BY A WEDGE OF CONCAVE SHAPE 

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#### Abstract

Multiple diffraction of short-wavelength electromagnetic waves by a wedge of concave shape is analysed with the uniform geometrical theory of diffraction (UTD). The resulting wave field is a superposition of a variety of edge-diffracted and specularly reflected waves, treated as "channels of multiple diffraction". The algorithms of UTD allow of describing birth and disappearance of diffraction channels under evolution of the wedge shape. Classification of diffraction channels is suggested and dominant channels are selected. The "four/five channels model" of scattering is proposed, which describes effectively diffraction of electromagnetic waves by steep and breaking water waves of mesoscale spectrum and embraces the main interference and polarization phenomena, characteristic of low-grazing angles of radar sounding, including enhanced backscattering, suppression of vertically polarized echoes due to Brewster effect and "super events"-gigantic spikes of radar cross-section (RCS). It is shown that RCS for plain individual mesowave of 30 cm height and of 1 m front width may reach as large value as $1-10 \mathrm{~m}^{2}$.


## INTRODUCTION

Experimental data, collected during radar imaging of ocean surfaces at low gazing angles, reveal that the intensity of horizontally polarised echoes exceeds that of vertically polarised echoes, besides, the radar echoes fluctuate strongly ("super events"), contrary to the classical theory of the resonant (Bragg) scattering [1-3].

These results suggest the existence of non-resonant mechanism of backscattering. Indeed such a mechanism can be found in multiple diffraction of electromagnetic waves by meso-scale water waves (mesowaves for short). Mesowaves are typically $10-30 \mathrm{~cm}$ high and 30 to 50 cm long.

Recent attempts to apply GTD to microwave scattering by steep and breaking mesowaves, modelled as non-perfectly conducting wedges of concave shape, were undertaken in [4-8]. The present paper intends to give a systematical description of the properties of EM waves, multiply diffracted from a non-perfectly conducting wedge with a concave face.

## ANALYSIS

## Single Diffraction: UTD Asymptotic Solution

Let a spherical wave (wavelength $\lambda$ ) fall on such a wedge. The curvature radius of the front face in the vertical plane is denoted by $a_{\mathrm{w}}$ and that in the horizontal plane is termed $a_{\mathrm{c}}$. In the radar frequency range there is $\lambda \ll a_{\mathrm{w}} \ll a_{\mathrm{c}}$. Hence, asymptotic methods like UTD can be used for studying scattering of electromagnetic waves by a mesowave.

According to the conventional UTD [9], an incident wave produces a first-order wave $E_{\mathrm{UTD}}^{(1)}(\mathbf{r})$ which consists of two
components

$$
\begin{equation*}
E_{\mathrm{UTD}}^{(1)}(\mathbf{r})=E_{\mathrm{eUTD}}(\mathbf{r})+E_{\mathrm{s}}(\mathbf{r}) \theta\left(\varphi_{\mathrm{B}}-\varphi\right), \tag{1}
\end{equation*}
$$

where $\theta(\xi)$ is the Heaviside step function: $\theta(\xi)=1$ for $\xi>0$ and $\theta(\xi)=0$ for $\xi \leq 0$. The first, proper diffraction component $E_{\text {eUTD }}(\mathbf{r})$, is an edge-diffracted wave, which leaves an edge point $\mathbf{r}_{\mathrm{e}}$ of the wedge, and the second, geometrical component $E_{\mathrm{s}}(\mathbf{r})$, describes a wave specularly reflected from the front wedge side.

Being interested in the monostatic radar cross-section of mesowaves, only normal incidence with respect to the wedge crest at the diffraction point need be considered. In this case, the edge diffraction causes no coupling between the horizontally and vertically polarized field components and hence these two polarizations can be studied separately. In the following, we do not use the polarisation super indices " $h$, v " when their meaning is clear from the context.

The reflected field $E_{\mathrm{s}}(\mathbf{r})$ is enhanced owing to focusing caused by the concave front side of the wedge. $E_{\mathrm{s}}(\mathbf{r})$ is proportional to the reflection coefficients $\Gamma^{\mathrm{v}, \mathrm{h}} .\left|\Gamma^{\mathrm{v}}\right|$ takes the smallest value at the Brewster angle, and reaches its maximum at grazing incidence; $\Gamma^{\mathrm{h}}$ remains, at least at low grazing angles, virtually a constant: $\Gamma^{\mathrm{h}} \approx-1$. Since [10] this fundamental fact has been an important element of all theories of electromagnetic wave scattering by a sea surface at low grazing angles.

In a neighbourhood of the light-shadow boundary, UTD solution (1) describes a joint field $E_{\mathrm{s}+\mathrm{e}}$, which converts into the single edge wave $E_{\mathrm{e}}$ in the shadow of the reflected wave and into a sum $E_{\mathrm{s}}+E_{\mathrm{e}}$ of the specularly reflected and edgediffracted waves in the lit region. The transformation process of the edge wave $E_{\mathrm{e}}$ into the sum $E_{\mathrm{s}}+E_{\mathrm{e}}$ when traversing the light-shadow boundary can be regarded as the birth of the specularly-reflected wave from the united UTD solution $E_{\mathrm{s}+\mathrm{e}}=E_{\mathrm{UTD}}(\mathbf{r}): E_{\mathrm{e}} \rightarrow E_{\mathrm{s}+\mathrm{e}} \rightarrow E_{\mathrm{s}}+E_{\mathrm{e}}$. Similarly, the inverse process $E_{\mathrm{s}}+E_{\mathrm{e}} \rightarrow E_{\mathrm{s}+\mathrm{e}} \rightarrow E_{\mathrm{e}}$ can be naturally considered as the disappearance of the specular-reflected wave.

## Double and Multiple Diffraction

This section deals with waves which experience multiple diffraction by a concave wedge. Different types of multiply diffracted waves will be treated as multiple diffraction channels.

Consider at first doubly diffracted channels. When falling on a curved wedge front side, the specularly reflected wave $E_{\mathrm{s}}(\mathbf{r})$ produces a doubly reflected field $E_{\mathrm{ss}}(\mathbf{r})$. Besides, the singly reflected wave $E_{\mathrm{s}}(\mathbf{r})$ falling on a wedge could generate the wave $E_{\mathrm{se}}$. The third diffraction channel, belonging to the class of double diffraction process, is the wave field $E_{\text {es }}(\mathbf{r})$ which is produced by the edge wave $E_{\mathrm{e}}(\mathbf{r})$, specularly reflected from the wedge front side. Properties of this wave are very similar to those of the $E_{\text {se }}$ wave discussed above.

Thus, in frame of the second-order UTD approximation, the diffraction wave $E_{\mathrm{UTD}}^{(2)}(\mathbf{r})$ generally consists of three terms:

$$
\begin{equation*}
E_{\mathrm{UTD}}^{(2)}(\mathbf{r})=E_{\mathrm{ss}}(\mathbf{r})+E_{\mathrm{se}}(\mathbf{r})+E_{\mathrm{es}}(\mathbf{r}) \tag{2}
\end{equation*}
$$

In principle each element of doubly diffracted wave field (2) is able to generate triple diffraction wave fields. Not all such waves, however, can be excited and observed in the far zone. Of relevance to this work is out of the triply diffracted wave fields merely $E_{\text {ses }}$.

## Enhanced Backscattering. Four/Five Channel Model of Scattering

If an observer (receiver) is located very close to the source, some diffraction channels become identical (coherent) by virtue of the reciprocity theorem. It concerns first of all the channels $E_{\mathrm{se}}$ and $E_{\text {es }}$. For scattering exactly in the backward direction, the sum of the wave fields $E_{\text {se }}$ and $E_{\text {es }}$ will be twice as much as $E_{\text {es }}$ or $E_{\mathrm{se}}: E_{\mathrm{se}}+E_{\text {es }}=2 E_{\text {es }}$. Therefore, the intensity $I_{\mathrm{es}+\mathrm{se}}=\left|E_{\mathrm{se}}+E_{\mathrm{es}}\right|^{2}$ is four times as much as $I_{\mathrm{es}}=\left|E_{\mathrm{es}}\right|^{2}: I_{\mathrm{es}+\mathrm{se}}=4 I_{\mathrm{es}}$, and is twice as large as the non-coherent sum $I_{\mathrm{se}}+I_{\mathrm{es}}=2 I_{\mathrm{es}}$. The important role of coherent channels in forming radar echo signal from large-scale
breaking waves was pointed out for the first time in [10]. Coherent phenomena for electromagnetic wave diffraction by sharp-crested mesoscale waves were analyzed in [4-8].

This phenomenon, known as the enhanced backscattering [11], concerns only coupled channels, which propagate along the same path, but in opposite directions. At the same time the channel $E_{\text {ses }}$ has no coherent partner (in [11] similar channels are regarded as idle) and therefore does not manifest the phenomenon of enhanced backscattering.

As made clear above, at the initial stage of the wedge evolution the following four channels are dominant: $E_{\mathrm{e}}, E_{\mathrm{es}}, E_{\mathrm{se}}$ and $E_{\text {ses }}$, the coupled channels $E_{\text {es }}$ and $E_{\text {se }}$ being identical. The far-zone wave field $E$ is given by

$$
\begin{equation*}
E=E_{\mathrm{e}}+E_{\mathrm{es}}+E_{\mathrm{se}}+E_{\mathrm{ses}}=E_{\mathrm{e}}+2 E_{\mathrm{es}}+E_{\mathrm{ses}}=E_{0} D_{\mathrm{four}} \sqrt{-2 \pi \mathrm{i} a_{\mathrm{c}} / k} \exp \left(\mathrm{i} k \rho_{\mathrm{e}}\right) / \rho_{\mathrm{e}} . \tag{3}
\end{equation*}
$$

When the slope $\nu$ of the wedge front side reaches the critical value $\nu_{\text {crit }}=\pi / 2-\gamma$, the specular channel $E_{\mathrm{s}}$ comes into play. In the transitional "four/five channels" regime where the fifth channel is appearing, the first term in eq. (3) should be replaced with the respective UTD solution $E_{\mathrm{UTD}}=E_{\text {e+s }}$ :

$$
\begin{equation*}
E=E_{\mathrm{e}+\mathrm{s}}+2 E_{\mathrm{es}}+E_{\mathrm{ses}}=E_{0} D_{\text {four } / \text { five }} \sqrt{-2 \pi \mathrm{i} a_{\mathrm{c}} / k} \exp \left(\mathrm{i} k \rho_{\mathrm{e}}\right) / \rho_{\mathrm{e}} \tag{4}
\end{equation*}
$$

At $\nu>\nu_{\text {crit }}$, the channels $E_{\mathrm{s}}$ and $E_{\mathrm{e}}$ split and can be observed separately, the four/five channel model (4) converts into a five-channel model

$$
\begin{equation*}
E=E_{\mathrm{e}}+E_{\mathrm{s}}+E_{\mathrm{es}}+E_{\mathrm{se}}+E_{\mathrm{ses}}=E_{\mathrm{e}}+E_{\mathrm{s}}+2 E_{\mathrm{es}}+E_{\mathrm{ses}}=E_{0} D_{\mathrm{five}} \sqrt{-2 \pi \mathrm{i} a_{\mathrm{c}} / k} \exp \left(\mathrm{i} k \rho_{\mathrm{e}}\right) / \rho_{\mathrm{e}} \tag{5}
\end{equation*}
$$

The expressions (3-5), generalising the results of the previous papers [4-8], describe the main interference as well as polarization phenomena, characteristic of radar backscattering from mesowaves at low grazing angles. They make it evident that in frame of "four/five channels" model the intensity of the backscattered wave field at horizontal polarization is undoubtedly larger than that at vertical polarization.

## Super Events

For the specular backscattering the radar cross-section is $\sigma_{\mathrm{s}}=\pi|\Gamma|^{2}\left|a_{\mathrm{w}}\right|\left|a_{\mathrm{c}}\right|$, typical of doubly curved smooth surfaces [12]. The radar cross-section due to the edge wave is $\sigma_{\mathrm{e}}=\lambda\left|a_{\mathrm{c}}\right||D|^{2}$.The ratio between the edge RCS and the specular RCS is then

$$
\begin{equation*}
\sigma_{\mathrm{e}} / \sigma_{\mathrm{s}}=\lambda\left|a_{\mathrm{c}}\right||D|^{2} /\left(\pi|\Gamma|^{2}\left|a_{\mathrm{w}} a_{\mathrm{c}}\right|\right) . \tag{6}
\end{equation*}
$$

After [12-13], the asymptotic techniques like GTD and UTD are then and only then applicable, when the length of the edge $L_{\mathrm{c}}$ is greater than the width of the Fresnel zone of the diffracted wave in the direction of the edge $2 a_{\mathrm{f}}$ with $a_{\mathrm{f}}=\sqrt{\lambda\left|a_{\mathrm{c}}\right|}$. If the contrary is true, one has to resort to the Kirchhoff-Fresnel diffraction integral. It follows from the Kirchhoff-Fresnel diffraction integral that the main contribution in forming the scattering stems from a "coherent" area $S_{\text {coh }}$, inside which the phase of the reflected field differs from its extremes by no more than $\pi$ (Fresnel criterion). This means that the scattered field is proportional to $S_{\mathrm{coh}} /(\lambda \rho)$ and therefore the radar cross-section $\sigma$ is proportional to $S_{\mathrm{coh}}^{2} / \lambda^{2}$ :

$$
\begin{equation*}
\sigma \sim S_{\mathrm{coh}}^{2} / \lambda^{2} \tag{7}
\end{equation*}
$$

The size of the effective area $S_{\text {coh }}$ in each of the two directions is determined by either the first Fresnel radius, or the size of the illuminated area, whatever is smaller. If the wave crest is a straight line and completely inside the first Fresnel zone [it means $L_{\mathrm{c}} \leq 2\left(\left|a_{\mathrm{c}}\right| \lambda\right)^{1 / 2}$ ], the coherent area $S_{\text {coh }}$, forming the edge wave, approaches $L_{\mathrm{c}} \lambda$ and therefore the radar cross-section can be estimated as

$$
\begin{equation*}
\sigma_{\mathrm{e}} \approx\left(L_{\mathrm{c}} \lambda\right)^{2} / \lambda^{2}=L_{\mathrm{c}}^{2} \tag{8}
\end{equation*}
$$

In a similar way one can calculate the cross-section $\sigma_{\mathrm{s}}$ for the specular wave, reflected from a concave cylindrical surface of length $L_{\mathrm{c}}$. In this case the coherent area forming the specular reflection is about $S_{\text {coh }} \sim L_{\mathrm{c}}\left(a_{\mathrm{w}} \lambda\right)^{1 / 2}$, so that

$$
\begin{equation*}
\sigma_{\mathrm{s}} \sim S_{\mathrm{coh}}^{2} / \lambda^{2} \sim L_{\mathrm{c}}^{2} a_{\mathrm{w}} / \lambda \tag{9}
\end{equation*}
$$

The specular cross-section is $a_{\mathrm{w}} / \lambda$ times the maximal value of the edge cross-section $\sigma_{\mathrm{e}}$, given by eq. (8).
According to (8), the coherently illuminated section of a wedge crest of $L_{\mathrm{c}} \approx 1 \mathrm{~m}$ by length provides a cross-section $\sigma_{\mathrm{e}} \approx 1 \mathrm{~m}^{2}$. The specular cross-section (9) under these conditions might be $a_{\mathrm{w}} / \lambda$ times as much as $\sigma_{\mathrm{e}}$, say, for $a_{\mathrm{w}}=30$ cm and $\lambda=3 \mathrm{~cm}$ one has $a_{\mathrm{w}} / \lambda=10$. Therefore, the radar cross-section, even very rarely encountered, could reach the value $\sigma_{\mathrm{s}} \approx 10 \mathrm{~m}^{2}$. Such large values of the radar cross-section are of significant practical interest, because they might be responsible for the so-called "super events", that is for rather strong echo signals, observed sometimes on the sea surface at low grazing angles [3].

## CONCLUSION

Analysis of multiple electromagnetic wave diffraction by a concave wedge allows of selecting dominant channels, responsible for forming the interference pattern under backscattering. The most important channels are unified in the "four/five channels" model of backscattering, which describes the main features of radar signals, backscattered from the sea surface at low grazing angles. This model embraces the main interference and polarization phenomena, characteristic of low grazing angles radar sounding, including enhanced backscattering, Brewster phenomenon and "super events", responsible for gigantic spikes of radar cross section, up to $1-10 \mathrm{~m}^{2}$ for individual mesowaves.

## REFERENCES

[1] Guinard N. W., Ransone J. T., Daley J. C.: Variation of the NRCS of the sea with increasing roughness. Geophys. Res., 1971, 76(6), pp. 1525-1538
[2] Rytov S. M., Kravtsov Yu. A., and Tatarskii V. I.: Principles of Statistical Radio Physics. Vol. 4: Wave Propagation through Random Media. Berlin: Springer-Verlag, 1989
[3] Lee P. H. Y. el al.: Wind-speed dependence of small-grazing-angle microwave backscatter from sea surfaces. IEEE Trans, 1996, AP-44(3), pp. 333-340
[4] Kravtsov Yu. A., Mityagina M.I. and Churyumov A.N.: Nonresonant mechanism of electromagnetic waves scattering by sea surface: scattering by steep sharpened waves. Radio Phys. Quant. Electron., 1999. 42(3), pp. 216-228
[5] Kravtsov Yu.A., Mityagina M.I., Churyumov A.N.: Electromagnetic waves backscattering by mesoscale breaking waves on the sea surface. Bulletin of the Russ. Acad. Sci. Physics, 1999, 63(12), pp. 1859-1865
[6] Churyumov A. N. and Kravtsov Yu. A.: Microwave backscatter from mesoscale breaking waves on the sea surface. Waves in Random Media, 2000, 10(1), pp. 1-15
[7] Churyumov A. N., Kravtsov Yu. A., Lavrova O. Yu., Litovchenko K. Ts., Mityagina M. I., Sabinin K. D.: Signatures of resonant and non-resonant scattering mechanisms on radar images of internal waves. Intern.J.Remote Sensing, 2002, 23(20), pp. 4341-4355
[8] Bulatov M.G., Kravtsov Yu.A., Lavrova O.Yu., Litovchenko K.Ts., Mitiagina M.I., Raev M.D., Sabinin K.D., Trokhi movskii Yu.G., Churiumov A.N., Shugan I.V.: Physical mechanisms of aerospace radar imaging of the ocean. PhysicsUspekhi, 2003, 46(1), pp. 63-79
[9] Pathak P. H. Techniques for High-Frequency Problems. Chapter 4 In Antenna Handbook: Theory, Applications, and Design (Van Nostrand Reinhold, New York, 1988)
[10] Trizna D.B., J.P. Hansen, P. Hwang, Jin Wu.: Laboratory studies of radar sea spikes at low grazing angles. J. Geo phys. Res., 1991, 96(C7), pp.12529-12537
[11] Barabanenkov Yu.N., Kravtsov Yu.A., Ozrin V.D., Saichev A.I.: Enhanced Backscattering in Optics. Progress in Optics (E.Wolf, Ed) Amsterdam: North Holland, 1991, 29, pp. 67-197
[12] Kravtsov Yu.A.: Geometrical Optics in Engineering Physics, Alpha Science International, UK, 2005
[13] Kravtsov Yu.A.: Rays and caustics as physical objects. Progress in Optics (E.Wolf, Ed), Amsterdam: North Holland, 1988, 26, pp.227-348

