A CLASS OF BENCHMARK PROBLEMS
FOR TRANSIENT SCATTERING BY METALLIC TARGETS

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ABSTRACT

In this paper we discuss a class of time harmonic scattering problems for metallic targets, for which a solution can be obtained via the method of regularisation. The resulting numerical algorithm is efficient, stable and of arbitrarily prescribed accuracy. These problems provide a benchmark for numerical models of transient and ultrawideband scattering and expose the strengths and weaknesses of a numerical method in both time and frequency domains. An illustrative example is given: the benchmarking of a time domain integral equation method.

INTRODUCTION

Most practical applications of electromagnetic (EM) waves involve scattering by complex targets; these may involve edges, cavities, inclusions, apertures, differing physical scales or differing material properties. No analytical solution of EM scattering by such complex targets is possible, in general, and a numerical solution must be sought instead. However, if any reliance is to be placed on numerical models of EM scattering from such complex targets then the numerical methods employed must be properly validated.

A range of canonical problems exist [1] which can be used to benchmark numerical models of EM scattering, for which analytical solutions may be determined, subject to certain simplifying assumptions. These canonical problems are of limited interest, however, since they involve scattering from relatively simple target geometries, with none of the complexity expected in a practical application.

In this paper we will discuss how the method of regularisation can be used to derive a class of benchmark scattering problems involving relatively complex obstacles with edges, cavities and inclusions. These benchmarks provide an accurate, easily computed, wideband solution which may be used to validate numerical models of both transient and ultrawideband EM scattering.

A CLASS OF BENCHMARK SCATTERING PROBLEMS

For the purposes of the present discussion, we restrict our attention to the problem of transient or ultrawideband scattering from metallic targets. The obstacles which we will discuss are based on two concentric, metallic spheres, from which metal may be removed to create open scattering surfaces. By removing all of the metal from the inner sphere the problem may be reduced to a simpler one involving a single spherical shell. Metal is removed from each spherical shell in such a way that the resulting target retains an axis of rotational symmetry. Some typical target geometries are depicted in Fig. 1.

A benchmark scattering problem may be defined for this type of target, when the incident field is a plane wave propagating along the axis of rotational symmetry, with a polarisation perpendicular to this axis. An accurate solution may be obtained for the case of a time-harmonic incident field, via the method of regularisation. An ultrawideband scattering response may be obtained for the target, by varying the frequency of the incident field, and a transient scattering response determined for the target, via Fourier transformation.

The range of mechanisms involved in scattering from such targets includes edge scattering, multiple body interactions and strong cavity resonances. It may be noted that a range of scattering targets may be generated, by varying the radius and the opening angles of each metallic surface. Since each of these parameters can be varied independently, the relative strength of different scattering mechanisms can be varied. This class of benchmark scattering problem has a direct physical relevance to the complex scattering mechanisms involved in many practical applications, in contrast to conventional canonical problems.

The Method of Regularisation

The EM scattering of a time harmonic field by this class of obstacles may be solved via the method of regularisation [2]. A detailed survey of this approach can be found in [3], its main features and advantages are as follows. In the case of the target class depicted in Fig. 1 a solution may be obtained by exploiting the underlying spherical geometry of the problem. The boundary conditions for the tangential electric field components on the metallic surface of each spherical shell, and the continuity conditions for the tangential magnetic field components on the non-metallic surface of each shell, are transformed to the corresponding boundary conditions for the Debye potentials and their derivatives. This yields two pairs of coupled dual series equations for the Fourier coefficients of the unknown electric and magnetic fields. This is in contradistinction to the usual Mie series solution [4] for the scattering of a perfectly conducting sphere, where the dual series equations are uncoupled. The resultant system of coupled equations is Fredholm type of the first kind. In order to provide a reliable benchmark, a transformation of this system is needed to obtain a set of equations which are Fredholm type of the second kind. This regularised system is well conditioned and may be solved by a numerical algorithm which is both stable and rapidly converging. This means that an ultrawideband solution may be reliably and efficiently computed. Furthermore, since estimates may be obtained of the rate of convergence of the regularised solution, a solution of arbitrarily prescribed accuracy can be determined. In the sense that a solution of arbitrary accuracy can be determined from the numerical solution of a coupled system of equations, the method of regularisation provides a semi-analytic solution to the time harmonic scattering problem.

EXAMPLE: BENCHMARKING A TIME DOMAIN INTEGRAL EQUATION METHOD

We next illustrate how this class of scattering problems can be used to benchmark numerical models of ultrawideband and transient scattering. For this purpose we consider one particular numerical method, based on a time domain solution of the electric field integral equation (EFIE).

The time domain EFIE algorithm

Assume that the axis of rotational symmetry, depicted in figure 1, is the z-axis of a cartesian coordinate system. Assume also that the field incident on any of these targets, $\underline{E}^i(\underline{r},t)$, is a Gaussian pulse which propagates along the z-axis from right to left of the figure. This field excites surface current and charge distributions on the target, $\underline{J}$ and ρ, which can be expressed in terms of the standard vector potentials, $\underline{A}$ and ϕ, in the usual manner [5]. Applying the boundary condition for a perfect conductor we obtain

$$\hat{n}_s \times \underline{E}^i(\underline{r},t) = \hat{n}_s \times \left[\exp\left(\frac{(t+z/c)^2}{\tau^2}\right)\hat{x}\right] = \hat{n}_s \times \left[\frac{\partial A}{\partial t}(\underline{r},t) + \nabla\phi(\underline{r},t)\right] \qquad (1)$$

which can be solved in conjunction with the continuity condition

$$\nabla_s \cdot \underline{J}(\underline{r},t) + \frac{\partial\rho}{\partial t}(\underline{r},t) = 0\ , \qquad (2)$$

where $\hat{n}_s$ and ∇_s denote a unit normal to the scattering surface and surface divergence, respectively.

We solve (1) and (2) using a time-marching algorithm, as discussed by Rynne [5], to which the reader is referred for further details. In essence, the scattering surface is approximated by a grid of triangular elements on which approximations to the surface current and charge densities can be defined. These can then be solved recursively for any desired time interval, given suitable initial conditions on the scattering surface. Having determined a solution for the approximate surface current and charge densities on the grid, approximations may then be obtained for the transient electric field scattered by the target, in either the near or far field. The ultrawideband response of the target may also be computed, via a fast Fourier transform of the resulting time series.

Results for a double-hemispherical reflector

In this section we compare some numerical results obtained by the time domain EFIE code with the semi-analytic results obtained via the method of regularisation. In both cases we consider a double-hemispherical target, comprising two concentric hemispheres aligned with their apertures facing the incident Gaussian pulse [6] with time constant $\tau = 1/2$. Both transient back scattered far fields and normalised back scattering cross sections are compared, in order to illustrate the utility of this approach as a benchmark for numerical models of both transient and ultrawideband scattering.

When the inner hemisphere has a radius only 1/4 that of the outer hemispherical shell, Fig. 2, the agreement in both time and frequency domains is good. Note that the agreement of the scattering cross sections is limited to a frequency range $kb < 6$ by the spectral content of the incident field assumed for the numerical calculation. When the radius of the inner hemispherical shell is increased to 3/4 that of the outer shell, Fig. 3, a pronounced disagreement is found. In the time domain we see an inability to resolve well the late time ringing expected of the target. This is because a relatively narrow cavity exists between the two hemispheres. This gives rise to a relatively high Q response which is apparent in the two resonant peaks of the scattering cross section. The time domain EFIE approach fails to resolve these resonances adequately and is evidently ill-suited to modelling such high Q structures.

If the radius of the inner hemispherical shell is increased, such that the cavity between the two shells narrows further, the response of the target becomes even more strongly resonant. It may be noted, in passing, that this type of resonant behaviour has potential practical applications. For example, the ponderomotive force associated with this resonant response in the microwave region is significant and provides a potential thrust mechanism for propulsion and levitation. The method of regularisation provides an accurate and robust method for studying such problems.

CONCLUSION

The benchmark scattering problems described above exhibit a range of scattering mechanisms with direct relevance to many practical applications. This is in contrast to standard canonical problems. Furthermore, since the parameters defining target geometry can be varied independently of one another, the relative strength of these scattering mechanisms can be varied. Since an accurate solution to the time-harmonic problem can be obtained efficiently, it can be used to benchmark numerical models of both transient and ultrawideband scattering, or to expose the strengths and weaknesses of any particular numerical method in both time and frequency domains.

In this paper we have described a class of metallic targets defined with reference to an underlying spherical geometry, comprising one or two concentric shells. We have also restricted our attention to the case in which the incident electric field is a plane wave and for which far field results are derived. It may be noted that none of these restrictions is necessary. Recently, results have been reported for a benchmarking problem involving an open spheroidal cavity excited by a dipole source [7]. Target geometries with impedance loaded surfaces or with dielectric regions also provide scattering problems to which the method of regularisation can be applied. The potential of this approach for solving scattering problems with complex features is discussed more fully in [3].

REFERENCES

[1] J.J. Bowman, T.B.A. Senior and P.L.E. Uslenghi, *Electromagnetic and acoustic scattering by simple shapes*, Revised Printing. New York: Hemisphere Publishing Corporation, 1987.

[2] S.S. Vinogradov, "Reflectivity of a spherical shield", *Radiophysics and Quantum Electronics*, vol. 26, pp 78-88, 1983.

[3] S.S. Vinogradov, P.D. Smith and E.D. Vinogradova, *Canonical Problems in Scattering and Potential Theory Part II: Acoustic and Electromagnetic Diffraction by Canonical Structures.* Boca Raton: Chapman and Hall/CRC, in press.

[4] M. Born and E. Wolf. *Principles of optics*, 7th ed. Cambridge: Cambridge University Press, 1999.

[5] B.P. Rynne, "Time domain scattering from arbitrary surfaces using the electric field integral equation", *J. Electro. Waves Applic.*, vol. 5, pp 93-112, 1991.

[6] S.M. Booker and S.S. Vinogradov, "A two-body benchmark problem for transient and ultrawideband scattering", *J. Electro. Waves Applic.*, vol. 16, pp 365-384, 2002.

[7] S.M. Booker, P.D. Smith, S.S. Vinogradov and E.D. Vinogradova, "Benchmark problems for transient and ultrawideband scattering by metallic targets", unpublished.

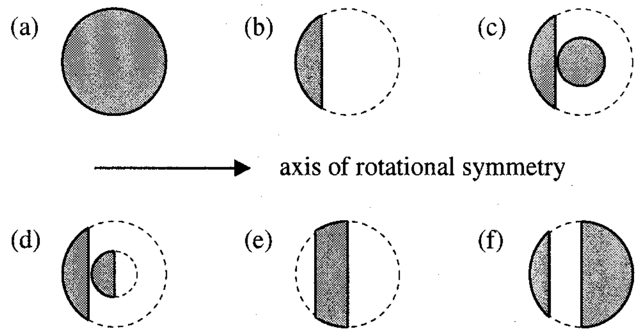

Fig. 1. A class of scattering targets for which a semi-analytic solution can be obtained via the Method of Regularisation.

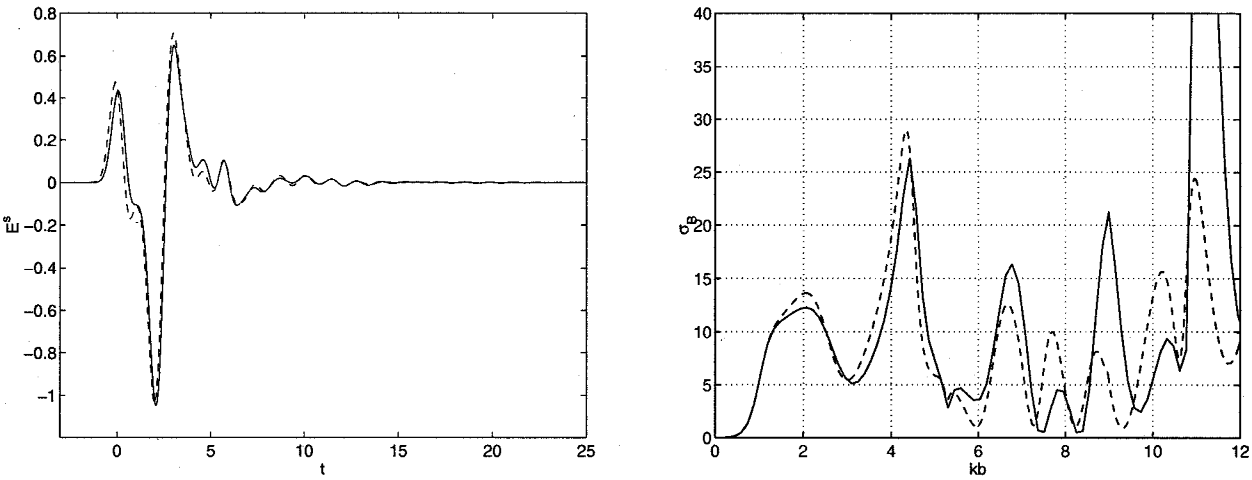

Fig. 2. Transient back scattered far field (left) and back scattering cross section (right) for a double-hemispherical target with radii in ratio 1:4, for a Gaussian pulse with parameter $\tau = 1/2$. Numerical results: solid line; semi-analytic results: dash line.

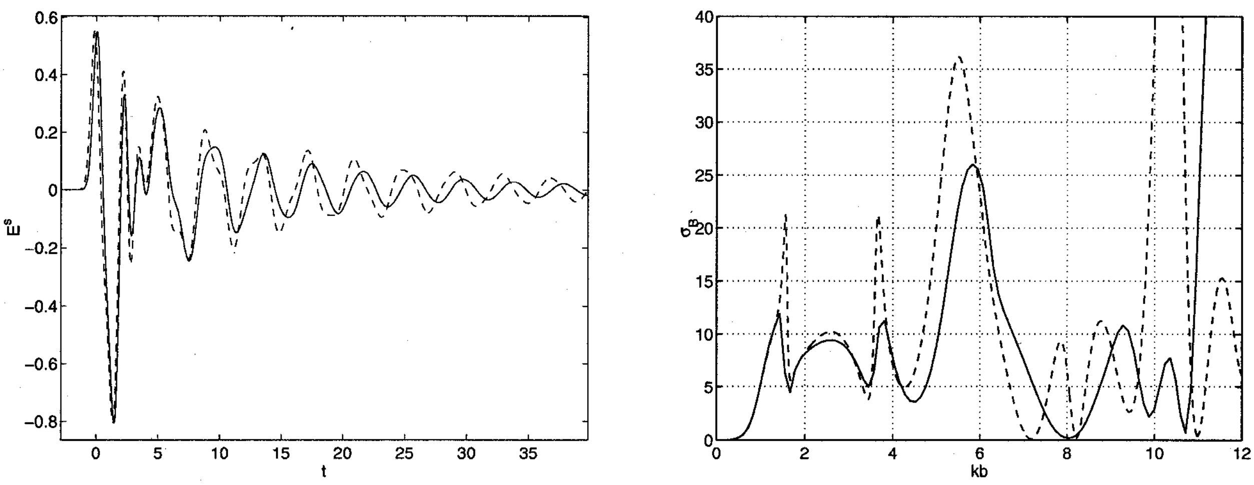

Fig. 3. Transient back scattered far field (left) and back scattering cross section (right) for a double-hemispherical target with radii in ratio 3:4, for a Gaussian pulse with parameter $\tau = 1/2$. Numerical results: solid line; semi-analytic results: dash line.