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# A Multi-Task Deep Neural Network for Joint Estimation of Direction of Arrival and Unknown Mutual Coupling Matrix for Structured Sparse Arrays

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## Abstract

In direction of arrival (DOA) estimation, structured sparse arrays provide improved performance over their uniformly-spaced array counterparts with the same number of elements. However, in practice when these elements are closely spaced, mutual coupling effects distort the spatial model underpinning standard DOA techniques. If the mutual coupling matrix (MCM) is not exactly known, both DOA and the MCM must be jointly estimated which results in a nonlinear problem. The advancement of neural networks and deep learning techniques offer efficient solutions to complex nonlinear and computationally intensive problems. In this paper, a neural network is trained to solve the joint DOA-MCM estimation problem and significantly reduce the computational load, making it amenable to real-time processing. We adopt a multi-task Deep-Learning (DL) approach to deal with the two-fold sets of unknowns underlying this estimation problem. The simulation results are presented for a nested sparse array.

## 1. Introduction

Direction of arrival (DOA) estimation is of high importance and well researched with applications in radar, sonar, acoustics, ultrasound, and wireless communications [1]. Conventional DOA techniques are based on beamforming, whereas high-resolution DOA methods [2] include Capon beamforming [3] Multiple Signal Classification (MUSIC) [4] Estimation of Signal Parameters via Rotational Invariant Techniques (ESPRIT) [5]. However, in practice, the general versions of these algorithms cannot account for and compensate the effects of mutual coupling (MC) between the array antenna elements if the MC is not known a priori [6].

Several existing methods account for approximately known MC and seek joint estimation of the DOA and mutual coupling matrix (MCM). However, these methods are developed specifically for uniformly spaced arrays and cannot be applied to sparse arrays [7]. When dealing with sparse apertures and arrays, global optimization techniques, such as those based on evolutionary and nature-inspired optimization techniques, have shown tremendous success for simultaneously estimating the DOA and MCM [8, 9]. These methods, however, are computationally intensive, not conducive to real-time execution, and may not be viable in some applications.

Neural networks have been proven beneficial in computationally intensive and nonlinear problems. These networks can be trained a priori and applied with relatively simple computations at runtime, as most of the computational load is done off-line. Deep neural networks, referred to as deep learning (DL), have been effectively applied for DOA estimation problems. Multiple architectures have been reported that can accurately resolve DOAs in low SNR environments, unknown number of signals, and with super resolution estimation [10, 11]. However, to the best of the authors' knowledge neural networks have not been applied to the problem of joint estimation of DOA and unknown mutual coupling in sparse arrays.

Compared with uniform arrays, the interelement spacing between two adjacent antennas in sparse arrays can be different. These arrays have shown increased capacity in wireless communications and improved radar functions. In essence, sparse arrays, in lieu of conventional uniform arrays, enable dealing with more targets and radio frequency (RF) emitters than the number of physical sensors [12, 13, 14, 15]. Structured sparse arrays are guided in their configurations by specific formulas, and they include, among many others, co-prime, minimum redundancy, and nested arrays [14, 16, 17]. Non-structured sparse array designs can be data-independent [13] or data-dependent [12, 18]. In this paper, we consider structured nested arrays [16]. Nested sparse array architectures have been shown to perform similar to uniformly spaced arrays with fewer physical elements in DOA estimation problems. The Nested sparse array design is based on the involvement of two uniform arrays but with different corresponding elements spacing which yields a difference coarray with more virtual sensors than physical sensors.

In this work, we train and evaluate a regression based multi-task neural network architecture, made of two parallel sub-networks, to estimate DOA and MCM for a nested sparse array. The input to the neural network is the received signal autocorrelation matrix and it is assumed that the number of incoming signals is known a priori. The trained neural network's DOA performance is compared to

MUSIC's performance on the same dataset, and it is shown that the proposed method performs significantly better than MUSIC with unknown mutual coupling. The MCM estimation task is evaluated using mean absolute error (MAE) for each individual real and imaginary component of the mutual impedance matrix. It is shown that the trained model is able to predict the unknown mutual impedances with relative accuracy.

The remainder of the paper is organized as follows. Section 2 presents the signal model used for the dataset generation. Section 3 outlines the multi-task neural network architecture. Section 4 discusses the training process of the neural network and delineates the results of the trained network with test data. Section 5 is the conclusions.

## 2. Signal Model

In the presence of mutual coupling, the received signal vector with N antennas and K impinging sources can be modeled as [6]

$$\mathbf{x}(t) = \mathbf{C} \cdot \mathbf{A}(\theta) \cdot \mathbf{s}(t) + \mathbf{n}(t) \quad (1).$$

Where $\mathbf{A}(\theta)$ is the N x K array steering matrix whose $k$th column $\mathbf{a}(\theta_k)$ is the steering vector of the $k$th impinging source. The $n$th element of the steering vector is defined as

$$[\mathbf{a}(\theta_k)]_n = e^{-jk(x_n sin\theta_k)} \quad (2).$$

Where $x_n$ is the x location of the $n$th element in the array and $\theta_k$ is the elevation angle of the $k$th source. $\mathbf{s}(t)$ is the source signal vector at snapshot t and $\mathbf{n}(t)$ is the noise vector. The sources are assumed to be narrowband and uncorrelated and the noise vector is spatially and temporally white with variance $\sigma_n^2$ . $\mathbf{C}$ is the mutual coupling matrix defined as $\mathbf{C} = \mathbf{Z}^{-1}$ where $\mathbf{Z}$ is the mutual impedance matrix [17]. The autocorrelation matrix is defined as

$$\boldsymbol{R}_{xx} = E\{\boldsymbol{x}(t) \cdot \boldsymbol{x}^H(t)\} \quad (3).$$

and is replaced by its estimate, sample autocorrelation matrix, incorporating the available number of snapshots [1].

## 3. Neural Network Architecture

The input to the neural network is defined as an NxNx3 matrix where N is the number of sensors, and the third dimension is the real, imaginary, and phase components of the autocorrelation matrix. The remainder of the neural network is designed similar to the architecture proposed in [10]. The input is then fed into two parallel series networks that are identical except for the final fully connected layer. The parallel networks first consist of a convolution stage where there are four convolution layers with 256 filters each, with each convolution layer followed by a batch normalization layer and ReLU activation layer. After the convolution stage there is first a flatten layer to prepare the data for a fully connected stage where there are three fully connected layers of decreasing size, each followed by a ReLU activation layer and a dropout layer with p = 0.2. There is then a final fully connected layer for each task where the output of the DOA branch consists of K outputs, and the output of the MCM branch consists of $2*N^2$ outputs corresponding to the real and imaginary components of the mutual impedance matrix. The outputs of each branch are then concatenated, and the final output is a regression layer. The architecture is shown in Figure 1. There are a total of 41,538,131 learnable parameters in this network.

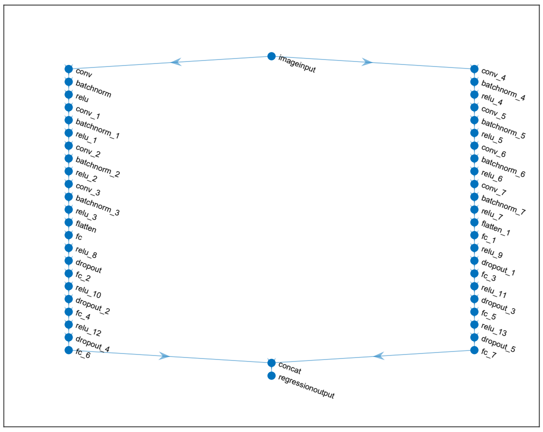


**Figure 1.** Multi-task DOA and MCM estimation neural network architecture

## 4. Results and Discussions

As a preliminary example of the effectiveness of this network, a dataset consisting of 150,000 training, 20,000 validation, and 30,000 testing autocorrelation matrices with the associated DOA and mutual impedance labels for a six-element nested array are generated. The nested array used for this example is shown in Figure 2, with the maximum number of resolvable signals being 11.

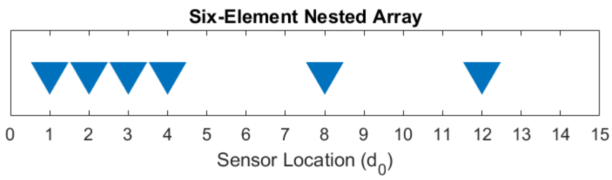


**Figure 2.** Six-element nested array antenna locations

The elements used for the data generation are planar $\lambda/2$ dipoles with a fundamental spacing of $d_0 = 0.5\lambda$. The theoretical mutual impedances are calculated using the induced EMF method [19] and each value varies independently and randomly within a +/-15% range of the calculated value for each data point to simulate a dynamic environment. Eleven equal power sources with SNR = 10dB are generated with directions of arrival randomly generated for each data point between -60° to 60° with at least 4° between them. The number of snapshots used for each data point is 10,000 which is used to compute the time-average estimation of the autocorrelation matrix from equation (3).

The neural network is trained using the ADAM solver with an initial learn rate of 0.001. A mini-batch size of 128 is used with 50 epochs where the mini-batches are shuffled every epoch. The two tasks share a root-mean-square error (RMSE) loss function during training. The training plot is shown in Figure 3.

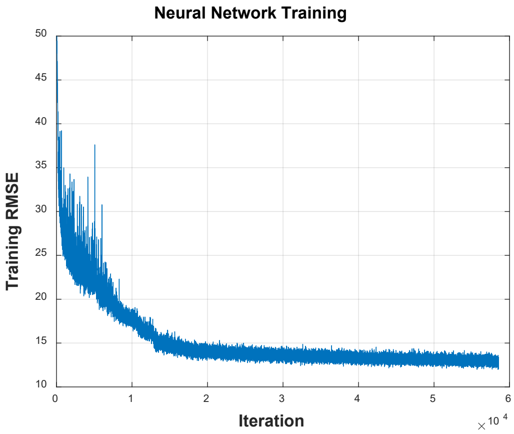


**Figure 3.** Multi-task DOA and MCM estimation neural network training plot

The test data is then used to evaluate the effectiveness of the trained network using data not previously inputted. For the DOA estimation task, an average RMSE of 0.71° is achieved with the training dataset. With this same dataset, MUSIC, which does not account for mutual coupling, achieves an average RMSE of 12.11° and only had 11 peaks 25,692 times out of 30,000. The distribution of RMSEs for both the neural network and MUSIC is shown in Figure 4 and Figure 5 respectively. Figure 6 provides an example of MUSIC's performance compared to the proposed deep neural network (DNN) where the red lines are the actual source locations, and the blue trace gives the methods' estimations. As evident from the figures, the proposed neural network performs significantly better in DOA estimation than MUSIC when mutual coupling is introduced.

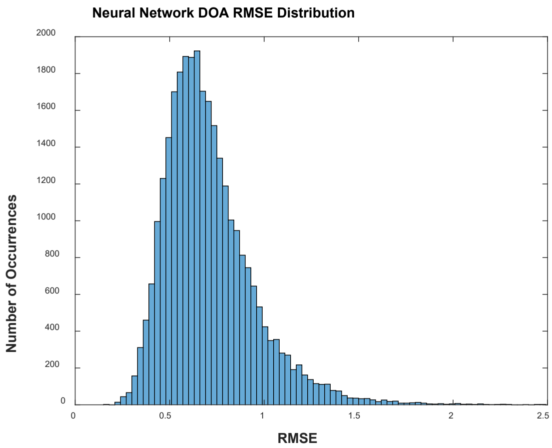


**Figure 4.** Neural network DOA RMSE distribution

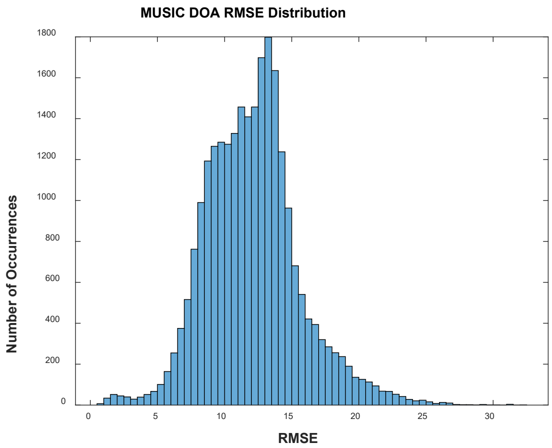


**Figure 5.** MUSIC DOA RMSE distribution

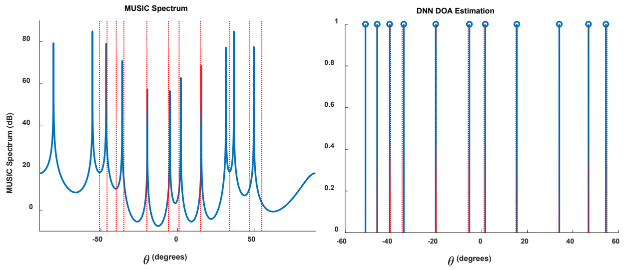


**Figure 6.** Example of MUSIC DOA estimation compared to DNN method

For the MCM estimation task, the performance metric mean absolute error (MAE) is used to evaluate the efficacy in this task. Table 1 shows the calculated mutual impedance matrix, without the applied +/-15% variance since this changes for each for each received signal vector generated. Table 2 shows the MAE for each real and imaginary component individually. The MAEs are calculated independently since each resistance and reactance vary independently for each received signal vector. From the tables, it can be seen that the neural network is able to determine the unknown and varying mutual impedance values with relatively good accuracy.

**Table 1.** Theoretical mutual impedance matrix without applied variance

| | | | | | |
|---|---|---|---|---|---|
| 68.160 - j0.056 | -12.497 - j29.918 | 3.996 + j17.725 | -1.880 - j12.290 | -0.3592 - j5.414 | -0.146 - j3.459 |
| -12.497 - j29.918 | 68.160 - j0.056 | -12.497 - j29.918 | 3.996 + j17.725 | 0.487 + j6.302 | 0.177 + j3.803 |
| 3.996 + j17.725 | -12.497 - j29.918 | 68.160 - j0.056 | -12.497 - j29.918 | -0.698 - j7.534 | -0.218 - j4.222 |
| -1.880 - j12.290 | 3.996 + j17.725 | -12.497 - j29.918 | 68.160 - j0.056 | 1.080 + j9.353 | 0.276 + j4.745 |
| -0.3592 - j5.414 | 0.487 + j6.302 | -0.698 - j7.534 | 1.080 + j9.353 | 68.160 - j0.056 | 1.080 + j9.353 |
| -0.146 - j3.459 | 0.177 + j3.803 | -0.218 - j4.222 | 0.276 + j4.745 | 1.080 + j9.353 | 68.160 - j0.056 |

**Table 2.** Mean absolute error of each mutual impedance term

| | | | | | |
|---|---|---|---|---|---|
| 1.408 + j0.006 | 0.929 + j2.224 | 0.298 + j1.318 | 0.142 + j0.930 | 0.028 + j0.409 | 0.013 + j0.260 |
| 0.940 + j2.249 | 1.657 + j0.019 | 0.931 + j2.218 | 0.301 + j1.334 | 0.037 + j0.472 | 0.020 + j0.292 |
| 0.305 + j1.335 | 0.933 + j2.224 | 1.693 + j0.002 | 0.948 + j2.244 | 0.061 + j0.570 | 0.018 + j0.318 |
| 0.145 + j0.928 | 0.298 + j1.319 | 0.929 + j2.220 | 1.595 + j0.021 | 0.089 + j0.703 | 0.021 + j0.357 |
| 0.030 + j0.408 | 0.037 + j0.471 | 0.052 + j0.577 | 0.084 + j0.704 | 1.498 + j0.004 | 0.087 + j0.705 |
| 0.013 + j0.260 | 0.016 + j0.289 | 0.017 + j0.319 | 0.021 + j0.361 | 0.085 + j0.701 | 1.532 + j0.013 |

## 5. Conclusions

The joint estimation of direction of arrivals (DOA) and mutual coupling matrix (MCM) elements for sparse arrays is a challenging problem that does not lend itself to the use of standard techniques such as MUSIC, and in general it requires utilization of computationally intensive non-convex stochastic optimization methods. In this work, we have used a dual-function (dual-task) deep neural network (DNN), made of two parallel sub-networks, to tackle this problem. Preliminary results for the example of a nested array consisting of six antenna elements demonstrate that the proposed DNN is able to accurately resolve 11 incoming signals and at the same time estimate and compensate for the unknown MCM elements. Results for examples of other structured arrays will be given in the presentation.

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