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# Some Advances and Emerging Perspectives for Inverse Scattering Solution Methods

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## Abstract

We focus on some recent advances in solution methods for the inverse scattering problem. In particular, attention is concentrated on the virtual experiments framework and the possibility of rewriting in a convenient fashion the basic scattering equations. Moreover, a new unconventional application of inverse scattering is discussed, which is the 'inverse design' framework. Finally, some considerations are given about the emerging paradigm of machine learning.

## 1. Introduction

The capability to solve in a reliable and accurate fashion inverse scattering problems is at the basis of very many existing and possible applications, including subsurface imaging, oil exploration, biomedical imaging, and many others. On the other side, these possibilities have to tackle considerable difficulties, the main (inter-related) ones being the non-linearity of the problem, its ill-posedness, the computational burden associated to the need of considering very many unknowns, as well as the need for appropriate and cumbersome measurement set-ups collecting as much information as possible. In this contribution, focus is on some recent advances in solution procedures and their innovative applications, with a particular emphasis on some results achieved in our research groups.

By the sake of simplicity, the following analysis is restricted to the case of the monochromatic electromagnetic 2D scalar problem, i.e. when the target to be imaged is invariant along a direction, and subject to impinging electric fields which are parallel to such an invariance direction (say z).

## 2. Basic Equations and Notation

In the case at hand (see also fig. 1) equations ruling the problem can be written as :

$$E_s(\underline{r}_m,\underline{r}_t) = A_e\left[\chi(\underline{r})E(\underline{r},\underline{r}_t)\right] \quad (1)$$

and

$$E\left(\underline{r},\underline{r}_t\right) = E_i\left(\underline{r},\underline{r}_t\right) + A_i\left[\chi(\underline{r})E(\underline{r},\underline{r}_t)\right] \quad (2)$$

where $E_i$ , $E_s$ and $E$ are respectively the incident, the scattered and the total fields, $\chi$ is the function encoding the target properties, while $\underline{r}$ spans the investigation domain $\Omega$ and $\underline{r}_m,\underline{r}_t$ identify the receiving and transmitting antenna positions on a given observation curve $\Gamma$. Finally, $A_e$ and $A_i$ are short notations of the external and internal radiation operators, respectively.

By introducing the so called 'Contrast sources' $W = \chi E$, the same equations can also be written as :

$$E_s(\underline{r}_m,\underline{r}_t) = A_e\left[W(\underline{r},\underline{r}_t)\right] \quad (3)$$

and

$$W\left(\underline{r},\underline{r}_t\right) = \chi(\underline{r})E_i\left(\underline{r},\underline{r}_t\right) + \chi(\underline{r})A_i\left[W\left(\underline{r},\underline{r}_t\right)\right] \quad (4)$$

In both cases, the aim is to retrieve the contrast function $\chi$ from the knowledge of incident $E_i$ and scattered $E_s$ fields. As well known, both the contrast function and internal fields (contrast sources) are unknown of the problem, which is therefore non-linear.

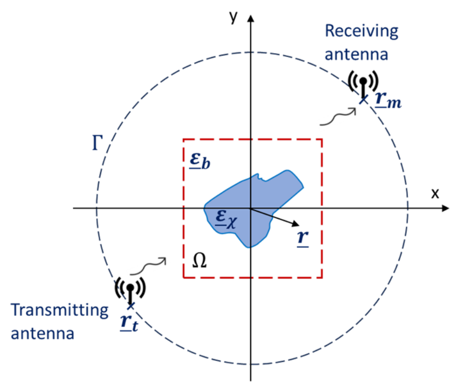


Fig. 1: Sketch of the 2D inverse scattering problem at hand

## 3. Some Recent Approaches to Solution

Non linearity of the inverse scattering problem implies that solution procedures may be trapped into the so called 'false solutions', which nothing have to do with the actual ground truth. Because of its very challenging nature, the quest for fast and reliable solution methods for quantitative inverse scattering problems continue to be the subject of very many scientific contributions, the last trend being the adoption of Machine Learning or even Deep Learning, with very many different options (see for example [1], [2], [3]). By the sake of space, we focus in the following on some recent more traditional approaches, and defer the consideration of some interesting open issues related to ML related approaches to the last Section.

## 3.1 The virtual experiments paradigm

By looking at eq.(1), a very obvious observation is that the knowledge of the internal field, or of the contrast sources, would give rise to linear a problem, so that issues arising from non-linearity would disappear (ill-posedness being still there).
At the same time, as a linear combination of incident fields give rise to the same linear combinations of the scattered fields, a weighted superposition of scattered fields can be conceived as the result of a 'virtual' scattering experiment arising from the same linear superposition of the incident fields. Then, the basic idea of the approach originally presented in [4] is to design these virtual experiments in such a way that the internal fields (contrast sources) exhibit some expected property. In such a way, the (even partial) knowledge of the internal field (contrast sources) allows to alleviate non linearity, and hence the occurrence of false solutions.
Obviously, the problem arises of which kind of fields one wants to deal with, and how to design these 'virtual experiments'. To this end, the realization of a focused internal field (and/or of a focused Contrast source) appears to be particularly attractive. In fact, it corresponds to a kind of 'local' interrogation of the region of interest, which is a way to keep under control mutual interactions amongst the different parts of the scatterer at hand, and hence non-linearity.
Very interestingly, in a large relevant set of cases, these internal fields focused around given 'pivot points' of the scatterer can be effectively achieved by means of the far field equation underlying the so-called Linear Sampling method [5], i.e.:

$$\int_\Gamma \alpha(\underline{r}_p, \underline{r}_t) E_s(\underline{r}_m, \underline{r}_t)\, d\underline{r}_t = G_b(\underline{r}_m, \underline{r}_p) \qquad (5)$$

where $r_p \in \Omega$ is the considered 'pivot' point, and $\alpha$ identifies the combination function required to implement the VE that give rise to the sought circular symmetry around the given pivot points $r_p$. Moreover, $G_b$ is the Green's function pertaining to the background medium, i.e., the field radiated by an elementary source located in $r_p$ and observed in $\underline{r}_m \in \Gamma$.
In fact, solving such an equation amounts to look for a combination of the scattered fields measured in the different experiments such that the field appears to emerge from a single (pivot) point. Then, the achieved combination can be assumed to be the required combination of incident (and total) fields, thus solving the virtual experiments design problem. Alternative convenient procedures can also be derived by using the so called 'Orthogonality Sampling Method' [6].
The method was originally conceived in [3] on the basis of the physical interpretation of the LSM given in [7]. Notably, it has also proven to be of interest when the target has holes, as one is still enforcing circularly symmetric fields, and take advantage from that.
It is also very interesting that, leveraging on the focused behavior of the virtual contrast sources, (as well as on the properties and expansions of Bessel functions), in very many cases one is able to reduce the problem to the solution of a diagonal system of third degree algebraic equations, which leads in turn to a non-iterative, and hence very fast, solution procedure [8]. While comparable in terms of computational burden to the Born approximation, the approach (very) largely exceeds the limitations of this latter.
The basic method of [3] has then successfully extended to the case where the background is non homogeneous, which has finally allowed to develop a very effective 'Distorted Iterated Virtual Experiments' (DIVE) solution method [9]. The more recent efforts aim to extend the range of applicability and accuracy of the method by keeping under control the focusing shift occurring in case of exceedingly large scatterers or contrast functions.

## 3.2 Effective rewritings of the scattering equations

The non-linearity of the inverse scattering problem is due to the mutual (or even self) interactions amongst the different (parts of the) scatterers. It can be mathematically represented through the state equation (Eq. (2) or (4)), also known as the Lippman-Schwinger equation. In particular, the norm of the product $\chi A_i$ can be seen as a 'measure' of the degree of non-linearity of the problem [10].
Then, the idea underlying some recently introduced inversion approaches relies on the possibility of effectively rewriting the state equation in such a way to reduce the degree of non-linearity, without adopting any kind of approximation.
A first attempt (see [10] and references therein) was by means of the so called Contrast Source Extended Born (CS-EB) model. In this latter, the state equation is recast by taking advantages from the behavior of the Green's function in lossy media. Another effective rewriting was introduced in [11]. Herein, the local effects of the induced Contrast sources are extracted in such a way to alleviate the amount of the non-linearity. In both models, the relevant internal radiation operators are substituted with new ones (denoted in the following as $A_i^{MOD}$), while the target properties are encoded in new auxiliary functions (denoted in the following as $p$). Interestingly, these substitutions allow for a lower norm of the product $pA_i^{MOD}$ with respect to the standard scattering formulation of the problem.
Finally, in [12] the 'Y0 model' was introduced. This model is based on a decomposition of the pertaining Green's function and the extraction of the main contribution of the radiating currents to the total field from the internal integral operator. In this case, not only the integral internal operator was replaced by a new one, but also the incident field by a new known 'modified incident' field, which can be easily computed from the data. As discussed therein, the Y0 model also exhibits a lower degree of non-linearity with respect to the standard scattering model.

Notably, hybridizations are possible between the virtual experiment framework described in Section 3.1 and the above rewritings of the state equations [10]. Also note the virtual experiments can also be seen as possible rewritings of the equations describing the different scattering experiments through proper combinations of the data.

## 4. A new task for inverse scattering tools: inverse design

Inverse scattering for solving retrieval problems continues to be a challenging problem, so that further relevant progress is expected in the forthcoming years (see also sect. 5 below). Also, any progress in procedures will give a further boost to the very many possible applications. At the same time, interest has recently arisen, with a very large number of publications in the literature, for the so-called 'inverse design' (i-Design) framework. In such a paradigm, knowledge from scattering theory allow to cast complex design tasks as a problem where the solution one is looking for is forced to have a set of desired properties and functional requirements.

Hence, dealing with optimal i-Design of electromagnetic devices, eventually based on graded index metamaterials, entails tuning a number of variables describing the device under test (DUT) and hence facing a very complex, non-linear, inverse problem. In this respect, very many i-Design procedures rely on brute force (trial and error) numerical optimization, while other approaches rely on approximated theories or simplifying hypotheses, such as homogenization theories [13] and local periodicity assumptions.

Interestingly, inverse scattering theoretical tools and actual procedures, as originally developed for retrieval problems, can efficiently contribute to the topic [14], [15]. In fact, by changing the data of the problem, i.e., by substituting measurements of the total field with given total field (e.g, aperture field) specifications, the solution of the corresponding inverse scattering problem corresponds to the design of a given DUT. Notably, one could also benefit from a series of useful characteristics:

i. the degrees of freedom of scattered fields [16] allow to set ultimate theoretical limitations and goals in the synthesis;
ii. non-linearity is not necessarily an issue, but it can become a possibility. In fact, having multiple solutions to the i-Design problem can simplify actual realization and reduce dimensions/costs;
iii. only amplitude (phaseless) specifications can add further degrees of freedom, and recently developed tools can be applied to manage the additional difficulties ;
iv. regularization strategies handling ill-posedness can be profitably explored as design facilitators to enforce spatial properties on the DUT and chose the best solution;
v. multi-purpose or reconfigurable devices can be seen as a multi-view/multifrequency inverse scattering problem.

All the above led our group to develop in [17] an i-Design procedure through a modified Contrast Source Inversion (CSI) method for the design of dielectric lens antennas, resembling the composition of a graded index metamaterial, realized by means of properly synthesized inclusions in void. As a matter of fact, by starting from a given aperture field, we were able to achieve performance close to the optimal theoretical ones for a device acting as a lens and able to realize two different patterns when subject to different impinging fields.

As a further step, we challenged ourselves to design metamaterials-like devices without using full wave numerical discretization or homogenization theories. To this end, we took advantage from the scattering matrix method (SMM), where each inclusion is conveniently managed by means of a matrix relating the coefficients of a basis for the incident fields to the coefficients of a representation for the scattered field. The approach, originally developed in [18] as a tool for the analysis of the scattering problems, was turned into a design tool. In particular, we were able to efficiently synthesize lens antennas [19] as well as guiding devices [20]. The i-SMM showed good performance also in comparison to the previous CSI-based approach, both in terms of specifications fitting and computational burden.

Our present activities are devoted to considering the additional presence of a planar Perfect Electric Conductor, so that I-SMM can be applied to the design of the nowadays very popular smart skins.

## 5. A very relevant trend, and some related issues

The vast majority of recent contributions for effective solutions methods of inverse scattering problems relies on the use of Machine Learning or even Deep Learning procedures. A good synthesis of the different general approaches can be found in [1], and although many other papers have appeared since then, a further general review is out of the scope of this contribution, which just focuses on some aspects of the extremely broad topic. Hence, on the basis of our present understanding, we focus below on some aspects, which we deem particularly relevant, concerning the relationship amongst ML based methods and more traditional inversion approaches.

Training is obviously a key point. In terms of inverse problems traditional tools, it can be seen as a form of regularization of the following actual inversion. In fact, it enforces a priori on the basis of some statistical inference (rather than a specific starting point or regularization term), a relevant difference being the setting of very many parameters rather than more rigid schemes. Such a circumstance suggests several interesting synergies, such as seeing the optimal training as a inverse problems itself, as well as an increased flexibility for more classical inversion techniques.

More important than that, the stimulating area of eXplainable (Machine, or even Deep) Learning could bring to completely new (and unexpected) ways of approaching the problem, thanks to the expected capability to understand the rationale of the performed processing.

In a reverse fashion (with respect to learning from the machines optimized behavior) there seems to be some consensus that Physics has somehow to enter in the scenario in order to optimize performances of machines. On the other side, it is still an open issue how Physics can enter in the processing in the best possible fashion. One interesting although conservative possibility is setting architectures having a preprocessing stage extracting some 'easy' characteristic, a central core (eventually simulating the forward problem, which can often be thought in terms of convolutions) and a post processing (further) enforcing the available a priori information.

Inverse design by means of ML or even DL enforces further issues, as in recovery problems one has a single ground truth to communicate (to set) in the training stage, while in inverse problems for design many different solutions could be equally adequate.

As it can be seen from these brief and incomplete notes, there is very much to be done, including very many possible hybridizations (see for example [21]) amongst all the above (and more). So that we skip a contestable list of future work.

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## References

[1] X. Chen, Z. Wei, L. Maokun, and P. Rocca, "A review of deep learning approaches for inverse scattering problems," *Electromagnetic Waves*, 167, 67-81, 2020.

[2] L. Li, L. G. Wang, F. L. Teixeira, C. Liu, A. Nehorai, and T. J. Cui, "DeepNIS: Deep neural network for nonlinear electromagnetic inverse scattering," *IEEE Trans. on Antennas and Propagation*, 67(3), 1819-1825, 2018.

[3] Á. Yago Ruiz, M. Cavagnaro, L. Crocco, "An Effective Framework for Deep-Learning-Enhanced Quantitative Microwave Imaging and Its Potential for Medical Applications", *Sensors.* 23(2), 2023.

[4] L. Crocco, I. Catapano, L. Di Donato, and T. Isernia, "The linear sampling method as a way to quantitative inverse scattering," *IEEE Transactions on Antennas and Propagation*, 60(4), 1844-1853, 2012.

[5] D. Colton, H. Haddar, M. Piana,'The Linear Sampling Method in Inverse Electromagnetic Scattering Theory", *Inverse Problems,* 19:105–137, 2003.

[6] M. T. Bevacqua, R. Palmeri, T. Isernia, and L. Crocco, "A simple procedure to design virtual experiments for microwave inverse scattering," *IEEE Transactions on Antennas and Propagation*, 69(12), 8652-8663, 2021.

[7] I. Catapano, L. Crocco, T. Isernia, "On simple methods for shape reconstruction of unknown scatterers," *IEEE Trans. on Antennas and Propag.,* 55(5), 1431-1436, 2007.

[8] M. T. Bevacqua, L. Crocco, L. Di Donato, and T. Isernia, "An algebraic solution method for nonlinear inverse scattering," *IEEE Transactions on Antennas and Propag.*, 63(2), 601-610, 2015.

[9] R. Palmeri, M. T. Bevacqua, L. Crocco, T. Isernia, and L. Di Donato, "Microwave imaging via distorted iterated virtual experiments," *IEEE Transactions on Antennas and Propag.*, 65(2), 829-838, 2016.

[10] M. T. Bevacqua, and T. Isernia, "Quantitative non-linear inverse scattering: A wealth of possibilities through smart rewritings of the basic equations," *IEEE Open Journal of Antennas and Propagation* 2, 335-348, 2021.

[11] Y. Zhong, M. Lambert, D. Lesselier, and X. Chen, "A new integral equation method to solve highly nonlinear inverse scattering problems," *IEEE Trans. Antennas Propag.,* vol. 64, no. 5, pp. 1788–1799, May 2016

[12] M. T. Bevacqua, and T. Isernia. "An effective rewriting of the inverse scattering equations via Green's function decomposition," *IEEE Trans. on Antennas and Propag.* 69(8), 4883-4893, 2021.

[13] S. Datta, C. T. Chan, K. M. Ho, and C. M. Soukoulis, "Effective dielectric constant of periodic composite structures," Phys. Rev. B, Condens. Matter, 48(20), 1993.

[14] R. Palmeri and T. Isernia, "Volumetric invisibility cloaks design through spectral coverage optimization," *IEEE Access*, 7, 2019.

[15] T. Brown, C. Narendra, Y. Vahabzadeh, C. Caloz, and P. Mojabi, P., "On the use of electromagnetic inversion for metasurface design," *IEEE Trans. Antennas Propag.,* 68(3), 2019.

[16] O. M. Bucci and G. Franceschetti. "On the degrees of freedom of scattered fields," *IEEE Trans. Antennas Propag.,* 37(7), pp. 918-926, 1989.

[17] R. Palmeri, M. T. Bevacqua, A. F. Morabito, and T. Isernia, "Design of artificial-material-based antennas using inverse scattering techniques," *IEEE Trans. Antennas Propag., 66(12), 2018.*

[18] D. Felbacq, G. Tayeb, D. Maystre, "Scattering by a random set of parallel cylinders*," J. Opt. Soc. Am. A, 11(9), 1994.*

[19] R. Palmeri and T. Isernia, "Inverse design of artificial materials based lens antennas through the scattering matrix method," *Electronics,* 9(4), 2020.

[20] R. Palmeri and T. Isernia, "Inverse design of EBG waveguides through scattering matrices," *EPJ Applied Metamaterials,* 7(10), 2020.

[21] M. T. Bevacqua, C. Ieracitano, N. Mammone, F. C. Morabito, T. Isernia, L. Di Donato, "Electromagnetic Inverse Scattering via Deep Learning Enhanced by Virtual Experiments," 2023 Prague Photonics & Electromagnetics Research Symposium (PIERS) Proceedings, pp.1231-1236