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Scattering from cylinders parallel to a perfectly conductive plane: an efficient analysis tool

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Abstract

The plane-wave scattering from a set of cylinders in presence of a totally reflecting plane is dealt with. In particular, starting from the scattering matrix method which treats the same scattering problem for the free-space, we exploit image principle arguments to model the additional scattering contributions due to the presence of the conductive plane while still adopting the free-space scattering matrix of each scatterer.

1. Introduction

The phenomenon of the 2D scattering of a plane wave by N arbitrary cylinders has been treated and widely explored in the literature with reference to different kinds of targets, ranging from few scatterers [1] to wires [2], small [3] or comparable to the wavelength [4] targets, either dielectric or metallic. Most of the analyzed scenarios are homogenous media, but the case of scattering in presence of an interface could be of interest in many practical applications, from optics to microwaves, such as metamaterials, metasufaces, smart skins and so on.
Regarding the analysis of this kind of devices, two comments are in order . First of all, their elementary cell, which represent the n-th scatterer in the model we are considering, is usually small with respect to the operative wavelength. Moreover, they can be composed by a huge number of cells (scatterers). In this respect, traditional analysis procedures based on numerical models could be computationally heavy, so it would be useful developing efficient tools gathering accurate results while keeping the computational burden as low as possible.
In 1994, Maystre and co-workers [5] developed the Scattering Matrix Method (SMM) for the evaluation of the field scattered by a random set of parallel cylinders in free space. Basically, that approach relies on the representation of the electromagnetic properties of the scatterers in terms of scattering matrices linking the coefficients of a suitable representation basis for the incident and scattered field. By so doing, the scattering problem can be solved through a system of linear equations in which the right-hand sides take into account the kind of incident field (e.g., plane or cylindrical wave) and the matrix relating data-unknowns embeds the mutual interactions between scatterers. It is worth noting that the computational complexity of the SMM grows with the sum of the perimeters of the different scatterers (and not with the sum of their areas); hence, this analysis tool allows a large saving in the number of unknowns with respect to mesh-based solvers.
By taking profit of the SMM advantages listed above, we focused on the evolution of such a tool to treat the plane wave scattering in presence of a plane conductive surface. The proposed approach still allows the adoption of the free-space scattering matrix of each scatterers. More in detail, the presence of the planar PEC is handled through "image matrices" and the contributions due to the PEC are incorporated in the linear system as field radiated by the fictitious scatterers plus the reflection of the impinging plane wave.

2. The Scattering Matrix Method (SMM)

Let us denote by $\mathbf{a}_l$ and $\mathbf{b}_l$ the finite dimensional vectors ordering the coefficients, respectively, of the incident and scattered field in the chosen basis, relative to the l-th scatterer. Notably, the bases can be safely truncated [6].
Let us also define the scattering matrix, S_l, associated to the l-th scatterer such that:

$$\mathbf{b}_l = S_l \mathbf{a}_l \tag{1}$$

This matrix is available in a closed form for scatterers with canonical shapes (even for stratified cylinders [7]) and it is symmetric in case of circular targets.
When the N (different) cylindrical scatterers are in free space, the incident field coefficients can be given as:

$$\mathbf{a}_l = \mathbf{Q}_l + \sum_{i=1, i\neq l}^{N} T_{l,i} \mathbf{b}_i \tag{2}$$

where $\mathbf{Q}_l$ are the coefficients of the expansion representing the primary incident field, while $T_{l,i}$ are the matrices accounting for the mutual interactions, so they just depend on the positions of the scatterers.
By substituting (2) in (1), one derives the following linear system:

$$\mathbf{b}_l - \sum_{i=1, i\neq l}^{N} S_l T_{l,i} \mathbf{b}_i = S_l \mathbf{Q}_l \tag{3}$$

that, once solved for $\mathbf{b}_l$, allows to compute the scattered field in any point of the space (outside the area of cylinders).

3. The modified SMM

In the free-space SMM the overall total field is computed as the sum of three terms, that are, the primary incident field, the scattered field from the l-th cylinder and the field scattered from all the other cylinders, which act as secondary incident field on the l-th cylinder [5].
Based on the above, the presence of a planar PEC implies a doubling of the contributions. In fact, for each scatterer we have to consider the reflected (primary) incident field, the reflected l-th scattered field and the reflected scattered field from all the other *N-1* cylinders, see Figure 1(a).
In order to incorporate these new contributions in the SMM model, we restore a free-space configuration by exploiting the "images" method, so that the reflected fields can be seen as fields radiated by fictitious entities, see Figure 1(b).
According to the above, the linear system (3) is recast as follow:

$$\mathbf{b}_l = S_l\mathbf{Q}_l + S_l\widehat{\mathbf{Q}}_l + \sum_{i=1,i\neq l}^{N} S_l T_{l,i}\mathbf{b}_i + \sum_{i=1}^{N} S_l \hat{T}_{l,i}\mathbf{b}_i \qquad (4)$$

where $\widehat{\mathbf{Q}}_l$ are the coefficients of the expansion representing the reflected primary incident field, while matrices $\hat{T}_{l,i}$ play a double role. In fact, they embed the change of the reference system from the i-th image scatterer to the l-th cylinder. Moreover, they take into account the fact that the expansion of the field radiated by the i-th image has to accommodate the annihilation of the tangential field on the PEC when paired to the actual scatterer, which is indeed the circumstance allowing to express the last summation of (4), by virtue of a proper analytical processing, in terms of the original $\mathbf{b}_i$ coefficients.

4. Numerical Validation

In this section, some numerical examples concerning the evaluation of the scattered field from parallel cylinders above a PEC planar surface are presented.
In the following, we treat metallic targets with circular cross-section. In particular, we investigate the case of multiple scatterers for normal incidence of the plane wave. The first considered scenario is composed of two scatterers with radius $a = \lambda/10$ and an inter-element spacing of $\lambda/2$, λ being the operative wavelength. In Figure 2 it is shown the amplitude of the total scattered field obtained with the modified SMM (left) and with the full-wave simulation performed in COMSOL Multiphysics (right).
The second scenario consists of three smaller scatterers, with radius $a = \lambda/12$ and an inter-element spacing of $\lambda/4$. Figure 3 still shows the amplitude of the total scattered field obtained with the modified SMM (up-left) and with the full-wave simulation performed in COMSOL Multiphysics (up-right).

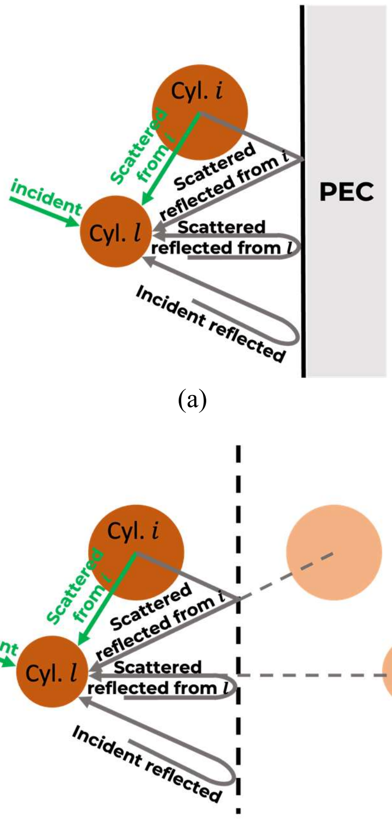

Figure 1. Schematization of the non-homogenous scenario at hand.

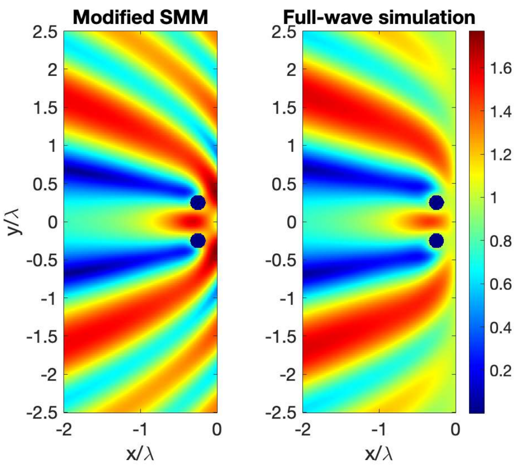

Figure 2. Amplitude of the total scattered field for the case of *N=2* scatterers with radius $\boldsymbol{a} = \boldsymbol{\lambda}/10$ and inter-element spacing of $\boldsymbol{\lambda}/2$. Comparison between the modified SMM (left) and full-wave simulation (right).

As it can be seen, in both cases the results are indeed consistent. The slight differences between SMM and full-wave simulation are due the corner diffraction occurring in the full-wave model at the interface between the PEC condition on the PEC side and the Perfectly Matched Layer at the other boundaries. To corroborate the above statement, we provide in the second row of Fig. 3 the field obtained by halving the vertical side of the investigation domain (note that the pictures pertaining to the COMSOL simulations just show the field in a sub-domain), from which it can be observed greater artifacts since the corner was moved closer to the scatterers.

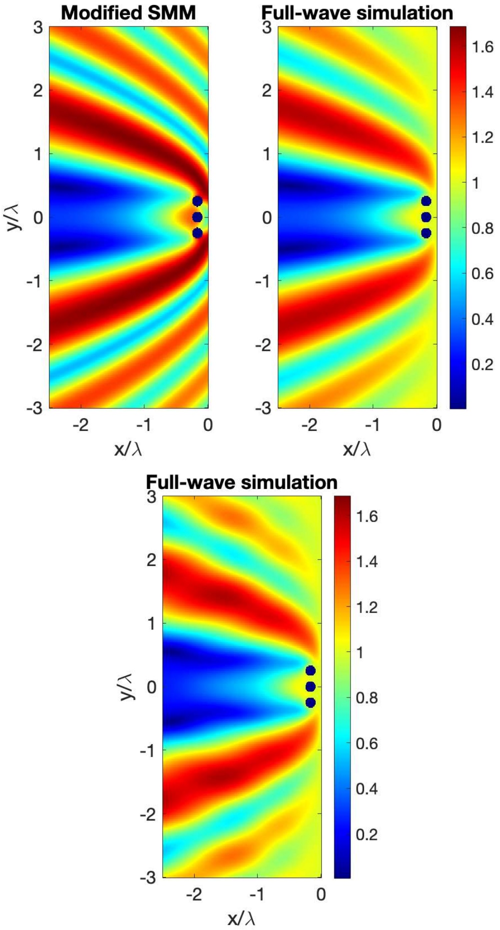

Figure 3. Amplitude of the total scattered field for the case of *N=3* scatterers with radius $\boldsymbol{a} = \boldsymbol{\lambda}/12$ and inter-element spacing of $\boldsymbol{\lambda}/4$. Comparison between the modified SMM (up-left), full-wave simulation (up-right) and full-wave simulation with vertically-halved simulation domain (bottom).

5. Conclusions

We presented a simple and effective method to treat the plane wave scattering from a set of arbitrary cylinders parallel to a perfect electric conductor plane. The tool, which can be easily extended to other kind of impinging fields, is based on a proper exploitation of the 'scattering matrix' associated to each scatterer. Then, a careful application of image principle arguments allows to handle the PEC discontinuity by means of the introduction of simple additional terms. The obtained preliminary results encourage to further investigate such a canonical problem that is of interest in many applications, such as for example smart skins, or metamaterial-based lenses (see for example [7]). In this respect, future work will concern the adoption of the proposed tool within an inverse design framework [9], as previously done by the authors for antenna design [8],[10], to synthesize structures realizing anomalous reflection or other kind of scattered fields. To this end, we can explore available skills in phase retrieval [11]-[13] and power pattern synthesis methods.

6. Acknowledgements

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References

[1] K. Hongo, ''Multiple scattering by two conducting circular cylinders,'' IEEE Trans. Antennas Propag. AP-26, 748–751 (1978).

[2] J. H. Richmond, ''Scattering by an arbitrary array of parallel wires,'' IEEE Trans. Microwave Theory Tech. MTT-13, 408–412 (1965).

[3] J. R. Wait, ''Reflection at arbitrary incidence from a parallel wire grid,'' Appl. Sci. Res. B 4, 393–400 (1954).

[4] N. Zitron and S. N. Karp, ''Higher-order approximations in multiple scattering. Two-dimensional scalar case,'' J. Math. Phys. 2, 394–406 (1961).

[5] D. Felbacq, G. Tayeb, D. Maystre, "Scattering by a random set of parallel cylinders," J. Opt. Soc. Am. A, vol. 11, no. 9, pp. 2526 2538, Sept. 1994.

[6] O. M. Bucci and T. Isernia, "Electromagnetic inverse scattering: Retrievable information and measurement strategies," Radio science, 32(6), pp. 2123-2137, 1997.

[7] O. Zetterstrom, N. J. G. Fonseca and O. Quevedo-Teruel, "Compact Half-Luneburg Lens Antenna Based on a Glide-Symmetric Dielectric Structure," in *IEEE Antennas and Wireless Propagation Letters*, 21(11), pp. 2283-2287, 2022.

[8] R. Palmeri and T. Isernia, "Inverse design of artificial materials based lens antennas through the scattering matrix method," Electronics, 9(4), p. 559, 2020.

[9] R. Palmeri, M. T. Bevacqua, A. F. Morabito, and T. Isernia, "Design of artificial-material-based antennas using inverse scattering techniques," *IEEE Transactions on Antennas and Propagation*, vol. 66, n. 12, pp. 7076-7090, 2018.

[10] R. Palmeri and T. Isernia, "Inverse design of EBG waveguides through scattering matrices," *EPJ Applied Metamaterials*, 7(10), 2020.

[11] G. M. Battaglia, R. Palmeri, A. F. Morabito, P. G. Nicolaci, and T. Isernia, "A non-iterative crosswords-inspired approach to the recovery of 2-D discrete

signals from phaseless fourier transform data," *IEEE Open Journal of Antennas and Propagation*, vol. 2, pp. 269-280, 2021.

[12] R. Palmeri, G. M. Battaglia, A. F. Morabito, and T. Isernia, "Reflector antennas characterization and diagnostics using a single set of far-field phaseless data and crosswords-like processing," *IEEE Transactions on Antennas and Propagation*, vol. 70, n. 9, pp. 8424-8439, 2022.

[13] G. M. Battaglia, A. F. Morabito, R. Palmeri, and T. Isernia, "Effective Non-Iterative Phase Retrieval of 2-D Bandlimited Signals With Applications to Antenna Characterization and Diagnostics," *IEEE Transactions on Antennas and Propagation*, vol. 71, n. 8, pp. 6444-6453, 2023.