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Numerical evaluation of toroidal dipolar moments in thin-wire knots

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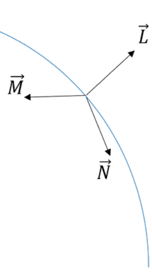
The study of the toroidal moments in electrodynamics was first introduced in the 1960s, and their existence as an independent multipole family to characterize electromagnetic sources, based uniquely in space-time parity arguments, is still a controversial topic nowadays. Rigorous mathematical derivations from dynamic multipolar expansions of scalar and vector potentials, when combined to seek invariant features in terms of varying coordinate systems, lead to solid conclusions about the infeasibility of separating toroidal multipoles from classical electromagnetic multipoles. However, there is an agreement regarding the extraordinary potential for the application of structures which support a dominant toroidal response to induce unique electromagnetic phenomena, such as the Aharonov-Bohm effect. A summary of the mathematical characterization of toroidal multipole moments for any bounded set of electromagnetic sources can be found in [1], while a review paper covering some experimental cases with toroidal responses was published in [2]. Interestingly, most of these examples are in the infrared and optical regimes, with only a few aimed at mimicking the response in the microwave regime by utilizing a thorough combination of electric and magnetic currents.

In this presentation we will explore the existence of dipolar toroidal polar moments in thin-wire knots at microwave frequencies. The unique properties of knotted geometries are still underexplored for microwave applications, either as scatterers [3] or antennas [4]. One of the reasons for this is the complexity of the mathematical description of knots, where small variations of parameters in the generating equations lead to substantial changes in the spatial knotted geometry, which subsequently hinders the extraction of any physical inferences through computer explorations. This presentation will focus on summarizing the numerical procedure to characterize the polar-toroidal dipolar multipole coefficient for thin-wire *(p-q)* torus knots, as geometries that live on the surface of a supporting torus. Intensive electromagnetic simulations, enabled by considering only thin-wire version of *(p-q)* torus knots, confirm them as *a-priori* ideal simple structures capable of supporting combined electric/toroidal responses.

$$\vec{L}_{lm} = \frac{1}{k}\nabla j_l(kr) Y_{lm}(\theta,\phi)$$

$$\vec{M}_{lm} = h_l^{(1)}(kr)\frac{1}{\sqrt{j(j+1)}}\mathcal{L}Y_{lm}(\theta,\phi)$$

$$\vec{N}_{lm} = \frac{\mathrm{i}}{k\sqrt{j(j+1)}}\nabla\times\left(h_l^{(1)}(kr)\mathcal{L}Y_{lm}(\theta,\phi)\right)$$



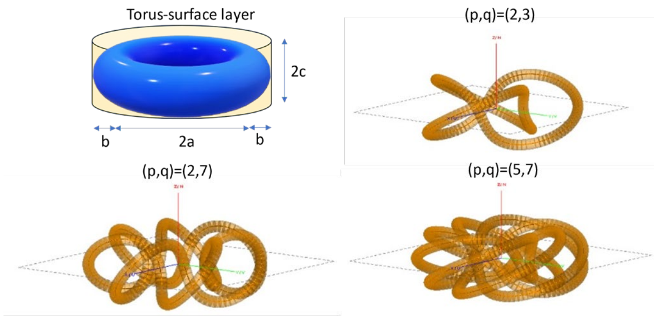


Figure 1. (Left) Vector Spherical Harmonic basis to calculate polar-toroidal response (Right) Some examples of polar *(p-q)* torus thin-wire knotted geometries.

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3. D. H. Werner, “Radiation and Scattering from Thin Toroidally Knotted Wires.” *IEEE Transactions on Antennas and Propagation*, **47**, 8, August 1999, pp. 1351–63, doi: 10.1109/8.791955.
4. S. Shukla, K. Saraswat, and A. R. Harish, “Knot as Antennas: A Brief Review,” in *2022 IEEE International Symposium on Antennas and Propagation and USNC-URSI Radio Science Meeting (AP-S/URSI)*, Denver, CO, USA: IEEE, Jul. 2022, pp. 1988–1989. doi: 10.1109/AP-S/USNC-URSI47032.2022.9886690.