



Fast Solvers Based on Generalized Source Integral Equations with Improved Kernels

Yaniv Brick⁽¹⁾, Arkadi Sharshesky⁽²⁾, Dror Zvulun⁽²⁾, Richard Kalhöfer⁽³⁾, Ludger Klinkenbusch⁽³⁾, and Amir Boag*⁽²⁾

(1) Ben-Gurion University of the Negev, Beer-Sheva, Israel, e-mail: yaniv.brick@gmail.com

(2) Tel Aviv University, Tel Aviv, Israel; e-mails: arkadish123@gmail.com, dorzvu@gmail.com, boag@tau.ac.il

(3) Kiel University, Kiel, Germany; e-mails: crk@tf.uni-kiel.de, lbk@tf.uni-kiel.de

Recent years have seen an increasing interest in the development of fast direct integral equation solvers. These do not rely on the convergence of iterative procedures for obtaining the solution. Instead, they compute a compressed factorized form of the impedance matrix resulting from the discretization of an underlying integral equation. The compressed form can then be applied to multiple right-hand sides, at a relatively low additional cost. The most common class of direct integral equation solvers exploits the rank-deficiency of the off-diagonal blocks of the impedance matrix, in order to express them in a compressed manner. However, such rank deficiency is inherent to problems of small size compared to the wavelength as well as to problems of reduced dimensionality, e.g., elongated and quasi-planar problems.

The present work proposes a class of Generalized Source Integral Equation (GSIE) formulations [1-3], which aim to extend the range of problems exhibiting inherent rank-deficiency. The new formulations effectively reduce the problem's dimensionality and, thus, allow for efficient low-rank matrix compression. When these formulations are used with hierarchical matrix compression and factorization algorithms, fast direct solvers are obtained. Shielded-source [1, 2] and multipole-based [3] types of directional kernels to be employed in the GSIEs are proposed. The computational bottlenecks introduced by the shielded-source kernels are reduced by using non-uniform grid (NG) sampling techniques. The NG techniques, originally developed for fast iterative solvers, are employed for efficient computation of fields produced by the shielded sources. On the other hand, recently developed multipole-based kernels facilitate simpler and more efficient field computation. The two formulations' properties and limitations are studied and their use for the development of fast direct solvers is showcased.

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2. D. Zvulun, Y. Brick, and A. Boag, "A Generalized Source Integral Equation for Enhanced Compression in Three Dimensions," *IEEE Trans. Antennas and Propagation*, vol. 71, no. 12, pp. 9316-9325, Dec. 2023, doi: 10.1109/TAP.2023.3242427.

3. R. Kalhöfer, L. Klinkenbusch, and A. Boag, "MoM Formulation for Large Impenetrable Scatterers Using Deep-Shadow Producing Multipole Kernels," *ICEAA '24 – International Conference on Electromagnetics in Advanced Applications*, Venice, Italy, October 2023.