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Scattering and Characteristic Modes of Nonreciprocal Particles

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Extended abstract

The concept of characteristic modes can be used to capture the dominating scattering from material structures. Although mostly associated with perfect electric conductor structures [1], recent developments have shown how they can be defined as eigenvectors for the scattering matrix or the transition matrix [2]. This formulation makes it possible to compute characteristic modes also for nonreciprocal structures [3]. In this paper, we demonstrate the implications of this theory for nonreciprocal particles.

The scattering matrix $\mathbf{S}$ maps a vector of ingoing spherical vector wave amplitudes $\boldsymbol{a}$ to a vector of outgoing spherical vector wave amplitudes $\boldsymbol{b}$ for a given scatterer as $\mathbf{S}\boldsymbol{a} = \boldsymbol{b}$. Characteristic modes are eigenvectors of the scattering matrix, $\mathbf{S}\boldsymbol{a}_n = s_n\boldsymbol{a}_n$. A convenient indicator of nonreciprocity in lossless scatterers for any excitation $\boldsymbol{a}$ is $\nu(\boldsymbol{a}) = |(\mathbf{S}-\mathbf{S}^{\mathrm{T}})\boldsymbol{a}|^2/(4|\mathbf{S}\boldsymbol{a}|^2)$, which is at most 1 and equal to 0 for a reciprocal scatterer (where $\mathbf{S} = \mathbf{S}^{\mathrm{T}}$) [3].

For particles small enough to be described by Rayleigh scattering, it is sufficient to consider pure dipole scattering where the electric and magnetic dipole moments $\boldsymbol{p}$ and $\boldsymbol{m}$ of the scatterer are computed from the polarizability dyadics $\boldsymbol{\gamma}_{\mathrm{ee}}$, $\boldsymbol{\gamma}_{\mathrm{em}}$, $\boldsymbol{\gamma}_{\mathrm{me}}$, and $\boldsymbol{\gamma}_{\mathrm{mm}}$, and the eigenvalue problem $\mathbf{S}\boldsymbol{a}_n = s_n\boldsymbol{a}_n$ boils down to an eigenvalue problem for the 6×6 polarizability dyadic as (with $t_n = (s_n-1)/2$ being the eigenvalue of the transition matrix $\mathbf{T} = (\mathbf{S}-\mathbf{I})/2$)

$$\begin{pmatrix} \boldsymbol{p}/\varepsilon_0 \\ \eta_0\boldsymbol{m} \end{pmatrix} = \begin{pmatrix} \boldsymbol{\gamma}_{\mathrm{ee}} & \boldsymbol{\gamma}_{\mathrm{em}} \\ \boldsymbol{\gamma}_{\mathrm{me}} & \boldsymbol{\gamma}_{\mathrm{mm}} \end{pmatrix} \begin{pmatrix} \boldsymbol{E} \\ \eta_0\boldsymbol{H} \end{pmatrix} \quad \Rightarrow \quad \frac{-\mathrm{j}k^3}{6\pi} \begin{pmatrix} \boldsymbol{\gamma}_{\mathrm{ee}} & \boldsymbol{\gamma}_{\mathrm{em}} \\ \boldsymbol{\gamma}_{\mathrm{me}} & \boldsymbol{\gamma}_{\mathrm{mm}} \end{pmatrix} \begin{pmatrix} \boldsymbol{E}_n \\ \eta_0\boldsymbol{H}_n \end{pmatrix} = t_n \begin{pmatrix} \boldsymbol{E}_n \\ \eta_0\boldsymbol{H}_n \end{pmatrix} \tag{1}$$

where ε_0 is the permittivity of free space and η_0 is the wave impedance in free space, and $\boldsymbol{E}$ and $\boldsymbol{H}$ are the electric and magnetic field strengths at the particle position, respectively. Reciprocity requires $\boldsymbol{\gamma}_{\mathrm{ee}} = \boldsymbol{\gamma}_{\mathrm{ee}}^{\mathrm{T}}$, $\boldsymbol{\gamma}_{\mathrm{mm}} = \boldsymbol{\gamma}_{\mathrm{mm}}^{\mathrm{T}}$, and $\boldsymbol{\gamma}_{\mathrm{em}} = -\boldsymbol{\gamma}_{\mathrm{me}}^{\mathrm{T}}$. For nonreciprocal particles, this symmetry is broken and it is seen that the eigenvectors $\boldsymbol{E}_n$ and $\eta_0\boldsymbol{H}_n$ (corresponding to the polarization of the characteristic modes) are in general complex valued, corresponding to elliptic polarization. It can further be shown that the nonreciprocity indicator $\nu \sim (ka)^6$, where k is the wavenumber and a is the radius of a sphere enclosing the particle. Hence, nonreciprocal scattering effects decrease in the low frequency limit.

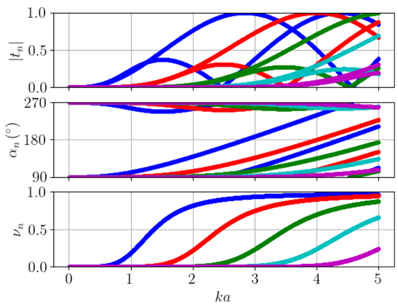

Figure 1. Example for a Tellegen sphere.

In the presentation we discuss nonreciprocal scatterers made of gyrotropic materials and bi-isotropic Tellegen materials, and extend the results beyond the Rayleigh limit by using Mie scattering and full wave simulations to compute the modal significance $|t_n|$, characteristic angle $\alpha_n = \arg(t_n)$, and nonreciprocity indicator ν_n as functions of frequency. An example is shown in Figure 1 for a Tellegen sphere with radius a and wavenumber k.

References

[1] R. Harrington and J. Mautz, “Theory of characteristic modes for conducting bodies,” *IEEE transactions on antennas and propagation*, vol. 19, no. 5, pp. 622–628, 1971.

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[3] N. Wingren, D. Sjöberg, M. Gustafsson, J. Lundgren, M. Capek, L. Jelinek, and K. Schab, “Characteristic modes of nonreciprocal structures,” *arXiv preprint arXiv:2312.14554*, 2023.