



From spacetime effective media to cascaded frequency conversion

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Abstract

Wave propagation in materials modulated in space and time has become a new paradigm for a full control of wave-matter interactions. Here, we will discuss our recent efforts in the field using time interfaces (i.e., a rapid change in time of the electromagnetic parameters of the medium where a wave travels). Applications include the use of time interfaces to enable space-time effective media, as well as frequency conversion in dielectric slab waveguides (single and cascaded waveguides) where the cladding material is time varying while the core does not change with time.

1. Introduction

The arbitrary control of wave propagation allowed by artificially engineered metastructures has enabled the proposal and demonstration of improved and new applications in fields such as computing, sensors, antennas, and even artificial intelligence, among others [1]–[6]. This has been possible as metamaterials and metasurfaces provide an unparalleled manipulation of their electromagnetic (EM) responses, which are not always easily available in natural media at a given desired spectral range (such as negative or near-zero values of permittivity, ϵ_r , and or permeability μ_r) [7]–[9]. Research and innovation in this field has shown how wave propagation can be controlled using spatial inhomogeneities.

In recent years, we have witnessed an increased interest into exploring the physical phenomena that occur when the EM response of the media where a wave propagates is modified not only in *space* but also in *time* (i.e., space-time media) [10–23]. An example of such time-modulation is time interfaces (or time boundaries) where the EM parameters of a material are rapidly changed in time from $(\epsilon_{r1}, \mu_{r1})$ to $(\epsilon_{r2}, \mu_{r2})$ at a time $t = t_0$ [24], [25]. In so doing, a time reflected (backward) and a time refracted (forward) wave are produced. Different from spatial interfaces between two media, for time interfaces the wave number and wavelength are not modified while the frequency of the wave(s) (forward/backward waves) inside such material is changed due to this process.

In this work, we will discuss some of our recent efforts in exploring four-dimensional media using time interfaces. It

will be considered that the permittivity of the medium where the wave travels is time-modulated while the permeability is not time-dependent. Several applications will be discussed including how a combination of spatially subwavelength multilayers (formed by two materials, A and B) can be engineered to develop effective medium concepts in space and time. This is achieved by considering that one of the filling materials (material B in this case) is time-dependent [26] while the other material (A) is not. Other applications will also be discussed in detail such as optical dielectric slab waveguides where their cladding material is modulated in time to induce a time interface at $t = t_0$. It will be shown how effective medium concepts also apply in this scenario demonstrating how frequency conversion is also possible even when only the cladding materials is time-dependent [27].

2. Space-time media

Schematic representations of some of the applications to be discussed using space and time interfaces along with effective medium concepts are shown in Fig. 1. In Fig. 1a, we consider the case of a parallel plate cavity filled with an isotropic dielectric medium having a relative permittivity value of ϵ_{r1} for times $t < t_0$. At $t = t_0$, a spatial subwavelength multilayer structure is generated inside the cavity by keeping some layers with the same value of $\epsilon_{rA} = \epsilon_{r1}$ while other layers are changed in time to $\epsilon_{rB} = \epsilon_{r2} \neq \epsilon_{r1}$. Various scenarios will be discussed during the conference such as $\epsilon_{rB}(t)$ being defined as a step function or multi-stepped periodic function, and the effect of inducing vertical or horizontal multilayers in time, among others.

As another application, the schematic representation of dielectric slab waveguides where their cladding is time-dependent is shown in Fig. 1b. Here, the core is time-independent with time interfaces induced only to the cladding, i.e., $\epsilon_{clad}(t)$. A full physical study of this scenario will be presented during the conference showing how the effective refractive index of the single mode that propagates within the waveguide is changed in time when applying the time interface to the cladding material. The effect of cascading, both in space and time, multiple dielectric slab waveguides will also be discussed. It will be

shown that cascaded frequency conversion is also possible with the proposed four-dimensional waveguides.

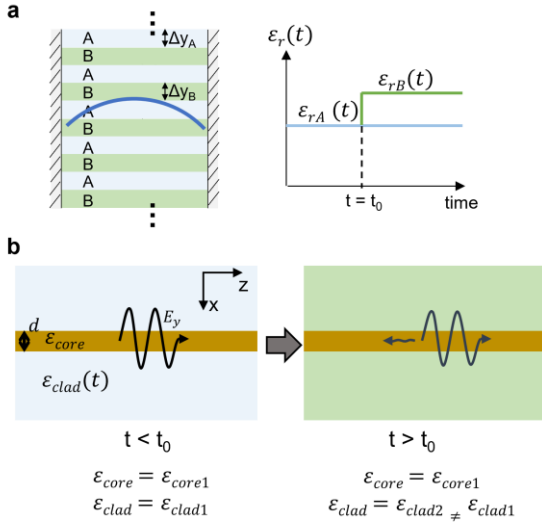


Figure 1. Effective medium concepts in space-time media (a) and dielectric slab waveguides involving time modulated cladding.

3. Conclusions

In summary, here we propose and study the effects of inducing time interfaces to create spatially subwavelength multilayer structures in time, and evaluate their response in terms of effective medium theory in four-dimensions (x, y, z, t) . It has also been discussed how space-time effective medium theory can also be exploited in dielectric slab waveguides where time interfaces are applied to their cladding material. These results may open new avenues for a full control of wave-matter interactions in four-dimensional metastructures.

4. Acknowledgements

V. P.-P. would like to thank the support of the Leverhulme Trust under the Leverhulme Trust Research Project Grant scheme (RPG-2020-316). N. E. acknowledge partial support from the Simons Foundation/Collaboration on Symmetry-Driven Extreme Wave Phenomena (grant 733684 (N.E.)). For the purpose of Open Access, V. P.-P. has applied a CC BY public copyright license to any Author Accepted Manuscript (AAM) version arising from this submission.

References

- [1] N. Engheta and R. W. Ziolkowski, *Metamaterials: Physics and Engineering Explorations*. 2006.
- [2] N. I. Zheludev and Y. S. Kivshar, "From metamaterials to metadevices," *Nat. Mater.*, vol. 11, no. 11, pp. 917–924, 2012.

- [3] T. Knightley, A. Yakovlev, and V. Pacheco-Peña, "Neural Network Design of Multilayer Metamaterial for Temporal Differentiation," *Adv. Opt. Mater.*, vol. 11, no. 5, p. 2202351, Mar. 2023.
- [4] D. C. Tzarouchis, M. J. Mencagli, B. Edwards, and N. Engheta, "Mathematical operations and equation solving with reconfigurable metadevices," *Light Sci. Appl.*, vol. 11, no. 1, pp. 1–13, 2022.
- [5] M. Nicolussi, J. A. Riley, and V. Pacheco-Peña, "Unidirectional transparency in epsilon-near-zero based rectangular waveguides induced by parity-time symmetry," *Appl. Phys. Lett.*, vol. 119, no. 26, p. 263507, Dec. 2021.
- [6] R. G. MacDonald, A. Yakovlev, and V. Pacheco-Peña, "Solving partial differential equations with waveguide-based metatronic networks," *arXiv:2401.00861*, pp. 1–23, Dec. 2023.
- [7] I. Liberal and N. Engheta, "Near-zero refractive index photonics," *Nat. Photonics*, vol. 11, no. 3, pp. 149–158, 2017.
- [8] Z. Zhou *et al.*, "Dispersion coding of ENZ media via multiple photonic dopants," *Light Sci. Appl.*, vol. 11, no. 1, p. 207, Dec. 2022.
- [9] M. Coppolaro, M. Moccia, V. Caligiuri, G. Castaldi, N. Engheta, and V. Galdi, "Extreme-Parameter Non-Hermitian Dielectric Metamaterials," *ACS Photonics*, vol. 7, no. 9, pp. 2578–2588, 2020.
- [10] C. Rizza, G. Castaldi, and V. Galdi, "Nonlocal effects in temporal metamaterials," *Nanophotonics*, vol. 3600, Feb. 2022.
- [11] J. C. Serra and M. G. Silveirinha, "Rotating spacetime modulation: Topological phases and spacetime Haldane model," *Phys. Rev. B*, vol. 107, no. 3, p. 035133, Jan. 2023.
- [12] V. Pacheco-Peña and N. Engheta, "Temporal metamaterials with gain and loss," *arXiv:2108.01007*, Aug. 2021.
- [13] V. Pacheco-Peña, D. M. Solís, and N. Engheta, "Time-varying electromagnetic media: opinion," *Opt. Mater. Express*, vol. 12, no. 10, p. 3829, Oct. 2022.
- [14] J. S. Martínez-Romero, O. M. Becerra-Fuentes, and P. Halevi, "Temporal photonic crystals with modulations of both permittivity and permeability," *Phys. Rev. A*, vol. 93, no. 6, pp. 1–9, 2016.
- [15] J. E. Vázquez-Lozano and I. Liberal, "Incandescent temporal metamaterials," *Nat. Commun.*, vol. 14, no. 1, pp. 1–11, 2023.
- [16] L. Stefanini, S. Yin, D. Ramaccia, A. Alù, A. Toscano, and F. Bilotti, "Temporal interfaces by instantaneously varying boundary conditions," *Phys. Rev. B*, vol. 106, no. 9, p. 094312, Sep. 2022.
- [17] M. S. Mirmoosa, G. A. Ptitcyn, V. S. Asadchy, and S. A. Tretyakov, "Time-Varying Reactive Elements for Extreme Accumulation of Electromagnetic Energy," *Phys. Rev. Appl.*, vol. 11, no. 1, p. 014024, Jan. 2019.
- [18] V. Pacheco-Peña and N. Engheta, "Antireflection temporal coatings," *Optica*, vol. 7, no. 4, p. 323, Apr. 2020.

- [19] V. Pacheco-Peña and N. Engheta, “Temporal aiming,” *Light Sci. Appl.*, vol. 9, no. 129, pp. 1–12, Dec. 2020.
- [20] H. Li, S. Yin, E. Galiffi, and A. Alù, “Temporal Parity-Time Symmetry for Extreme Energy Transformations,” *Phys. Rev. Lett.*, vol. 127, no. 15, p. 153903, 2021.
- [21] R. Tirole *et al.*, “Double-slit time diffraction at optical frequencies,” *Nat. Phys.*, vol. 19, no. 7, pp. 999–1002, Jul. 2023.
- [22] Z. Li, X. Ma, A. Bahrami, Z.-L. Deck-Léger, and C. Caloz, “Space-Time Fresnel Prism,” *arXiv:2307.05613*, pp. 1–18, 2023.
- [23] E. Galiffi *et al.*, “Photonics of time-varying media,” *Adv. Photonics*, vol. 4, no. 01, pp. 1–32, Feb. 2022.
- [24] F. Morgenthaler, “Velocity Modulation of Electromagnetic Waves,” *IEEE Trans. Microw. Theory Tech.*, vol. 6, no. 2, pp. 167–172, Apr. 1958.
- [25] R. Fante, “Transmission of electromagnetic waves into time-varying media,” *IEEE Trans. Antennas Propag.*, vol. 19, no. 3, pp. 417–424, May 1971.
- [26] V. Pacheco-Peña and N. Engheta, “Merging Effective Medium Concepts of Spatial and Temporal Media: Opening new avenues for manipulating wave-matter interaction in 4D,” *IEEE Antennas Propag. Mag.*, vol. 65, no. 4, pp. 39–49, Aug. 2023.
- [27] V. Pacheco-Peña and N. Engheta, “Spatiotemporal cascading of dielectric waveguides,” *arXiv:2312.12233*, Dec. 2023.