Characteristic Modes of DB and D′B′ Objects

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Characteristic mode (CM) analysis of a lossless object gives a set of characteristic eigencurrents $\mathbf{J}_n$ and/or $\mathbf{M}_n$ and their corresponding characteristic eigenvalues λ_n. These eigencurrents radiate power-orthogonal far fields and the eigenvalues measure how well each mode radiate. There are today several good CM formulations for perfectly conducting objects, dielectric objects, composite objects, etc. This contribution presents a new CM formulation for objects with the DB or D′B′ boundary condition [1].

For DB and D′B′ objects, the ordinary scattering problem with a given incident field $\mathbf{E}^{\mathrm{inc}}, \mathbf{H}^{\mathrm{inc}}$ can be formulated as a combination of the tangential electric field integral equation (EFIE) and tangential magnetic field integral equation (MFIE) on the surface of the scatterer as [2]

$$\underbrace{\begin{bmatrix} \eta\mathcal{T} & -\mathcal{K}^{+} \\ \mathcal{K}^{+} & \frac{1}{\eta}\mathcal{T} \end{bmatrix}}_{\mathcal{L}} \underbrace{\begin{bmatrix} \mathbf{J} \\ \mathbf{M} \end{bmatrix}}_{\mathbf{f}} = -\begin{bmatrix} \mathbf{E}^{\mathrm{inc}} \\ \mathbf{H}^{\mathrm{inc}} \end{bmatrix}_{\mathrm{tan}} \qquad \text{where } \mathbf{J} \text{ and } \mathbf{M} \text{ are} \begin{cases} \text{solenoidal for DB} \\ \text{irrotational for D}'\text{B}' \end{cases} \tag{1}$$

and $\eta = \sqrt{\mu/\varepsilon}$ is the intrinsic impedance of the surrounding space. The definition of the surface integral operators $\mathcal{T}$, $\mathcal{K}$, and $\mathcal{K}^{+}[\mathbf{J}] = \mathcal{K}[\mathbf{J}] + \frac{1}{2}\mathbf{n}\times\mathbf{J}$ are the same as in [2], except that we for simplicity include the tangential trace operator in $\mathcal{T}$ and $\mathcal{K}$. Using the surface integral operator $\mathcal{L}$, we get the new CM formulation as the generalized eigenvalue equation

$$\mathcal{L}[\mathbf{f}_n] = (1 - i\lambda_n)\mathcal{W}[\mathbf{f}_n], \qquad \mathcal{W} = \begin{bmatrix} \eta\,\mathrm{Re}\{\mathcal{T}\} & -i\,\mathrm{Im}\{\mathcal{K}\} \\ i\,\mathrm{Im}\{\mathcal{K}\} & \frac{1}{\eta}\,\mathrm{Re}\{\mathcal{T}\} \end{bmatrix} \tag{2}$$

where the selected weight operator $\mathcal{W}$ is properly related to radiated power [3].

In both (1) and (2) we formally have the same equation for DB and D′B′ and the boundary condition is enforced by selecting the appropriate function space for the unknown surface currents $\mathbf{J}$ and $\mathbf{M}$. To get matrix equations from (1) and (2), we expand the surface currents using RWG loop functions for DB and RWG star functions for D′B′, and in both cases use Galerkin testing.

As initial verification, we compare the results for a sphere with the Mie-series based analytical solution

$$\lambda_n^{\mathrm{DB}} = \frac{\chi_n(x)}{\psi_n(x)}, \qquad \lambda_n^{\mathrm{D'B'}} = \frac{\chi_n'(x)}{\psi_n'(x)} \tag{3}$$

where $x = ka$ is the size parameter of the sphere, and $\psi_n(x) = x\, j_n(x)$ and $\chi_n(x) = -x\, y_n(x)$ are the Riccatti–Bessel functions.

References

[1] I. V. Lindell and A. H. Sihvola, “Electromagnetic boundary conditions defined in terms of normal field components,” *IEEE Transactions on Antennas and Propagation*, **58**, 4, April 2010, pp. 1128–1135, doi: 10.1109/TAP.2010.2041149.

[2] P. Ylä-Oijala and S. Järvenpää, “New trends in frequency-domain surface integral equations,” in *New Trends in Computational Electromagnetics*, Ö. Ergül, Ed. Scitech Publishing, Jan. 2019, Chapter 3.

[3] P. Ylä-Oijala, “Generalized theory of characteristic modes,” *IEEE Transactions on Antennas and Propagation*, **67**, 6, June 2019, pp. 3915–3923, doi: 10.1109/TAP.2019.2905794