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# Performance Comparison of *AT1* Algorithm for a Smaller Ensemble of Atomic Clocks

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## Abstract

Accurate and traceable precision timing information is essential for numerous applications. The current standards for timing accuracy are linked to atomic standards, particularly Cesium clocks. Typically, National Metrology Institutes (NMIs) maintain an ensemble of atomic clocks, utilized in deriving the respective national time submitted to the Bureau of Weights and Measurement (BIPM) for traceability. Various techniques exist for obtaining this ensemble timing. This paper compares the very widely used *AT1* algorithm to the *Linear Regression* model for a smaller ensemble of only 3 clocks. The results show a significant improvement in accuracy when the *AT1* algorithm is applied.

## 1 Introduction

Precision timing information finds applications in various fields, such as high-speed financial trading, synchronization for the Internet of Everything, and advancements in 5G - 6G communications, among others [1]. Generally, the precision timing information is generated by the National Metrology Institutes (NMIs) and is traceable to the Universal Coordinated Time (UTC) maintained by the Bureau of Weights and Measurement (BIPM) [2]. NMIs commonly operate multiple atomic clocks, forming an ensemble that contributes to the derivation of the *UTC(k)* for a specific NMI, denoted as $k$.

Ensemble timing capitalizes on the advantages offered by each clock within the ensemble. In instances where a clock exhibits suboptimal behavior, it is excluded from the system. Various techniques exist for calculating ensemble time. Conceptually, an ensemble represents a weighted average of all clocks, and a larger ensemble enhances system reliability, providing ample options for selecting the best-performing clocks. *AT1* is a very reputed method that has been adopted by many NMIs. ALGOS and Kalman filter-based methods are a few other techniques to generate the ensemble timings [3, 4]. This paper aims to explore the performance of *AT1* in the context of a relatively smaller ensemble, which typically exits in less developed NMIs or backup time labs. Given a limited number of clocks, options for removing clocks with abnormal performance from the ensemble are constrained, and a straightforward algorithm may prove effective. This study will specifically evaluate the performance of *AT1* against a simple *Linear Regression* model.

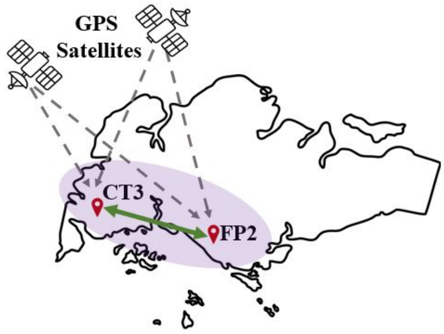


**Figure 1.** Map showing the location of the two labs in Singapore.

As technology advances and the demand for heightened precision in timing grows, NMIs are increasingly investing in the resilience of their timing services to ensure uninterrupted services. In Singapore, the National Metrology Centre (NMC) serves as the custodian for time-keeping services, and, in the pursuit of building a resilient timing network, the establishment of a business-continuity laboratory is underway. Fig.1 illustrates the locations of these two facilities in Singapore, with *CT3* designated as the main laboratory and *FP2* as the business-continuity laboratory. The experiments conducted for this paper pertain to the business-continuity laboratory, and the obtained results will undergo verification by comparison with data from the main laboratory.

## 2 Data and Data Processing

The business-continuity Time and Frequency lab of Singapore in FP2 has a small ensemble of atomic clocks which consists of two Cesium atomic clocks (5071A) and one is the Active Hydrogen Maser (MHM-2020). Fig.2 shows the images of the hardware installed and Fig.3 shows the system flow diagram. As illustrated by Fig.3, the measurements of the atomic clocks are recorded by a multi-channel measurement system (MMS) (VCH-315M). The MMS records the time differences between a reference clock and a member clock of the ensemble. For our case, we use H-maser as the reference. Therefore MMS records

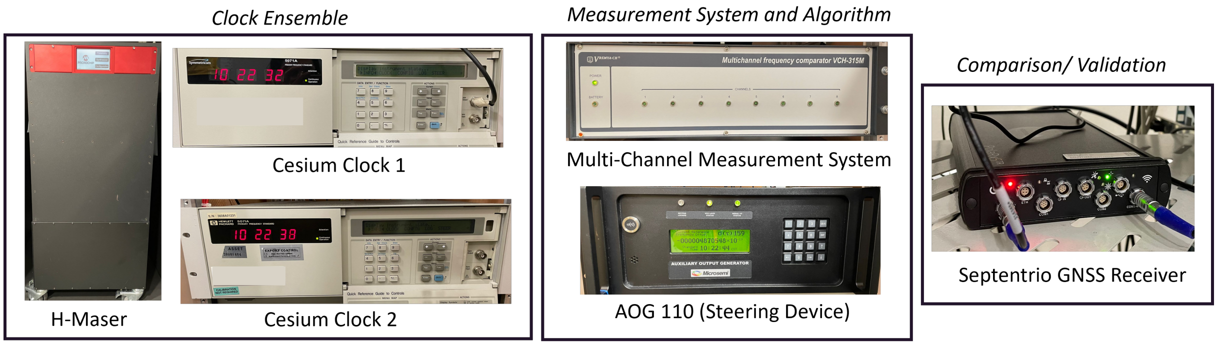


**Figure 2.** Pictures of the hardware from the business-continuity lab at FP2.

two sets of differences $x_r - x_1$ and $x_r - x_2$, differences between H-maser and Cesium clock 1 and H-maser and Cesium clock 2 respectively.

These recorded differences are then used by a time scale algorithm to calculate the ensemble time. The study involves testing two distinct algorithms, namely the *AT1* algorithm and a *Linear Regression* model. The resulting ensemble output is then fed into a time steering algorithm to adjust our reference clock. For this purpose, a phase stepper, AOG 110, is employed. The output of the AOG is the Coordinated Universal Time (UTC) generated by our business-continuity lab at FP2 [named as *UTC(FP2)*]. Here, data from our main lab at CT3, which maintains the *UTC(SG)* will be used to compare with our business-continuity lab and establish the traceability for this paper. The measurement system for this setup is working in 5MHz domain.

In the system diagram depicted in Fig.3, a specific section within the dotted box has not been implemented yet. This particular segment is responsible for inputting the differences between AOG and *UTC(SG)* into the AOG's input, thereby facilitating correction. The integration of this component ensures a more accurate output. This aspect will be addressed and implemented in our subsequent work.

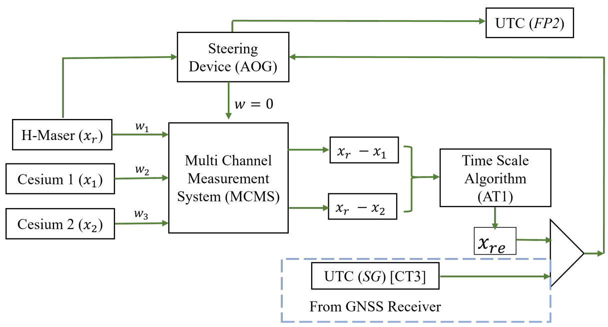


**Figure 3.** Schematic of System setup for the business-continuity Time & Frequency lab in Singapore (at FP2)

Table 1 shows the data that has been used for this paper. A total of 45 days' data has been used for the analysis ($14^{th}$ Oct to $27^{th}$ Nov 2023).

| Clocks | Location | MJD |
|---|---|---|
| H-maser | FP2 | 23287 - 23331 |
| Caesium 1 | FP2 | 23287 - 23331 |
| Caesium 2 | FP2 | 23287 - 23331 |
| UTC (SG) | CT3 | 23287 - 23331 |

**Table 1.** Specifications on Clocks and Data

## 3 Methodology

### 3.1 Time Scale Algorithm

All time scale algorithms share a common foundation, expressed by the general equation depicted in Eq. 1 [3]. The equation shows the fundamental relationship used to estimate the behavior of a clock $j$ with respect to an ensemble $e$, at the time stamp $k$ given the condition of clock at *k-1*. Here $x_{je}$ is the phase difference, $y_{je}$ is the frequency difference, and $d_{je}$ is the frequency aging of clock $j$ with respect to the ensemble $e$.

$$\hat{x_{je}}(k) = x_{je}(k-1) + y_{je}(k-1)\tau + 0.5d_{je}(k-1)\tau^2 \quad (1)$$

*AT1* algorithm is a recursive algorithm that can be used in the estimation of the clock behavior. The procedural steps involved in the realization of the *AT1* algorithm have been described in our earlier paper [6]. The implementation of the algorithm results in the estimates of all clocks in the system with respect to the ensemble time i.e. $x_{1e}$ (AT1), $x_{2e}$ (AT1), and $x_{re}$ (AT1) for Cesium clock 1, Cesium clock 2 and H-maser respectively. The algorithm estimates the weight of each clock based on its estimation error. The clock with a lower error is accorded a higher weight in the ensemble.

A fundamental estimation of clock behavior can also be accomplished by directly applying the *Linear Regression* equation represented by eq. 1. This direct implementation leads to estimates for $x_{1e}$ (LR), $x_{2e}$ (LR) and $x_{re}$ (LR). Subsequently, the *AT1* estimates will be subject to a comparative analysis with the *Linear Regression* estimates within the context of this ensemble, which comprises only 3 clocks.

### 3.2 Common View Method

Once the ensemble estimates are generated, the ensemble can be compared to the reference time scale, *UTC(SG)* which is a physical time signal maintained at the the main lab in our case. This is achieved by implementing the common view method. The underlying principle of the common view method is based on the visibility of one or more common satellites from two GPS ground stations or receivers. Time differences between these two receivers can be calculated by considering the common satellites in view [7]. In the specific case where all satellites irrespective of whether they are common or not are used in the difference calculation, it is referred to as the *All-in View* method. In this method, the data from all satellites are averaged first before calculating the difference [8]. For this paper, the common view method is employed. In the context of this study, given the relatively short distance between the two receiver locations (FP2 and CT3), there is minimal distinction between the two methods, as the same satellites are visible to both stations for the majority of the time.

The difference between *UTC(SG)* and FP2 is termed as $x_{rutc}$. To assess the performance of the *AT1* and *Linear Regression*, we compare the ensemble time estimated from these methods to the *UTC(SG)* data i.e. we calculate $x_{eutc}$ from the two methods. Eq. 2 show the respective calculations.

$$\begin{aligned} x_{eutc}(AT1) &= x_{rutc} - x_{re}(AT1) \\ x_{eutc}(LR) &= x_{rutc} - x_{re}(LR) \end{aligned} \tag{2}$$

## 4 Results and Discussion

### 4.1 Time Series Analysis

Fig.4 shows the time series of differences when using *AT1* and *Linear Regression* model. The curves in solid lines represent the results of the *AT1* algorithm: $x_{je}$ (AT1) and the dashed line represents the results of *Linear Regression*: $x_{je}$ (LR) for atomic clocks: Cesium clock 1, Cesium clock 2 and the H-maser. It can be observed that the results of the *Linear Regression* follow a linear pattern as it is implemented from eq. 1. Whereas the results of *AT1* show characteristic changes attributed to the dynamic nature of the weights assigned to each clock in the *AT1* model. These weights evolve based on the performance of each clock, with higher noise leading to lower weights: $w_j \propto 1/N_j$, Where $N_j$ is the individual clock's estimation error. The observed differences in the estimated values between the two methods will be further quantified in the subsequent section.

### 4.2 Histogram Analysis of the Differences

Fig.5 shows the histogram of differences between *UTC(SG)* and estimates of the ensemble from *AT1* and *Linear Regression*. The graph clearly shows that the $x_{eutc}$ values for the case of *AT1* are smaller in magnitude and have a lesser spread. Most of the differences for *AT1* are within $\pm 50$ ns. Whereas for the *Linear model*, the differences can go beyond 200 ns. The ensemble clock created using *AT1* closely aligns with *UTC(SG)* compared to just the *Linear Regression*.

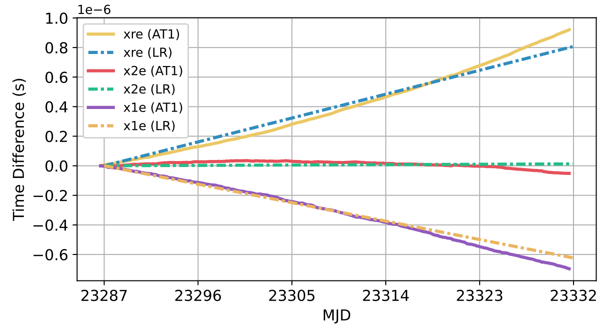


**Figure 4.** Time series plot of the estimates when using *AT1* and *Linear Regression* model.

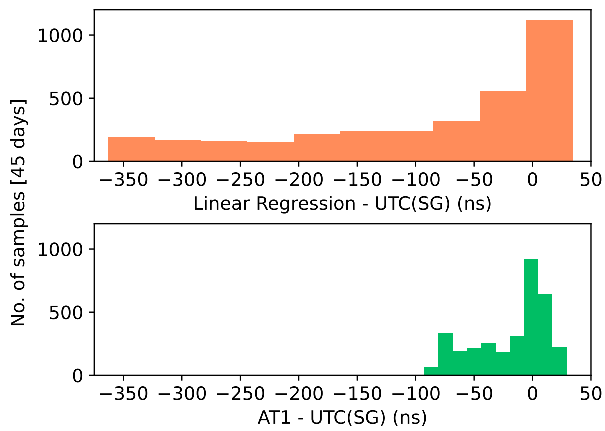


**Figure 5.** Histogram of differences with respect to the UTC(SG).

## 5 Conclusion

This paper implemented two methods to obtain clock estimates for a smaller ensemble comprising only 3 atomic clocks. The time series graphs for both the *AT1* and *Linear Regression* models exhibit close alignment. However, upon comparing the differences with *UTC(SG)*, it becomes evident that the *AT1* algorithm outperforms the *Linear Regression* model. This underscores the effectiveness of the dynamic weighting scheme for clocks within the ensemble, even in cases where the ensemble is relatively smaller. The subsequent phase involves steering the phase stepper, such as AOG, to obtain the physical signal post the implementation of the *AT1* algorithm.

# 6 Acknowledgements

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