Reduced-order modeling of time-harmonic systems in electromagnetics using efficient error estimation

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Extended Abstract

Numerical discretization of time-harmonic Maxwell's equations usually results in large-scale parametric systems. Repeated simulation of such systems at many samples of parameters is necessary in many design tasks, in order to get optimal performance of devices. As a result, the computational time spent on such simulations is very long. Many efforts have been made to speed up the design process by accelerating the simulation. Model order reduction (MOR) is one of the most promising techniques that aim to reduce the simulation time of large-scale parametric systems. For projection-based MOR of a parametric system, representative solutions of the systems at certain samples of parameters are first computed by direct simulation. Then basis vectors of the subspace spanned by those solution snapshots are generated to construct a small reduced-order model (ROM). The ROM is finally used as a surrogate of the original large-scale system to achieve fast simulation. How to choose those representative solution snapshots to obtain ROMs with guaranteed accuracy remains challenging for many problems.

A posteriori state error bounds [1] have been used in a greedy process for ROM construction. However, this necessitates computing the so-called inf-sup-constants at quite a few parameter samples in a training set. Evaluating the inf-sup-constant at a single parameter needs to compute the smallest singular value of a large matrix, which is expensive. Moreover, for some problems whose inf-sup-constants are close to zero, the error bound becomes rather rough. Such systems are nevertheless typical in computational electromagnetics. The residual induced by the ROM is also heuristically used as an error indicator in order to avoid computing the inf-sup constants. An inf-sup-constant-free randomized error estimation approach has been recently proposed in [2]. However, this approach still has relatively high computational complexity. We present a new inf-sup-constant-free error estimation approach [3]. Compared with the existing inf-sup-constant-free error estimation, our approach has much less computational complexity without sacrificing high accuracy. The proposed tight error estimator not only yields a compact ROM with small size using less greedy iterations in the offline phase, but also could be reliably used in online certification of the ROM in applications such as digital twins. We present numerical tests on several examples in electromagnetics including narrow and wideband antennas, a coax-fed dielectric resonator filter consisting of many resonances over the frequency range, and a three-parameter inline dielectric resonator filter. All numerical results show that the proposed inf-sup-constant-free error estimator is a tight bound of the true error and generates accurate and small ROMs. It outperforms all other closely related error estimators, w.r.t. to runtime, computational complexity and accuracy.

References

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