



Study on Lunar Calendar

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Abstract

The calendar formed on the Earth cannot be copied to the Moon. The moon is located in a hierarchically nested multilevel coordinate system. Lunar observers choose different references from different levels of coordinate systems, and different rotation periods and revolution periods will be measured. On the surface of the moon, observers can be divided into: the observers facing the Earth, the observers situated on the lunar far side, halo observers, and the observers of the lunar equator. These four types of observers see different phenomena of celestial cycle. The lunar calendar is periodicity and localized. The Earth is the best reference for the moon's rotation and revolution cycle. Compared to the Earth, the moon has no rotation, only swing. Compared with the sun and the stars, the revolution of the moon around the earth can be regarded as the rotation of the moon, but the motion of speed from the moon is not uniform.

Key words: Calendar; Lunar calendar; rotation; revolution; Moon's path; ecliptic

1. Introduction

The observers on the Moon (hereinafter referred to as "observers"), such as lunar surface astronauts, observe celestial motion (phenomena) cycles and their cumulative relations, known as the "lunar calendar." Traditional "calendars" typically refer to the observed cycles seen by Earth observers and the carry relations of three celestial phenomena, the tropical year, synodic month, and solar day. The calendar formed on Earth cannot be directly applied to the Moon. However, the basic principles followed by the calendar are universal.

Presently, astronomical constant tables provide the synodic month period as 29.5305881 days. This illustrates the remarkable stability of the Moon's revolution around the Earth. Consequently, the period of these synodic phenomena can serve as the fundamental cycle for lunar calendars, facilitating cumulative calculations.

Using the Moon as a reference for the Earth's rotational period enabled a more precise measurement of the relationship between "seconds" and celestial cycles compared to using the Sun as reference directly. In 1960,

the International System of Units (SI) defined the ET second, designating the orbital period around the Sun as the benchmark for time, considering it more stable and uniform than the Earth's rotational period^[1]. However, this conclusion is not absolute, as this article will discuss how selecting different celestial bodies as references yields different observational periods

Satellites for navigation systems used in future lunar settlements will require local atomic clocks. Scientists are currently calculating the timekeeping measures that will be maintained in those settings. In 2022, the European Space Agency convened experts to discuss issues surrounding lunar time^[2].

1 The relationship of coordinate Systems

This region centers around the lunar center-of-mass coordinate system, encompassing both the lunar surface and orbiting spacecraft. Hierarchically nested coordinate systems originating from local surface observers, depicted in Figure 1, include the Lunar Surface Observer Coordinate System (L), Lunar Center-of-Mass Coordinate System (M), Earth-Moon Conjunction Coordinate System (C), Solar System barycenter Coordinate System (O2), Galactic Center System (O3), and the Local Star System Center-of-Mass Coordinate System (O4). In the Earth-Moon Conjunction Coordinate System (C-O1-M), also called Earth-Moon synodic coordinate system, the center-of-mass C lies along the line connecting the Earth center O1 to the lunar center M. The Moon's path plane represents the orbital plane of the Earth-Moon system. C orbits along the ecliptic plane within the heliocentric coordinate system, termed as the ecliptic plane. The inclination between the Moon's path plane and the ecliptic plane is approximately 5.15°, and the intersection of these two planes rotates clockwise around the C axis about once every 18.6 years.

2 The periodicity of lunar calendar

The reference celestial bodies visible from lunar surface observers are categorized into four types, originating from different hierarchical coordinate systems.

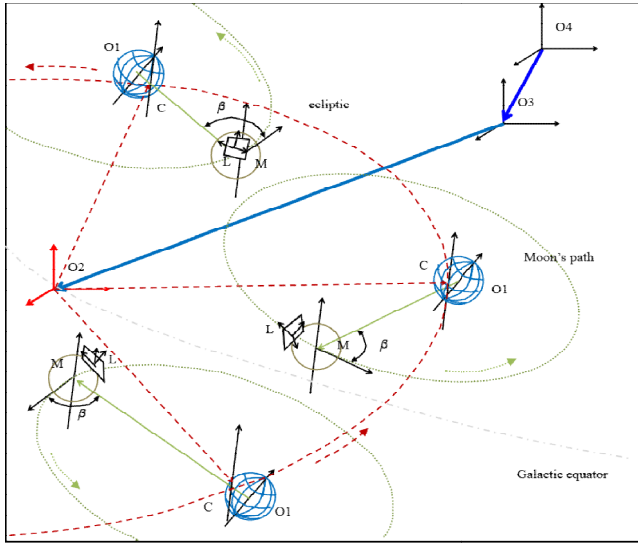


Figure 1. Movement of the moon in a hierarchically nested coordinate frame

2.1 Earth as a Reference

Observers on the lunar surface, utilizing a camera fixed over an extended period toward the Earth, through overlapping images, have made the following discoveries:

- 1) For the observers facing the Earth, referred to as observer A, they perceive the Earth's azimuth (direction angle) oscillating back and forth concerning a fixed direction in the L coordinate system. The azimuth of the Earth alternates approximately $\pm 3.2^\circ$. However, as the Moon progresses around the Earth along an elliptical orbit, viewing the Earth from various arc segments yields distinct azimuth angles, as depicted by the β angle in Figure 1.
- 2) Additionally, observer A discerns a conical oscillation of the Earth's rotational axis, resembling a circular motion along a conical surface around the Earth's center. The polar regions of the Earth alternate visibility, exhibiting a swinging period of approximately (27.32166×86400) seconds, expressing the Earth's cyclical movement in the M coordinate system. As depicted in figure 2.
- 3) Observer A also observes an additional oscillation in the direction of the Earth's rotational axis. Notably, markers on Earth, such as laser generators, reappear to Observer A in a cycle nearly equivalent to the Earth's rotational period, approximately $[86400 \times (1 + 1/29.5305881)]$ s.
- 4) Observer B, situated on the lunar far side, never perceives the Earth due to the Moon's lack of rotation relative to the Earth, similar to how Earth observers never witness the lunar far side.
- 5) Within the region between Observers A and B, termed the "halo," Observer C on the lunar surface observes the Earth rising and setting along the lunar marginal line with

periodicity.

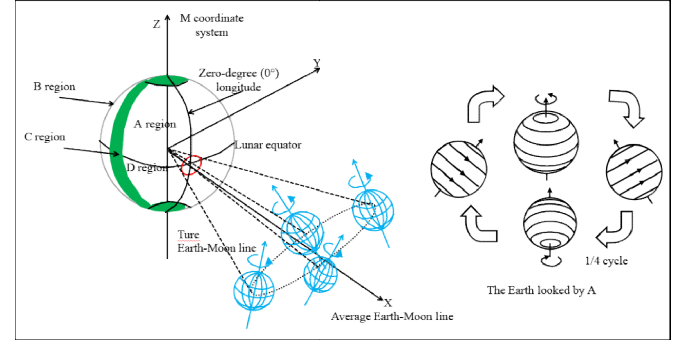


Figure 2. Subregions of observers named A, B, C and D and the movement of the Earth looked by A

In summary, for lunar observers, the Earth resides within the upper-tier coordinate systems and serves as the most consistent periodic reference. The Earth's celestial cycles, including the azimuthal oscillation periods, rotational axis oscillation cycles, and alternating visibility of the North and South poles, can all serve as temporal indicators. Considering the highly stable lunar orbit around the Earth, in sync with its rotation, it becomes a fundamental cycle for lunar calendrical systems. Referring to Earth's rotation, lunar timekeeping can involve cumulative calculations based on the lunar rotational period.

2.2 Sun as a Reference

Observers on the lunar surface can see the Sun and refer to ancient astronomical timing methods, sundials are used on the Moon to measure longitude and gnomons for latitude.

- 1) Observer A on the lunar surface can witness sunrise and sunset, Earth's phase is similar to lunar phases. During the Earth's lack phase, the Sun passes by the back of the Earth, and the Earth situates between the Sun and the Moon. Observer A only have a look the Earth's nighttime scene. At the Earth's round phase, the Moon positions itself between Earth and the Sun, leaving observer A in lunar darkness but observing Earth brightly illuminated as daylight. Observer A's phases equate to Earth observer's round and lack cycles, approximately 29.53 Earth days. This is the most direct and easily observed celestial event, serving as a prime reference event for time transmission between Earth and the Moon, potentially the optimal choice for lunar calendrical units.
- 2) Observer A can also observe the Sun pass above Earth for several cycles, followed by the Sun passing below Earth for subsequent round and lack cycles. During these alternations, partial or total eclipse events occur when the Moon partially or fully blocks the Sun from observer A's viewpoint, leading to 2-3 solar eclipses annually on Earth. Additionally, observer A observes the Moon's shadow cast on Earth, forming a small circular patch, varying in size. The projection of this patch on Earth at the same latitude recurs approximately every 18 years, 11 days, and

8 hours 【Saros cycle 6585.3 days】 .

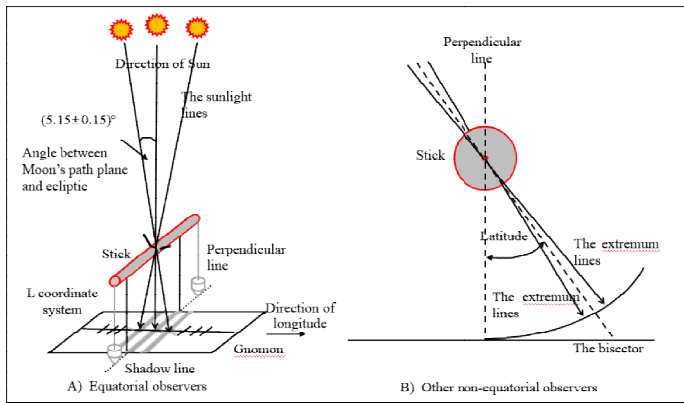


Figure3.The gnomon on the equator of the Moon and it's shadow line

3) Observing the shadow trail cast by sunlight on the gnomon, we can determine the equatorial and latitudinal circles on the lunar surface. Observer D, situated at the lunar equator, witnesses the shadow's movement on the gnomon's scale during local solar noon. The shadow line oscillates symmetrically around the perpendicular line, with the farthest deviation from this perpendicular known as the extremum line. The extremum line's to-and-fro movement constitutes a lunar regression cycle, closely resembling Earth's tropical year. The intersection of the ecliptic plane and the Moon's path plane rotates within the ecliptic coordinate system with a period of approximately 18.6 Earth tropical years. When the distance from the gnomon's perpendicular line to the two extremum lines is equal, the latitude at that location is 0 degrees. Utilizing this definition, observers A, B, C, and D can determine the lunar equator's position. Other non-equatorial observers can determine their latitude by measuring the angle between the bisector of two extremum lines and the perpendicular line on the gnomon.

4) Lunar observers can also use sundials to determine their longitude. Once the zero-degree (0°) longitude is established, the shortest projected time of the sundial on the zero-degree meridian is defined as noon. The 180° meridian on the lunar far side marks the starting point of the Sun's cycle. Since the Sun is not the reference in the near-lunar coordinate system, noon moments may not be evenly spaced along uniform angular meridians, similar to Earth's true solar day versus mean solar day relationship. This is because, on elliptical orbits, the Sun's position varies when observed from different segments, leading to variations in angles.

2.3 Stars as Reference

Observers on the lunar surface can witness the stars in the celestial sphere. By employing a fixed-angle camera and capturing timed photographs, they can detect the periodic trajectories of stars in these images, revealing various cyclic patterns, which serve as a primary means for lunar

calendar research:

1) Observer A can utilize the Earth as a reference, analogous to a pointer on a dial, with the starry background resembling the dial. As the local geocentric coordinate system rotates, Observer A perceives the stellar chart behind Earth shifting rightward or Earth seemingly advancing leftward against the backdrop. When Earth completes one revolution, causing the identical starry background to overlap, this constitutes the stellar cycle of the Moon's rotation, shorter than the Earth's phase of round and lack.

2) Observer B lacks Earth as a reference but can observe the sky from a fixed perspective. By utilizing the local vertical and meridian plane as references, Observer B can still determine the time from the star chart dial.

Photographing images at equidistant intervals using local proper time or indexing time with it to construct a stellar ephemeris reveals that the stars' apparent angular speed are not uniform, and their periods are unstable. This phenomenon arises from the stars' position in the O3 coordinate system, as depicted in Figure 1, where lunar observers traverse both the C and O2 coordinate systems to observe the stars. The Moon's movement along the curved path within the O2 system causes different positions for observing the same star. The effect of the elliptical orbit's curvature alters the angular separation between these two positions, influencing the measured periodic intervals. This phenomenon can also be explained by aberration: on a curved orbit, the light aberration varies at different times, resulting in variations in the measured stellar periods.

2.4 Pulsars as Reference

Setting up an X-ray astronomical telescope on the lunar surface enables the observation of pulsars. Employing the pulse number of pulsars to construct a space time-keeping system, distinct from celestial observations. Observations from at least three locations near the Moon, depicting the pulse number and arrival time of pulsars, are referenced back to the lunar center. This allows the reproduction of coordinate time through pulse numbers, thereby achieving temporal unification within the lunar vicinity. Further details can be found in the referenced literature. ^[3, 4]

3 The local nature of lunar calendrics

Localness is a fundamental requirement of Einstein's general principle of Relativity: "All reference frames are equally valid, and physical laws remain invariant in any coordinate system, exhibiting general covariance." The concept of "equality" refers to the relationship of measurement standards between different coordinate systems; standards cannot be transferred across coordinate systems. Meanwhile, "localness" implies that measurement standards should be reproducible within the

local coordinate system. In the lunar vicinity, a unified local time should be employed, utilizing locally occurring natural cyclic phenomena as timing units in the calendar system, directly tied to the rotational period of the local environment.

3.1 Local time

The SI second replicated locally should be utilized for timekeeping, the proper time. Atomic clocks at different locations are subject to relativistic effects, causing variations in their tick rates between different space. Relative to the Earth-Moon synodic coordinate system, the Moon doesn't rotate. In this conjunction system, all points on the lunar surface are motionless relative to the Moon's center of mass. Measurements of proper time at various locations can be calculated to the lunar center, establishing local time within the lunar vicinity, known as Lunar Time (TL), which is also the coordinate time of the lunar center coordinate system. If laser gyroscopes are employed to measure the Moon's rotation or oscillation, it's crucial to use local time (TL). Otherwise, if timing signals received from Earth are accepted as standard, uneven time scales may occur, leading to measurement noise or errors. Using TL as the independent variable of time for observing various cyclic phenomena such as Earth's phases, Earth's axial motion, solar cycles, stellar cycles adheres to the requirement of "localness."

3.2 Local celestial reference objects

For observers A, B, C, and D on the lunar surface, positioned within a multi-tiered coordinate system, only the Earth lies within the near-upper-level coordinate system, coexisting with the Moon within the Earth-Moon synodic coordinate system. For lunar observers, the cyclical celestial events of Earth unfold within this system and are unaffected by the curvature of the Earth-Moon center on the ecliptic plane or the Barycenter center on the Galactic plane. Hence, Earth's cyclical celestial events serve as a uniform and stable reference.

However, tacking the Earth as a reference point, the lunar rotation cannot be observed, only periodic oscillations can be detected. As depicted in Figure 1(β), this oscillation refers to the varying angle (around $90^\circ \pm 3.2^\circ$) between the velocity direction and the position vector direction at different locations along the Moon's elliptical orbit, assuming a lunar eccentricity of $e=0.0554$, calculated using the flight path angle formula.

The lunar observer's perception of phenomena such as "sunrise, sunset, and stellar movements" isn't a result of the Moon's rotation. The fact that one side of the Moon consistently faces Earth demonstrates a null lunar rotational angular momentum within the Earth-Moon system C-M-O1. The rotation of the system C-M-O1 within the system O2 is primarily responsible for the periodicity of the solar and stellar cycles. The conservation of orbital angular momentum in the C-M-O1

system stabilizes the periodicity of the C system's rotation. Moreover, due to the conservation law of orbital angular momentum, derived from elliptical orbits and Kepler's area theorem, the observed speeds of stars and the Sun vary unevenly across different orbital arc segments. This variation leads to an unevenness in stellar periods when the Sun is used as a reference and, conversely, an unevenness in solar periods when stars are used as a reference. If one were to traverse beyond the Earth-Moon synodic coordinate system to seek references in the Solar System or the Galaxy, the uneven orbital curvature would be attributed to irregularities in the Moon's revolution.

3.3 The local calendars of various celestial phenomena

Defining the observed periodic celestial events by lunar surface observers—A, B, C, and D—as timekeeping units, establishing their cumulative rules or radix relationships, and continually refining them through long-term, consecutive observations to adjust these rules, forms various local calendars for different celestial phenomena. While using the Sun or stars as references lacks strict uniformity, the long-term average of these periods remains stable. Kepler's area theorem informs us that "uniformity in time on elliptical orbits cannot coexist with uniformity in space." Uniform intervals on the lunar surface do not equate to uniform angular sectors traversed by celestial bodies; conversely, dividing uniform angular sectors on the lunar surface does not equate to celestial bodies passing overhead at a uniform rate across the local meridian.

4 Conclusion

Applying the fundamental principles of cyclical and localized time to lunar calendrics has been preliminarily analyzed for certain characteristics of the lunar calendar. Scientists and space exploration governing bodies have yet to reach a consensus on how to define a standardized time system for lunar surfaces. The lunar calendrical rules proposed in this paper offer a reference point for researchers involved in lunar calendrical studies.

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