Nearly Unidirectional Microwave Networks and Quantum Graphs

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A quantum graph Γ is a one-dimensional complex system with the Laplace operator $L(\Gamma) = -\frac{d^2}{dx^2}$ defined in the Hilbert space of square integrable functions. A quantum graph consists of one-dimensional leads l_i which are connected at the vertices v_k. The propagation of a wave along the one-dimensional lead is described by the Schrödinger equation. The boundary conditions are implemented on the wave functions entering and leaving the vertices. Here, we will consider a commonly used Neumann vertex boundary conditions. The Neumann boundary condition imposes the continuity of waves propagating in the leads meeting at the vertex v_k. and vanishing of the sum of outgoing derivatives at v_k. Quantum graphs can be simulated by microwave networks. It is possible because of the equivalence of the stationary Schrödinger equation describing quantum graphs and the telegraph equation describing microwave networks [1]. In this report we discuss the spectral statistics of nearly unidirectional quantum graphs [2] and the transport properties of nearly unidirectional microwave networks. A scheme of the unidirectional network is shown in Figure 1.

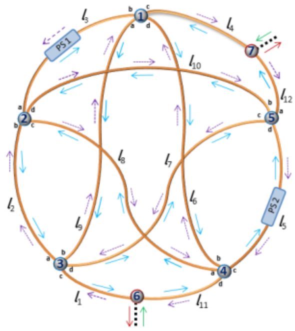

Figure 1. A scheme of the nearly unidirectional network. It contains five 4-port microwave hybrid couplers, vertices ($v_1 - v_5$), and two T-joints v_6 and v_7. The direction of propagating waves are shown by the arrows.

The eigenvalues of the nearly unidirectional graphs are found numerically by using the pseudo-orbits method [3] which leads to the equation:

$$\det[I_{2N} - LS_G] = 0, \tag{1}$$

where I_{2N} is $2Nx2N$ identity matrix, N is the number of the internal edges of the graph, L is the length matrix and S_G is the lead scattering matrix, containing scattering conditions at the graph vertices.

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References

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