Evolution of magnetic field and velocity gradient tensor invariants in various turbulent regions of the heliosphere

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The investigation of the gradient tensor invariant statistics is useful to characterise the complex topologies of magnetic and velocity field in turbulent space plasmas. Numerous studies have been published in the past on the statistical characteristics of the gradient tensor invariants at magnetohydrodynamic and ion scales in turbulent space plasmas using data from multipoint space missions such as NASA-MMS and ESA-Cluster in various near-Earth and interplanetary environments [1, 2, 3, 4]. The main objective of this presentation is to evaluate the relative weight of the various terms present in the evolution equation for the gradient tensor invariants of the velocity and magnetic fields. This will help determine which dynamical regime is more appropriate for governing the evolution of the line topology of the magnetic and velocity fields. This work builds on a previous study in which the evolution equations of gradient tensor invariants in the magnetohydrodynamic domain have been derived [5], i.e.,

$$\begin{cases} \dot{Q} + 3R + \frac{1}{\mu_0 \rho}\mathrm{Tr}(\Xi \cdot \mathbf{Z} \cdot \mathbf{A}) = 0 \\ \dot{R} - \frac{2}{3}Q^2 + \frac{1}{3\rho\mu_0} Q\,\mathrm{Tr}(\Xi^2) + \frac{1}{\rho\mu_0}\mathrm{Tr}(\Xi \cdot \mathbf{Z} \cdot \mathbf{A}^2) = 0 \\ \dot{X} = 0 \\ \dot{Y} = 0 \end{cases} \tag{1}$$

where

$$\mathrm{Q} = -\frac{1}{2}A_{im}A^{mi} \qquad \mathrm{R} = -\frac{1}{3}A_{im}A^{m}_{k}A^{k}_{i}, \tag{2}$$

$$\mathrm{X} = -\frac{1}{2}Z_{im}Z^{mi} \qquad \mathrm{Y} = -\frac{1}{3}Z_{im}Z^{m}_{k}Z^{k}_{i}. \tag{3}$$

Here, $(\mathbf{A})_{ij} = \partial_i v_j$ and $(\mathbf{Z})_{ij} = \partial_i b_j$ are the velocity and magnetic field gradient tensors, and $\Xi = \mathbf{Z} - \mathbf{Z}^\dagger$ is the skew-symmetric part of the magnetic field tensor which is associated with the current, i.e., $\Xi^{ik} = -\varepsilon^{ikh} j_h$. Specifically, we will demonstrate how distinct dominant components govern the evolution of gradient tensor invariants in the solar wind and magnetosheath plasma turbulence [6]. This result provides an alternate way to the classical ones to study space plasma turbulence and is explained in relation to the distinct nature of the observed turbulence.

References

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