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Transmission through an Isorefractive Parabolic-Cylinder Radome

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A radome whose interfaces with the surrounding medium are confocal parabolic cylinders with the same symmetry plane is considered. The medium on both convex and concave sides of the radome is linear, homogeneous, lossless and isotropic; it is characterized by a real positive electric permittivity ε_1 and a real positive magnetic permeability μ_1 or, equivalently, by a refractive index $n=\sqrt{(\varepsilon_1\mu_1)}$ and an intrinsic impedance $Z_1=\sqrt{(\mu_1/\varepsilon_1)}$. The radome is made of a linear, homogeneous, lossless and isotropic material characterized by a real positive permittivity ε_2 and a real positive permeability μ_2 such that $\varepsilon_1\mu_1=\varepsilon_2\mu_2$, meaning that the media outside and inside the radome have the same refractive index, but different intrinsic impedances: $Z_2=\sqrt{(\mu_2/\varepsilon_2)} \neq Z_1$. The primary field is a plane wave symmetrically incident on the concave side of the radome, and the analysis is conducted in the phasor domain, with a time-dependent factor $\exp(+j\omega t)$ omitted throughout.

A possible approach to the solution of the boundary-value problem consists in expanding the electromagnetic field in each of the three space regions into infinite series of parabolic-cylinder functions and in determining the expansion coefficients by imposing the boundary conditions at the material interfaces. However, this approach leads to results that are not amenable to an easy physical interpretation, and therefore it will not be pursued herein.

Instead, the method introduced by Lamb in the study of reflection by a convex metallic parabolic cylinder (H. Lamb, "On Sommerfeld's diffraction problem; and on reflection by a parabolic mirror", *Proc. London Math. Soc.*, vol. 4, pp. 190-203, 1907) and extended by Uslenghi to the reflection by a concave mirror (P.L.E. Uslenghi, "Reflection by a concave parabolic mirror", *IEEE Antennas Wireless Prop. Lett.*, vol. 11, pp. 419-422, 2012; ---, "Scalar and vector diffraction by a concave parabolic cylinder with a mixed boundary condition", *Proc. ICEAA-IEEE APWC Conference*, Aruba, 3-8 August 2014) and in several conference papers by Uslenghi dealing with the transmission through a single parabolic-cylinder interface separating two different media, is utilized in this work.

The field in each of the three regions is expressed in closed form as the sum of two plane waves propagating in opposite directions parallel to the symmetry plane, and the sum of two functions that are the product of a plane wave times a Fresnel integral. The coefficients of these various functions are determined by imposing the boundary conditions across the parabolic-cylinder boundaries, the conditions at infinity, and the continuity condition of the field across the focal line, resulting in an exact and closed form solution to the boundary-value problem. The obtained exact solution is simplified asymptotically when the radii of curvature of the interfaces are large compared to the wavelength.