3rd URSI AT-AP-RASC, Gran Canaria, 29 May – 3 June 2022

A Direct Tensor Train Decomposition Scheme for Electrostatic Problems in Two Dimensions

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Extended Abstract

Matrix equation formulations are widely used in dynamical simulations of electromagnetic systems, owing to their stability and precision across a broad range of problem classes. However, these matrix methods demand highly optimized solution algorithms, as naive implementations will frequently yield prohibitive scaling in computational complexity. Indeed, for the analysis of a system having N unknown quantities, basic solution schemes will commonly yield $\mathscr{O}\left(N^2\right)$ or even $\mathscr{O}\left(N^3\right)$ scaling in runtime and memory complexity. This complexity can often be improved significantly by exploiting various properties of the system. Currently, the most advanced matrix solution algorithms in electromagnetics are capable of providing $\mathscr{O}(N)$ or $\mathscr{O}(N\text{polylog}N)$ runtime and memory complexity[1, 2, 3], with the precise scaling being dependent on the structure of the system under analysis.

Here we focus on the tensor train decomposition (TTD) approach to optimization. The TTD expresses a tensorized version of the interaction matrix, A, as a product of tensor cores G_p:

$$A_{j_1 j_2 \cdots j_d} = G_1(j_1)\, G_2(j_2) \cdots G_d(j_d), \tag{1}$$

where d is the length of the tensor train. Each $G_p(j_p)$ is commonly taken as a matrix of size $r_{p-1} \times r_p$, with $r_0 = r_d = 1$. These values of r_p are known as the ranks of the tensor cores. Reference [2] shows that, for scatterers possessing many regions which are similar in geometric and material properties, the tensor cores are highly rank-deficient. By exploiting these rank deficiencies, the train of tensor cores can be stored and applied with significantly greater efficiency than the original interaction matrix. For low-frequency regimes, the conjugate gradient-TT (CG-TT) scheme used in [2] yielded $\mathscr{O}(N)$ scaling in both runtime and memory. In the high-frequency regime, the method provided $\mathscr{O}\left(r^2 N \log N\right)$ scaling, where r is the highest tensor core rank.

The complexity scaling provided by the CG-TT solution scheme is highly favorable, but the method suffers from ambiguity in the definition of the tensor core ranks. The tensor cores—and hence their ranks—are determined by an iterative decomposition procedure, and as a consequence, the resources necessary to solve a given system cannot be predicted. The focus of our current work is the development of a scheme in which the tensor core ranks can be predicted. We study the decomposition of the 2-D Laplace kernel, wherein the interaction between elements located at points ρ and ρ' is given by $\log|\rho - \rho'|$. The expression of tensor core elements as terms of truncated series expansions of the log function allows us to formulate cores of predetermined size that, when multiplied, produce an approximation to the desired interaction. The precision of this approximation can then be controlled by the size used for the tensor cores.

References

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