3rd URSI AT-AP-RASC, Gran Canaria, 29 May – 3 June 2022

A two-way split-step wavelet method for the 2D tropospheric propagation

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Abstract

Modeling the long-range propagation of electromagnetic waves in the troposphere is a topic of interest for many applications in surveillance, communication, and remote-sensing. The split-step Fourier method is widely used in this context, allowing wide steps along the propagation direction. Recently, a wavelet-based method with lower complexity and efficient in terms of memory storage has been proposed. Both methods are one way and compute the forward propagation in the troposphere. Based on the Fourier two-way method, which computes the backward propagation when reaching an obstacle, we propose here a two-way split-step wavelet method. Besides, we show that the stopping criterion is intrinsic using compression in the wavelet domain. Numerical experiments have been performed (in S band) to validate the method and highlight its advantages.

1 Introduction

A precise model for electromagnetic waves propagation in the troposphere over large distances is necessary to study and optimise the performance of many systems, such as radar. In this context, we need to take into account the ground composition, relief, and refraction.

Due to the size of the scenario, rigorous methods are prohibited. Therefore, asymptotic methods are used in this context. One one the most common model in this case is the parabolic wave equation (PWE) [1]. Based on the Helmholtz equation, this latter only model the forward propagation from the source in a paraxial cone around the propagation direction [1]. This model is well suited for long-range propagation since it can take into account the effects of the refraction and the relief [1].

A widely used method to solve the PWE in 2D is the split-step Fourier (SSF) method [1]. This latter compute the field iteratively by going back and forth from the spatial domain to the spectral domain. The propagation is performed with two steps. First, we propagation in free-space in the Fourier domain. Second, the effects of the refraction are taken into account through a phase screen. Relief can also be considered with this method with various techniques [2]. Furthermore, the ground condition is efficiently incorporated with the discrete mixed Fourier transform (DMFT) [3]. In recent years, to avoid spurious solutions a self-consistent SSF method has been developed, discrete SSF (DSSF) [4].

To obtain an efficient method in terms of memory and computation time, in particular for generalisation to 3D, wavelet-based methods have been proposed for electromagnetic propagation [5, 6]. In particular, a fast and memory-efficient split-step wavelet (SSW) has recently been proposed [5, 6]. Indeed, a low complexity method has been obtained from the compression allowed by wavelets and the low complexity of the fast transform associated. Also, the sparse signal and propagator obtained in the wavelet domain improve the memory efficiency [6, 7].

These methods are one way and only compute the forward propagation and do not take multipath into account. To tackle this problem two-way recursive SSF methods [8] have been proposed. In this case, a condition on the magnitude of the field is applied to stop the algorithm and be efficient.

In this article, based on the compression introduced in the wavelet domain, we propose a two-way recursive SSW method without a stopping condition on the magnitude of the field.

The remaining of this paper is organized as follows. Section 2 introduces the two-way model based on the parabolic wave equation. Section 3 gives a brief reminder on the 1D discrete wavelet transform. Section 4 describes the computation method. Section 5 is devoted to the numerical experiments. The last section concludes the paper and gives perspectives.

2 Model and discretization

2.1 The two way parabolic equation model

In this article, we assume an $\exp(j\omega t)$ time dependence. We also assume that the refractive index n is slowly-varying.

In this article, we are interested in the 2D Cartesian configuration along x – the propagation direction – and z – the altitude. The initial field at $x = 0$ is known, since the source is placed at $x_s \leq 0$ and we compute the propagation for $x \geq 0$. The ground is at the altitude $z = 0$. Thus, the propagation in the domain $[0, x_{\max}] \times [0, z_{\max}]$ is computed. With respect to z, the field can decomposed in transverse magnetic (TM) or electric (TE) components. In this article we focus on the

TE case, nevertheless calculations remain the same for the TM case. We denote by ψ the TE components, which is solution of the Helmholtz equation

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial z^2} + k_0^2 n^2 \psi = 0, \tag{1}$$

with n the refractive index and k_0 the free-space wavenumber.

As in [8], we describe the total field with a reduced forward, u_F, and backward, u_B, propagation part along x

$$\psi = \exp(jk_0 x)u_F + \exp(-jk_0 x)u_B. \tag{2}$$

This leads to two equations for the forward and backward propagation

$$\begin{aligned} \frac{\partial^2 u_F}{\partial x^2} + 2jk_0 \frac{\partial u_F}{\partial x} + k_0^2(n^2-1)u_F = 0, \\ \frac{\partial^2 u_B}{\partial x^2} - 2jk_0 \frac{\partial u_B}{\partial x} + k_0^2(n^2-1)u_B = 0. \end{aligned} \tag{3}$$

Note that both equations are the same if we change the sign of k_0. Therefore, we only study the forward part, since calculations remain the same for u_B. Accounting only for the forward propagation of u_F and applying the wide angle approximation to the first equation of (3) leads to

$$\frac{\partial u_F}{\partial x} = -j\left(\sqrt{\frac{\partial^2}{\partial z^2} + k_0} - k_0\right)u_F - jk_0(n-1)u_F. \tag{4}$$

Note that we can obtain the same equation for u_B by changing k_0 to $-k_0$. In this article, we thus aim at solving the equations for u_F and u_B using a split-step method, whereas one way method only focus on u_F.

2.2 Discretization

For numerical reasons, the domain is discretized. First, along the z-axis, a discretization of N_z points is performed. The grid is given by

$$z_{p_z} = p_z \Delta z \quad \text{with} \quad p_z \in \{0, \cdots, N_z\}, \tag{5}$$

with $\Delta_z = z_{\max}/N_z$ the step size. At a position x, the discrete field is denoted by $u_x[p_z]$. Second, the x-axis is also discretized on N_x points and a step size Δx. Computations are now performed only in this discrete domain.

3 Brief overview of the discrete wavelet transform

In this part, we give a brief reminder on the 1D discrete wavelet transform. First, a wavelet is a short length oscillating function localised both in frequency and space. As for the Fourier atoms for the Fourier transform, we need to define a wavelet basis. First, we construct a wavelet family by dilating, on L levels, and translating of p along z a zero-mean function ψ – called the wavelet mother. The daughter wavelets $\psi_{l,p}$, with $l \in [1, L]$, are thus obtained. While dilations allow to cover the spectrum from the upper part to the lower part with L increasing, with the translation we cover the spatial domain. This functions correspond to a wavelet family. Nevertheless, since the wavelet are of zero-mean, a last function $\phi_{L,p}$ – the scaling function – is added to cover the lowest part of the Fourier domain and to obtain an orthonormal basis.

We can now decompose the any field u_x on this basis with the discrete wavelet transform. This latter is defined [9] as follows

$$u_x[\cdot] = \sum_{p=0}^{N_z/2^L-1} a_{L,p}\phi_{L,p}[\cdot] + \sum_{l=1}^{L}\sum_{p=0}^{N_z/2^l-1} d_{l,p}\psi_{l,p}[\cdot], \tag{6}$$

where $a_{L,p}$ and $d_{l,p}$ corresponds to the approximation and details coefficients, respectively. The first are the one corresponding of the decomposition of field on the scaling function while the second corresponds to the decomposition on the wavelet functions. These latter are computing using the fast wavelet transform (FWT) [9].

Different wavelet basis can be used. Important parameters for the choice are the size of the support and the number of vanishing moments n_v [9]. This latter defines how well a smooth signal can be described with few coefficients. In this article, we choose the symlet family with $n_v = 6$ since these wavelets are of minimal support for a given n_v and almost symmetric. For the maximum level, I choose $L = 3$. These choices are discussed in [5].

4 The two way split-step wavelet method

In this section, we desccribe the two-way wavelet-based split-step method. First, we focus on the one way method to solve the parabolic equation for u_F. Then, the two way version of the method is introduced.

4.1 Overview of the one way SSW method

First, we focus on the one way SSW method to compute u_F. This latter is an interative methods that solve (4) as follows:

1. The FWT, denoted $\mathbf{W}$, and a compression, denoted C, with hard threshold V_s are applied to u_x to obtain the sparse vector
$$U_x = \mathbf{CW}u_x \tag{7}$$
corresponding to the wavelet coefficients.

2. The coefficients are propagated in the wavelet domain using a wavelet-to-wavelet propagation operator, denoted $\mathbf{P}$, as
$$U_{x+\Delta x} = \mathbf{P}U_x. \tag{8}$$
Two main methods exist to compute this propagation. One consist in computing the matrix that stores all the

wavelet propagation [5]. The other [6] computes a reduce propagator and only the non-zero coefficients of U_x are propagated. In both cases, a compression with threshold V_p is performed to have a sparse propagator.

3. By applying the inverse FWT, denoted by $\mathbf{W}^{-1}$, to $U_{x+\Delta x}$ we obtain

$$u^{fs}_{x+\Delta x} = \mathbf{W}^{-1} U_{x+\Delta x}, \tag{9}$$

that corresponds to the free-space propagated field.

4. Refraction and relief are taken into account in the space domain through operators $\mathbf{R}$ – phase screen – and $\mathbf{L}$, respectively. Therefore, we obtain

$$u_{x+\Delta x} = \mathbf{L}\mathbf{R} u^{fs}_{x+\Delta x}. \tag{10}$$

To efficiently take into account the PEC ground composition with the wavelet method, we use the local image method [5]. To model the relief, a staircase model [1] is used.

4.2 The recursive two-way method

Now that the one-way method has been fully described, we introduce the two-way SSW method. In this latter we propagate both u_F, with a Δx, and u_B, with a step $-\Delta x$, along the propagation direction. Since both propagation equation are the same (3), except for the sign of k_0, it can be shown that both propagator are the same [8].

Therefore the two-way SSW consists in iterating back-and-forth the SSW algorithm described in Section 4.1 for u_F, and for u_B when reaching an obstacle. This allows to take into account multiple reflections and multipath in the model.

Nevertheless, for the backward propagation and the multiple reflections, we need the initial field at the obstacle, position x_o. To obtain a relation between $u_B(x_o)$ and $u_F(x_o)$, we use the condition on the transverse field at the obstacle. Since we use a staircase model for the relief, the condition is [8]

$$\psi(x_o) = 0 = e^{jk_0 x_o} u_F(x_o) + e^{-jk_0 x_o} u_B(x_o). \tag{11}$$

Thus, using (11) we can compute the initial field for the recursive two-way SSW when reaching any obstacle.

Note that, if only one obstacle is present, the method is easy since only one forward and backward propagation is computed. Indeed, we can compute the backward propagation until the beginning of the domain. Nevertheless, for multiple obstacle, we need a stop condition on the multiple forward and backward propagation. In [8], a condition on the magnitude of the field is introduced as a stopping condition of the recursive algorithm. Here, we differ from this approach by showing that the compression applied on the field in the wavelet domain works as an intrinsic stopping condition.

First, we define the operator norm as

$$\|\mathbf{P}\|_{\text{op}} = \frac{\|\mathbf{P}u\|_2}{\|u\|_2}. \tag{12}$$

If the propagation is computed in free-space then we have $\|\mathbf{P}\|_{\text{op}} = 1$. Otherwise, since the energy can only leave the domain, we have $\|\mathbf{P}\|_{op} \leq 1$. In all generality, we can thus keep

$$\|\mathbf{P}\|_{\text{op}} \leq 1. \tag{13}$$

Furthermore, note that the signals we are dealing with are smooth.

Using (13), we know that

$$\forall n \geq 0 \qquad \|u_n\|_2 \leq \|u_0\|_2. \tag{14}$$

This relation is true in both way the norm of all propagated field is smaller or equal to the norm of the initial field. Then, we use the Moyal relation [9], or energy conservation in the wavelet domain, to obtain

$$\|U_n\|_2 \leq \|U_0\|_2. \tag{15}$$

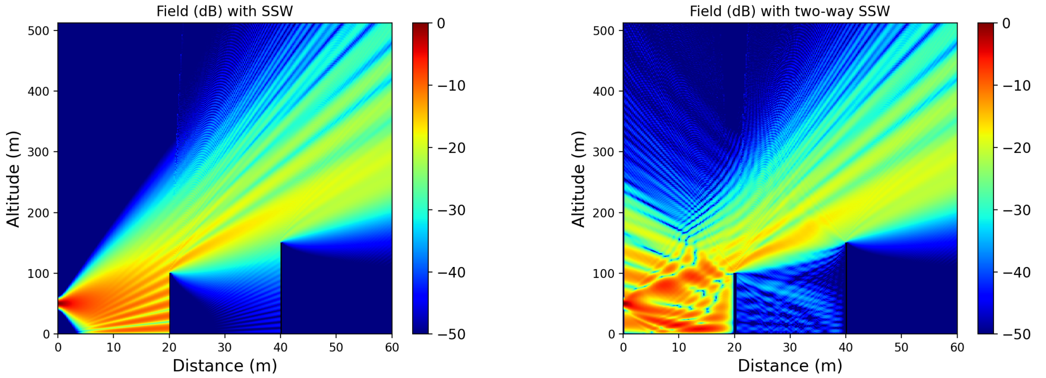
Using the relation (15) and that the signals are smooth we have that the wavelet coefficient are decreasing with the propagation. Thus, since a compression with a threshold $V_s = v_s \|U_0\|_\infty$ is applied on the wavelet decomposition at each step then the relative threshold v_s has the same effect as the stopping criterion on the field in [8]. Therefore, the stopping condition is intrinsic in two-way SSW. Besides, for each recursive propagation we can change this threshold to reduce the computation time. Also, the error due to the compression is managed through a theoretical formula [10].

5 Numerical test

The goal of the numerical experiment is to show that two way SSW works well. A comparison with the one-way version is proposed.

We compute the propagation from a complex source point (CSP) at $f_0 = 3$ GHz. The parameters of the source are as follows: $x_s = -50$ m, $z_s = 50$ m and $W_0 = 5$ m. The propagation is performed over 60 km and 512 m with a grid size of $\Delta x = 200$ m and $\Delta z = 0.1$ m. To validate the method, we consider a PEC ground condition and $n = 1$. Two knife edge obstacles at 20 km and 40 km of size 100 m and 150 m are considered. The threshold are set so as to obtain a maximum error of -20 dB with DSSF, for the one way SSW, with the theoretical formula [10]. The same wavelet parameters are used for the two-way SSW.

The normalised field obtained with the one way and two-way SSW are represented in Figure 1 (a) and (b), respectively. Note that, for the one way method the error with DSSF is below -45 dB.



(a) One-way SSW

(b) Two-way SSW

Figure 1. Propagation of the normalized field u with one-way and two-way SSW.

First, with the one way method the effects of the relief consist in shadow zones, and is limited, since multipath are not taken into account. Second, we can see in Figure 1 (b) that the two-way method the results of propagation differ from the one way method, Figure 1 (a), since multipath effects and interference patterns are taken into account. Also, after the second relief, the results of both methods are the same, as expected. The results are in line with the one in [8]. Therefore, we are confident that the method works well. Note that here the stopping criterion is intrinsic to the method, whereas in [8] they need a condition on the magnitude of the field to stop the method.

6 Conclusion and perspectives

In this article, we propose a recursive two-way split-step wavelet method, with an intrinsic stopping criterion.

First, the two-way model based on the parabolic wave equation has been introduced. Second, the 1D discrete wavelet transform has been briefly reminded. Third, the forward and backward model are solved with a SSW two-way method. With this latter, the stopping criterion is intrinsic through the compression performed on the field wavelet coefficients. Finally, numerical tests have been performed to show that the method works well and highlight its advantages in comparison with the one-way version of the method.

Further works include comparison with geometrical optic and uniform theory of diffraction. Other numerical experiments will also be considered with other type of relief and in other frequency range (such as X band).

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