Resistivity of copper at the initial stage of an electric explosion process.

Ali Alhammadi(1,2), Evgeny Gurnevich(1), John J. Pantoja(2), Felix Vega (2), and Chaouki Kasmi(2)
(1) Institute for Nuclear Problems, Belarusian State University, 220006, Minsk, Belarus.
(2) Technology Innovation Institute, Abu Dhabi, United Arab Emirates, http://www.tii.ae.

Abstract

We developed a circuit model to calculate the temporal dependence of the resistivity of a copper wire in an exploding-wire array. The model determines the resistance of the wire as a function of the specific absorbed energy, by using published data on the copper's heat capacity and resistivity at various temperatures. Experimental tests show excellent agreement with the proposed model.

1. Introduction

In this paper we present a procedure to model the electrical wire array (EWA) explosion due to high intensity pulsed currents. This model can be used to design experiments based on inductive energy storage with electrically exploded wire array. In some applications, such as high-power microwave generation, an accurate description of initial explosion stages (before vaporization and explosion) is necessary to provide the required amplitude and shape of the pulse to the load.

There are several conflicting requirements imposed on the design of EWA. Firstly, the length, cross-section and number wires must thus be selected, to provide the load with a voltage pulse of the required amplitude and shape. Secondly, the design of the EWA unit must have sufficient electric breakdown strength to withstand the generated voltage. Normally, this is solved by increasing EWA length and/or by pressurizing all critical structural elements (and wires also). Thirdly, with all this, the EWA unit should be as compact as possible, to provide minimal values of inductances and to ensure operational requirements. For these reasons, a wire array explosion model is necessary to get a design that complies with the list of restrictions and requirements.

The proposed procedure couples the basic RLC circuit equations with zero-dimensional thermodynamic calculations that track the effect of heating on the heat capacity and resistance of the wire [1].

2. Theoretical method

The current in exploding copper wire array as a function of time when a capacitor bank \((C)\) is discharged into the wire array through a series inductance, as shown in Fig. 1, can be calculated using (1)

\[
\frac{dI}{dt} = -\frac{R}{L}I - \frac{1}{LC}q ,
\]

where \(dI/dt\) is the derivative of current as a function of time, \(R\) is the resistance, \(L\) is the inductance, \(C\) is the capacitance, and \(q\) is the charge.

The electrical resistivity of copper at a given time depends on wire temperature \(T\), and, as result, on specific energy absorbed by wire \(\Delta H\):

\[
\Delta H(t) = \frac{1}{m} \int_0^t R_{\text{wire}}(t) I(t)^2 dt
\]

where \(\Delta H\) is the heat generated per unit mass of the wire (it is referred hereinafter as specific absorbed energy or more simple specific energy), and \(m\) is the mass, and

\[
R_{\text{wire}} = \rho(T) \frac{L}{\pi r^2}
\]

\(\rho\) is the resistivity of wire as a function of the temperature \(T\), \(r\) is the radius of the wire, and \(L\) is wire's length. Dependencies of copper resistivity on temperature and on
specific energy can be approximated by 2nd order polynomials, as explained in detail in [1,2].

The total resistance in the RLC circuit can be calculated as the sum of \(R_{wire}\) and the additional resistance of the system \(R_{system}\), that is
\[
R = R_{wire}(H) + R_{system}.  \tag{4}
\]

The current as a function of the time is finally calculated by solving (1)-(4). The data on the thermodynamic characteristics of the copper and the effect of heating on the heat capacity of copper presented in [1] was used for the calculations.

3. Experimental and calculated results

An experiment on a copper exploding wire array based on the setup represented in Figure 1 was developed. The parameters of the setup are summarized in Table 1.

<table>
<thead>
<tr>
<th>Table 1 The parameters of RLC circuit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Parameters</td>
</tr>
<tr>
<td>Capacitor</td>
</tr>
<tr>
<td>Inductance</td>
</tr>
<tr>
<td>Resistor</td>
</tr>
<tr>
<td>Initial capacitor voltage U0</td>
</tr>
<tr>
<td>Diameter of exploding wire</td>
</tr>
<tr>
<td>Length of the wire</td>
</tr>
<tr>
<td>Copper core</td>
</tr>
</tbody>
</table>

The raw data of the current in the wires obtained from the Rogowski coil is shown in Fig. 2. The current calculated from the raw data is presented in Fig. 3.

3.1 Results obtained by the proposed model

The model here developed was applied to the values presented in Table 1. The current obtained is presented in Fig. 4, together with the experimental results. As it can be seen, both are in excellent agreement. We can see a slight difference in the peak current at the melting phase, which is somehow to expect given the phase change in the material.

4. Conclusion

A model for initial stage of copper wire electrical explosion was presented and validated with experimental results. This model involves description of the heating stages of solid and liquid phases of copper and the melting process. The basic RLC circuit equations are coupled with temperature-dependent thermodynamic calculations to track how heat capacity of copper changes with increasing temperature. Other approaches do not consider this change but this detail can be extremely valuable for building
complex models that describe the explosive evaporation stage of a metal.

References
