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Radiation of limited-diffractive Bessel beams in cylindrical metallic cavities

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Abstract

In this work, the radiation inside a cylindrical metallic cavity of a limited-diffractive Bessel beam in near-field is analyzed both theoretically and numerically, showing a remarkable agreement between the model proposed and full-wave simulations. This study is motivated by possible applications of focused beams, such as microwave heating or plasma excitation in cavities.

1 Introduction

Limited-diffractive beams in near-field can be considered interesting for several applications in engineering and physics, spanning from near-field communications (NFC) to wireless power transfer (NF-WPT) [1], but also from ground penetrating radar (GPR) [2] to imaging for diagnostics. Among limited-diffractive beams, Bessel beams are undoubtedly of paramount importance since their early introduction in optics [3], due to their remarkable property of field collimation dominantly in their main lobe [4, 5], inside which the local wavefront is planar. Although ideal Bessel beams, i.e., a solution of Helmholtz equation in cylindrical coordinates, are able to maintain their collimation allover the space, in real applications they can be radiated only up to a certain distance from the finite radiating aperture, called nondiffractive range (NDR) [5, 6]. However, by properly designing the imposed radial wavenumber on the aperture (by acting on both aperture size and so-called axicon angle), it is possible to properly predict the NDR and to enlarge it in such a way to satisfy the requirements dictated by specific applications.

As it can be inferred from the literature on Bessel beams at microwaves and millimeter waves [7], specific high-efficiency launchers have been successfully synthesized and fabricated by exploiting different but complementary approaches, such as holography on radial line slot arrays (RLSA), or leaky-wave theory, just to mention some of them. Moreover, mature technologies, such as for instance PCB, are available to fabricate prototypes, so that realistic applications of Bessel beams at microwaves and millimeter waves do not represent only an idealization, but instead a promising reality. On the other hand, theoretical and numerical models of Bessel beam radiation have been mainly performed in free-space. However, it is sometimes convenient to consider Bessel beams radiating in metallic cavities, especially for microwave heating applications. To this aim, in this contribution we consider both analytical and numerical modeling of the radiation of a limited-diffractive Bessel beam inside a metallic cylindrical cavity, in such a way to highlight physical insight and relevant wave phenomena connected to beam radiation in bounded domains. Indeed, a suitable model needs to be developed to fully take into account Bessel beam expansion and radiation, reflections from cavity walls, stationary waves arising from the bounded domain, and also excitation of spurious modes. The proposed analysis will also pave the way to future applications of Bessel beams for microwave heating or for plasma excitation. The paper is divided as follows; in Section 2, the analytical formulation of the radiation of a magnetic current distribution that normally radiates a Bessel beam in free-space in a cylindrical metallic cavity is presented. Then, in Section 3 the presented model is compared versus full-wave simulations. Finally, conclusions are drawn.

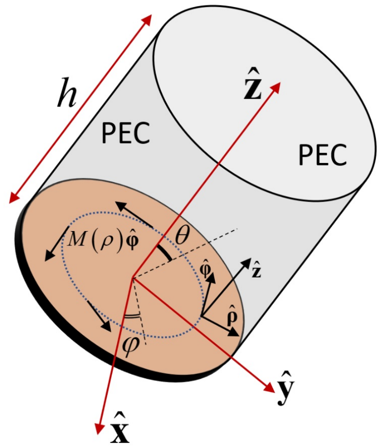

Figure 1. Geometry of the problem. An azimuthally-invariant magnetic current distribution shaped as a first order Bessel function radiates inside a cylindrical metallic cavity, thus exciting a specific field distribution inside it.

2 Problem Formulation

The geometry of the problem is depicted in Fig. 1. A finite azimuthally-invariant magnetic current distribution of the form

$$\mathbf{M}(\rho) = J_1(k_{\rho a}\rho)\Pi\left(\frac{\rho}{2b}\right)\hat{\phi}, \tag{1}$$

is impressed at $z = 0$ and radiates inside a cylindrical metallic cavity of radius b and height h. In (1), $\Pi(\cdot)$ is the rectangular function, $k_{\rho a} = k \sin\theta_a$ the imposed radial wavenumber, k the free-space wavenumber at the operating frequency, and θ_a the so-called *axicon angle*. For the sake of simplicity, cavity walls are assumed to be made of perfect electric conductors (PEC). As shown in detail in [4, 5], the current distribution in (1) is able to radiate a longitudinally-polarized Bessel beam in free-space, i.e., a Bessel beam whose electric field z-component is shaped as a zeroth order Bessel function. The objective of this paper is to analyze how the radiated beam is perturbed by the presence of a cylindrical metallic cavity, since in general the EM field inside it can be written as a superposition of a numerable infinite set of modes, hence the spectrum of the eigenvalues associated to Laplacian operator $\mathscr{L} = \nabla^2$ is discrete.

According to Marcuvitz-Schwinger formalism, the electromagnetic (EM) field by impressed current distributions (either electric, $\mathbf{J}$, or magnetic, $\mathbf{M}$) can be decomposed in its transverse $(\mathbf{E}_t, \mathbf{H}_t)$ and longitudinal (E_z, H_z) components with respect to a prescribed direction. For the problem at hand, it is convenient to use the z-axis. So that, the transverse EM field components can be written as superposition of transverse electric (TE) and magnetic (TM) modes [8], namely

$$\mathbf{E}_t(\mathbf{r}) = \sum_{m,n} V_{mn}^{TM}\mathbf{e}_{mn}^{TM}(\rho) + \sum_{m,n} V_{mn}^{TE}\mathbf{e}_{mn}^{TE}(\rho), \tag{2}$$

$$\mathbf{H}_t(\mathbf{r}) = \sum_{m,n} I_{mn}^{TM}\mathbf{h}_{mn}^{TM}(\rho) + \sum_{m,n} I_{mn}^{TE}\mathbf{h}_{mn}^{TE}(\rho), \tag{3}$$

in which $\mathbf{e}_{mn}^{TM,TE}$ ($\mathbf{h}_{mn}^{TM,TE}$) are the TE/TM electric (magnetic) vector mode functions, whereas $V_{mn}^{TM,TE}$ ($I_{mn}^{TM,TE}$) refer to complex modal amplitudes, to be suitably determined. Due to ϕ-invariance and azimuthal polarization of the impressed magnetic current distribution (1), the EM field inside the cavity can be represented only in terms of TM modes, so that $V_{mn}^{TE} = I_{mn}^{TE} = 0$. Moreover, the radiated EM field expansion in (2)-(3) can be reduced to a single n-dependent series. Hence, for the problem at hand the transverse field expressions reduce to

$$\mathbf{E}_t(\mathbf{r}) = \sum_{n} V_{0n}^{TM}\mathbf{e}_{0n}^{TM}(\rho), \quad \mathbf{H}_t(\mathbf{r}) = \sum_{n} I_{0n}^{TM}\mathbf{h}_{0n}^{TM}(\rho). \tag{4}$$

The longitudinal EM field component can be calculated by differentiation of (4) as

$$E_z(\mathbf{r}) = (j\omega\varepsilon)^{-1}[\nabla_t \cdot (\mathbf{H}_t \times \hat{\mathbf{z}}) - J_z], \tag{5}$$

$$H_z(\mathbf{r}) = (j\omega\mu)^{-1}[\nabla_t \cdot (\hat{\mathbf{z}} \times \mathbf{E}_t) - M_z], \tag{6}$$

being J_z (M_z) the z-component of electric (magnetic) current, that in the following will be assumed to vanish, since (1) represents a purely transverse current distribution. TM_z vector mode functions can be written as the transverse gradient of a normalized set of scalar mode functions ψ_{mn}^{TM}, so that

$$\mathbf{e}_{mn}^{TM}(\rho) = -\frac{\nabla_t \psi_{mn}^{TM}(\rho)}{k_{t,mn}^{TM}}, \qquad \mathbf{h}_{mn}^{TM}(\rho) = \hat{\mathbf{z}} \times \mathbf{e}_{mn}^{TM}(\rho), \tag{7}$$

being $k_{t,mn}^{TM}$ the transverse wavenumber associated to TM modes. For the special case of circular cylindrical cavity cross-section, bounded by a PEC rim, ψ_{mn}^{TM} can be expanded in terms of m-th order Bessel functions, hence vector modes functions (7) become

$$\mathbf{e}_{mn}^{TM}(\rho) = -\frac{\exp(-jm\phi)}{b\sqrt{\pi}k_{t,mn}^{TM}J_{m+1}(k_{t,mn}^{TM}b)} \cdot$$
$$\cdot \quad \left[\frac{m\sqrt{2}}{\rho}J_m(k_{t,mn}^{TM}\rho)\hat{\mathbf{p}} - k_{t,mn}^{TM}J_{m+1}(k_{t,mn}^{TM}\rho)\hat{\rho}\right], \tag{8}$$

$$\mathbf{h}_{mn}^{TM}(\rho) = -\frac{\exp(-jm\phi)}{b\sqrt{\pi}k_{t,mn}^{TM}J_{m+1}(k_{t,mn}^{TM}b)} \cdot$$
$$\cdot \quad \left[j\frac{m\sqrt{2}}{\rho}J_m(k_{t,mn}^{TM}\rho)\hat{\mathbf{p}} - k_{t,mn}^{TM}J_{m+1}(k_{t,mn}^{TM}\rho)\hat{\phi}\right], \tag{9}$$

in which $\hat{\mathbf{p}} = (\hat{\rho} - j\hat{\phi})/\sqrt{2}$ and $k_{t,mn}^{TM} = \chi_{mn}/b$, being χ_{mn} the zeros of m-th order Bessel functions, and $n \in \mathbb{N}_0$.

To calculate the z-dependence of the forced EM field inside the cavity by (1), a modal transmission-line approach is exploited [8]. An equivalent modal voltage generator v_{mn}^{TM}, whose amplitude is given by

$$v_{mn}^{TM} \equiv v_{0n}^{TM} = \iint_S \mathbf{M}(\rho)\cdot\mathbf{h}_{mn}^{*}(\rho)dS = \frac{2\sqrt{\pi}}{J_1(k_{t,0n}^{TM}b)} \cdot$$
$$\cdot \frac{k_{t,0n}^{TM}J_1(k_{\rho a}b)J_0(k_{t,0n}^{TM}b) - k_{\rho a}J_0(k_{\rho a}b)J_1(k_{t,0n}^{TM}b)}{k_{\rho a}^2 - (k_{t,0n}^{TM})^2}, \tag{10}$$

being S the cross-section surface, is placed at the section $z' = 0$ of a z-oriented transmission line, terminated on a short-circuit at $z = h$. It is worth noting that, by calculating (10), only modal voltage coefficients of order $m = 0$ contribute (i.e., do not vanish) to EM field expansion (2)-(3), so that the double series used for field expansion can be simply replaced by a single n-dependent series. So that, in the following the replacement $mn \to 0n$ will be adopted to take into account such a result. By using (10), modal voltages V_{0n}^{TM} and currents I_{0n}^{TM} are then calculated at section z as

$$V_{0n}^{TM} = \frac{2\sqrt{\pi}}{J_1(k_{t,0n}^{TM}b)}\left[\cos(k_{z,0n}z) - \cot(k_{z,0n}h)\sin(k_{z,0n}z)\right] \cdot$$
$$\cdot \frac{k_{t,0n}^{TM}J_1(k_{\rho a}b)J_0(k_{t,0n}^{TM}b) - k_{\rho a}J_0(k_{\rho a}b)J_1(k_{t,0n}^{TM}b)}{k_{\rho a}^2 - (k_{t,0n}^{TM})^2}, \tag{11}$$

$$I_{0n}^{TM} = \frac{2j\sqrt{\pi}}{J_1(k_{t,0n}^{TM}b)}\left[\sin(k_{z,0n}z) + \cot(k_{z,0n}h)\cos(k_{z,0n}z)\right] \cdot$$
$$\cdot \frac{k_{t,mn}^{TM}J_1(k_{\rho a}b)J_0(k_{t,mn}^{TM}b) - k_{\rho a}J_0(k_{\rho a}b)J_1(k_{t,mn}^{TM}b)}{k_{\rho a}^2 - (k_{t,mn}^{TM})^2}, \tag{12}$$

being $k_{z,0n} = -j\sqrt{(k_{t,0n}^{TM})^2 - k^2}$ the longitudinal modal wavenumbers, whose branch-cut is chosen in such a way to guarantee the damping of non-propagating modes (i.e., evanescent) away from the source region.

Finally, by substituting (11)-(12) in (2)-(3), the transverse EM field excited inside the cavity by the current distribution given by (1) is

$$\mathbf{E}_t(\mathbf{r}) = -\frac{2}{b}\hat{\rho}\sum_n \left\{ \frac{J_1(k_{t,0n}^{TM}\rho)}{J_1^2(k_{t,0n}^{TM}b)} \cdot \left[\cos(k_{z,0n}z) - \cot(k_{z,0n}h)\sin(k_{z,0n}z)\right] \cdot \frac{k_{t,0n}^{TM}J_1(k_{\rho a}b)J_0(k_{t,0n}^{TM}b) - k_{\rho a}J_0(k_{\rho a}b)J_1(k_{t,0n}^{TM}b)}{k_{\rho a}^2 - (k_{t,0n}^{TM})^2} \right\}, \quad (13)$$

$$\mathbf{H}_t(\mathbf{r}) = \frac{2j}{b}\hat{\phi}\sum_n \left\{ \frac{J_1(k_{t,0n}^{TM}\rho)}{J_1^2(k_{t,0n}^{TM}b)} \cdot \left[\sin(k_{z,0n}z) + \cot(k_{z,0n}h)\cos(k_{z,0n}z)\right] \cdot \frac{k_{t,0n}^{TM}J_1(k_{\rho a}b)J_0(k_{t,0n}^{TM}b) - k_{\rho a}J_0(k_{\rho a}b)J_1(k_{t,0n}^{TM}b)}{k_{\rho a}^2 - (k_{t,0n}^{TM})^2} \right\}. \quad (14)$$

Similarly, by using (5) the EM field longitudinal component (i.e., z-component) can be calculated as

$$E_z(\mathbf{r}) = -\frac{2}{b\omega\varepsilon}\sum_n \left\{ \frac{k_{t,0n}^{TM}J_0(k_{t,0n}^{TM}\rho)}{J_1^2(k_{t,0n}^{TM}b)} \cdot \left[\sin(k_{z,0n}z) - \cot(k_{z,0n}h)\cos(k_{z,0n}z)\right] \cdot \frac{k_{t,0n}^{TM}J_1(k_{\rho a}b)J_0(k_{t,0n}^{TM}b) - k_{\rho a}J_0(k_{\rho a}b)J_1(k_{t,0n}^{TM}b)}{k_{\rho a}^2 - (k_{t,0n}^{TM})^2} \right\}, \quad (15)$$

$$H_z(\mathbf{r}) = 0. \quad (16)$$

The previous equations reveal interesting features of radiated field inside the cavity. Indeed, transverse (longitudinal) field component is shaped as a first (zeroth) order Bessel function in the radial direction, thus they vanish (exhibit a maximum) on the z-axis. Moreover, the effects of cavity top wall on radiated EM field (i.e., stationary waves due to multiple reflections) are taken into account by the z-dependent term. On the other hand, the contribution of cavity PEC rim (due to side walls) to radiated field is instead considered by the b-dependent fraction occurring in (13)-(14) and (15)-(16), that in general diverges when $k_{t0n} \to k_{\rho a}$.

To conclude, it is worth mentioning that the use of (13)-(14) and (15)-(16) allows developing a very fast numerical algorithm for field evaluation by forced currents known *a priori*, and it is suitable for further theoretical generalizations. In the next section, a comparison of our analytical model versus full-wave simulations is provided, to prove correctness of the proposed approach.

3 Numerical results

In this section, the theoretical model developed previously is validated by comparing it versus a full-wave simulator (COMSOL Multiphysics). For the sake of convenience, the magnetic current distribution in (1) is characterized by free-space wavenumber normalized to the operating wavelength ($k = 2\pi/\lambda$, $\lambda = 1$), and by an axicon angle equal to $\theta_a = 15°$. Since the proposed current distribution radiates a limited-diffractive Bessel beam in free-space [4, 5], in Fig. 2 the simulated electric field components (longitudinal, on the left; transverse, on the right) of the imposed magnetic current radiating in free-space are shown for comparison. As it is apparent, the z-component exhibits an amplitude modulation on the z-axis, due to the interference of edge-diffracted rays from the aperture rim. On the other hand, the transverse electric field component vanishes on the z-axis, for symmetry.

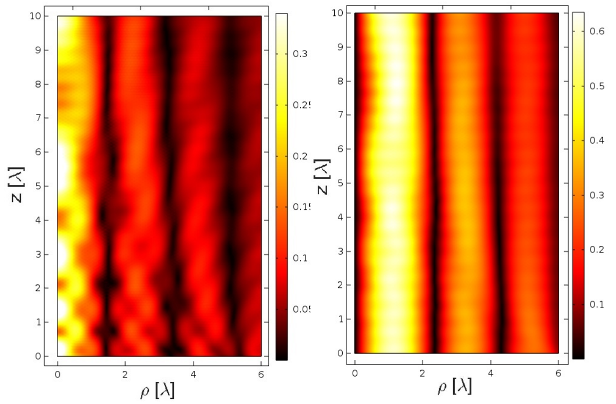

Figure 2. Electric field components (longitudinal, on the left; transverse, on the right) radiated in free-space by the magnetic current distribution in (1), with $k_{\rho a} = k\sin\theta_a$, being $\theta_a = 15°$. Such a field distribution resembles that of a longitudinally-polarized limited-diffractive Bessel beam, as proven in [4, 5].

If the current distribution radiates in a PEC cavity of radius $b = 6\lambda$ and heigth $h = 10\lambda$, cavity mode discretization unavoidably occurs, as shown theoretically in (13)-(14) and in (15)-(16). Hence, the Bessel beam radiated in free-space is perturbed by the presence of the cavity; in particular, stationary patterns due to multiple reflections by PEC boundaries arise, that should be carefully considered in the description of radiated EM field. In Fig. 3 (Fig. 4), the electric field radial (longitudinal) component inside the cavity is compared by considering both our analytical model (on the left) and full-wave simulations performed by COMSOL Multiphysics, showing a remarkable agreement between them. The stationary patterns are well-reproduced, together with field maxima and minima. It is worth noting that the proposed algorithm is really fast, since it is dominantly analytic, and requires only a summation over a few n orders (in the presented example, $N_{max} = 20$ is more than sufficient for achieving high accuracy). To conclude, the proposed theoretical model has been validated numerically and can be used as a starting point for fast design of suitable current distributions able to radiate properly inside metallic cylindrical cavities, that sometimes occur in practical applications related, for instance, to microwave heating.

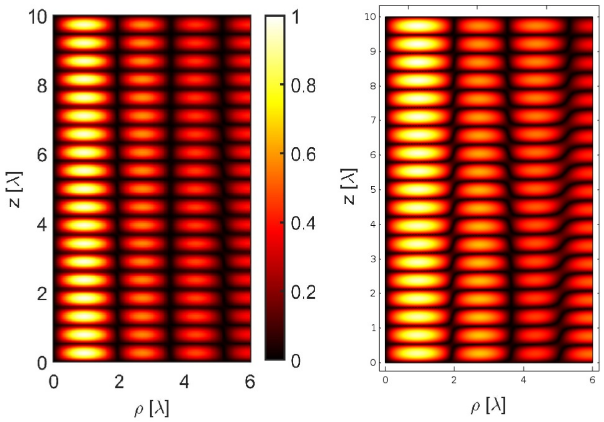

Figure 3. Normalized electric field radial component radiated by the magnetic current distribution in (1), with $k_{\rho a} = k \sin\theta_a$, being $\theta_a = 15°$, inside a metallic cylindrical cavity of radius $b = 6\lambda$ and height $h = 10\lambda$. On the left (right), analytical model (simulation by COMSOL Multiphysics).

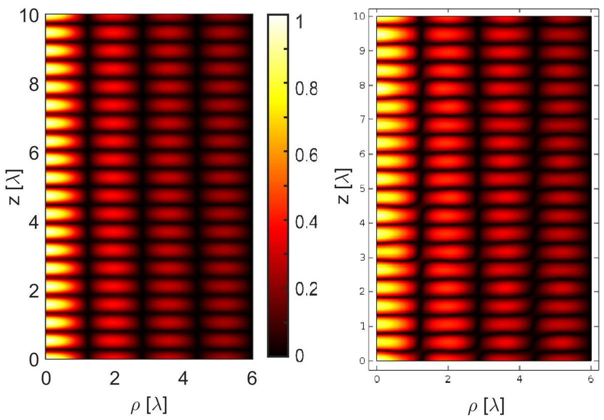

Figure 4. Normalized electric field longitudinal component radiated by the magnetic current distribution in (1), with $k_{\rho a} = k \sin\theta_a$, being $\theta_a = 15°$, inside a metallic cylindrical cavity of radius $b = 6\lambda$ and height $h = 10\lambda$. On the left (right), analytical model (simulation by COMSOL Multiphysics).

4 Conclusions

In this contribution, we discussed the radiation inside a cylindrical metallic cavity of a limited-diffractive Bessel beam in near-field, by developing an analytical model of the physical phenomena involved. Moreover, a numerical validation of the analytical model by a full-wave simulator (COMSOL Multiphysics) has been presented, showing a really fair agreement between the two approaches. The proposed analysis is motivated by possible applications of focused Bessel beams in near-field, such as microwave heating or plasma excitation in cavities.

5 Acknowledgements

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