3rd URSI AT-AP-RASC, Gran Canaria, 29 May – 3 June 2022

Multi-system All-sky Spherical Harmonic Transit Interferometry

Michael A. Kriele*(1)(2)(3), Randall B. Wayth(1)(2), Mark J. Bentum(3)(4), Budi Juswardy(1), and Cathryn M. Trott(1)(2)
(1) International Centre for Radio Astronomy Research (ICRAR), Curtin University, Bentley, WA, 6102
(2) ARC Centre of Excellence for All-Sky Astrophysics in 3 Dimensions (ASTRO3D)
(3) Eindhoven University of Technology, 5612 AZ Eindhoven, Netherlands
(4) ASTRON, the Netherlands Institute for Radio Astronomy, 7991 PD Dwingeloo, Netherlands

Abstract

We present multi-system spherical harmonic transit interferometry, an extension to the m-mode formalism, allowing for the inclusion of multiple interferometric systems to account for missing information on the sky; alleviating the need for prior fitting and/or regression. We simulate data with two model topologies to show we can indeed recover a genuine all-sky map. Furthermore, we explain how including multiple systems affects point-spread functions (PSFs) across declination, as well as how this method helps improving spatial coverage on the sky.

1 Introduction

Recently we've demonstrated the generation of diffuse all-sky maps at 159 MHz [1], using the engineering development array 2 (EDA2) [2], based on the Tikhonov regularised spherical harmonic transit-interferometry m-mode formalism by Shaw *et al.* [3] and Eastwood *et al.* [4]. While these maps are valid, they do require regularisation and the use of a prior to correct for bias due to missing information on the sky. Even with horizon-to-horizon widefield interferometers, depending on the latitude of the array, one does not necessarily see the full celestial sphere; which needs to be corrected for.

In this paper we show that the m-mode imaging methods can be extended to combine multiple systems on different hemispheres to achieve a genuine all-sky map. The linear nature of the formalism naturally lends itself for the inclusion of these systems as it does not discriminate on the basis of antenna location. The true power of this formalism therefore lies in the ability to expand both the measured visibilities on the sky, as well as the beam-transfer functions (beam × fringe) into spherical harmonic basis on a baseline-by-baseline manner.

2 Methods

2.1 Combined Definition

In this section we will briefly discuss the basics of the m-mode formalism, for a more extensive overview of the method and exact ordering of the coefficients please refer to [3, 4, 1]. In its purest form, the m-mode definition is defined in [3] as

$$\mathbf{v} = \mathbf{B}\mathbf{a}, \tag{1}$$

with $\mathbf{v}$ being the Fourier transformed measured visibilities m-mode vector, $\mathbf{B}$ the block-diagonal spherical harmonic beam-coefficient matrix, and $\mathbf{a}$ the spherical harmonic coefficient vector of the real-valued sky. Recovering these sky coefficients is achieved by inverting the beam-coefficient matrix. As discussed in [4], lack of information on the sky results in $\mathbf{B}$ not being full rank, requiring some form of prior constraints or regularisation.

The rows in $\mathbf{v}$ and the rows in a block of $\mathbf{B}$, however, are only a function of relative baseline separation. Since the coefficients act as weightings relative to functions that operate across the full sky, this feature can be leveraged, as it allows us to stack beam-coefficients and m-modes of multiple systems independent of physical location. This means we can reframe $\mathbf{v}$ and $\mathbf{B}$ in (1) for each absolute sorted $|m|$, such that

$$\mathbf{v}_{|m|} = \begin{bmatrix} \mathbf{v}_{|m|,\mathrm{sys1}} \\ \mathbf{v}_{|m|,\mathrm{sys2}} \end{bmatrix}, \qquad \mathbf{B}_{|m|} = \begin{bmatrix} \mathbf{B}_{+m,\mathrm{sys1}} \\ \mathbf{B}_{+m,\mathrm{sys2}} \\ --- \\ \mathbf{B}_{-m,\mathrm{sys1}} \\ \mathbf{B}_{-m,\mathrm{sys2}} \end{bmatrix}, \tag{2}$$

with the only requirement that the individual systems each measure a full sidereal day. The ordering in (2) not only lends itself to fill in missing regions on the sky, and therefore fixing the rank of $\mathbf{B}^{\dagger}\mathbf{B}$ (with † denoting the conjugate transpose), but also automatically adapts its sidelobe structure on the sky to account for regions of overlap between systems. In other words, point-spread functions (PSFs) on the sky are now a function of declination and system. Furthermore, multiple systems can also be included to fill in missing spatial scales on the sky that a single system would not be sensitive to.

2.2 Prior Information

Solving for missing regions on the sky and missing spatial scales are two separate matters. A system can assure the rank of $\mathbf{B}^\dagger\mathbf{B}$ by filling in missing parts of the sky, but still miss the largest scales due to lack of resolution. Conversely, using multiple systems to solve for missing angular scales does not necesarilly make $\mathbf{B}^\dagger\mathbf{B}$ full rank if the systems do not cover the full celestial sphere.

In the first case where large angular scales are missing, a prior sky can still be used without affecting the rank of $\mathbf{B}^\dagger\mathbf{B}$. The method of including a prior does not differ from [4, 1], however regularisation is no longer required

$$\hat{\mathbf{a}} = \left(\mathbf{B}^\dagger\mathbf{B}\right)^{-1}\mathbf{B}^\dagger\left(\mathbf{v} - \mathbf{B}\mathbf{a}_{\text{prior}}\right) + \mathbf{a}_{\text{prior}}, \tag{3}$$

with $\mathbf{a}_{\text{prior}}$ being the prior sky coefficients. In the cases where regions of the sky are still missing, some form of regularisation with a prior has to be done *e.g.* as shown in [4, 1].

2.3 Joint Spherical Harmonic Beam Coverage

As discussed in [1], the spherical harmonic beam coverage (SHBC) is a powerful tool to poll the sensitivity to spatial scales of a system in spherical harmonic space. In the case of multi-system spherical harmonic transit interferometry this still applies as the contribution of the absolute coefficients is additive in nature, such that the percentual spatial contribution in spherical harmonic space for a multi-system scenario is defined as

$$\mathcal{B}_{lm} = \sum_{n=1}^{N_{\text{sys1}}} |b^n_{lm,\text{sys1}}| + \sum_{n=1}^{N_{\text{sys2}}} |b^n_{lm,\text{sys2}}|, \tag{4}$$

with l, m denoting spherical harmonic order and rank respectively, N denoting the maximum number of baselines for that specific system, n denoting the baseline index, and b^n_{lm} describing the beam coefficient for its order, rank, and index.

3 Simulated Data

As a proof of concept, to verify the methods from Section 2, we generated two array topologies inspired by the EDA2 [2], and LOFAR's LBA central tile [5] at their latitudes of $\sim$ -26.7° and $\sim$ 52.9° respectively. To reduce computational cost we selected 32 elements of the outer ring of EDA2 and removed the outer elements of the LBA station. This, to ascertain both arrays have enough spatial coverage and are similar in sensitivity to angular scales. Both arrays are assumed to be homogeneous in response, sharing the same beam patterns as in [1]. However, this is only for simplicity. The methods described in Section 2 and [3, 4, 1] allow for the inclusion of unique antenna responses between elements and systems. The topologies are depicted in Figure 1.

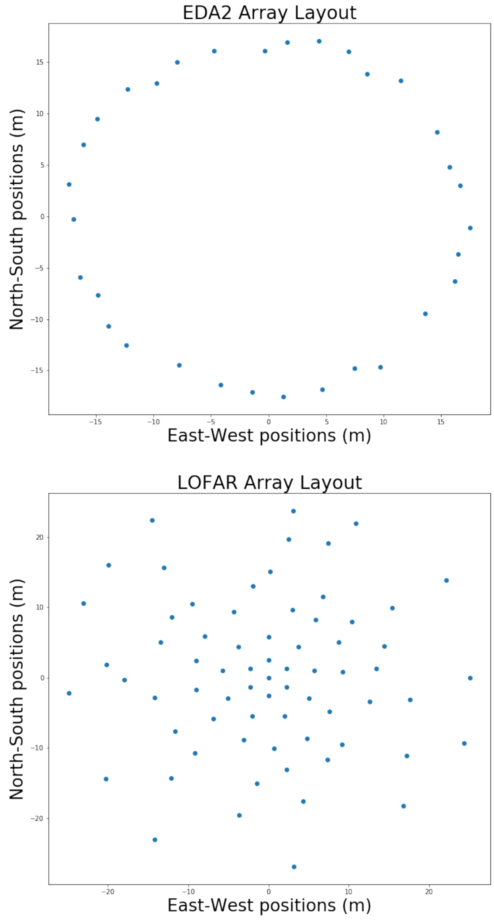

Figure 1. Generated homogeneous array topologies inspired by EDA2 and LOFAR LBA. Top: EDA2 32-element outer ring, Bottom: LOFAR LBA tile, capped at 70 elements. Elements have been selected to allow for similar spatial scales, but with sufficient spatial coverage.

Using the system topologies in Figure 1, along with the reprocessed destriped Haslam map [6] rescaled to 159 MHz as sky model, we generated two sets of visibilities using Miriad [7]. We let the simulated sky drift overhead, providing two sets of visibilities consisting out of 1440 data points each with 1 minute integration per sample; resulting in 24 hours worth of sky coverage for each set. These visibilities are then converted to m-modes. For all data a noiseless sky was assumed. Beam-transfer functions for both arrays were generated following [3] and converted to spherical harmonic beam coefficients. Both m-modes and beam-coefficients were stacked as per (2). Imaging was performed following [1], but without regression. HEALPix [8] is used as coordinate system due to its equal area projection and natural compatibility with spherical harmonics.

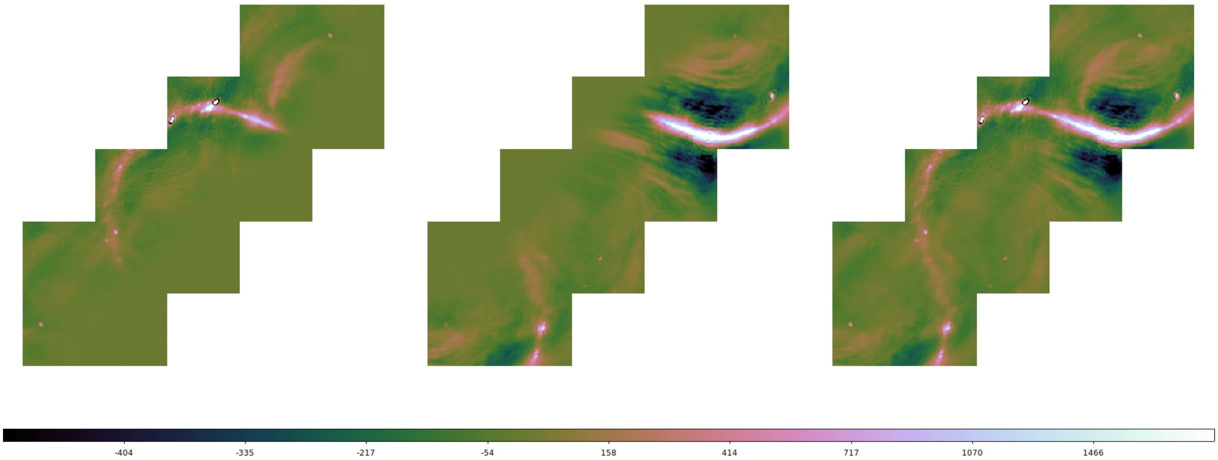

Figure 2. Dirty image of simulated sky. Left: Northern hemisphere dirty image using LOFAR LBA topology with Tikhonov regularisation, Middle: Southern hemisphere dirty image using EDA2 topology with Tikhonov regularisation, Right: All-sky dirty image using both the LOFAR LBA and EDA2 topologies following the multi-system spherical harmonic transit interferometry approach. Log-scaling.

4 Results

The resulting all-sky map is presented in Figure 2 (right image). The sky maps are dirty images where sources are convolved with PSFs of both systems; since we do not have uniform sky coverage on all angular scales. Negative values are due to the limited number of elements in our test arrays, resulting in lack of unique baseline orientations and lengths to sample the larger angular scales. Increasing the number of elements in the array will improve this, as shown in [1]. We've also sampled the sky for both array topologies separately, using Tikhonov regularisation to account for missing regions on the sky. It can be seen we can sucessfully recover the same information as the sky images on individial hemispheres with the muli-system approach and without the need for regularisation.

We also generated PSFs at 3 different declinations in order to see how they change based on declination. The PSFs are shown in Figure 3, and are generated at declinations of $\sim$ 55.9°, $\sim$ 14.5°, and $\sim$ -26.7° respectively. These declinations have been selected such that the PSF at $\sim$ 55.9° is only a function of the LBA topology, the PSF at $\sim$ 14.5° is affected by both systems (as they both see this declination), and the PSF at $\sim$ -26.7° is only a function of the EDA2 topology. It can clearly be seen that not only the shape of the central lobe of the PSFs change with declination, but that the sidelobes for the LBA PSF and the joint PSF are reduced. This is expected as more elements contribute to the generation of it. This means that overlapping regions will be better constrained and sidelobes on point sources will be less apparent. This shows that combining multiple systems has a positive impact on the overall response on the sky. Since these PSFs are only still declination dependent, deconvolution can still be done in either coefficient space [4], or image space [1].

Furthermore, we've generated the individual SHBCs as well as the joint SHBS to poll the overall improvement of the system. The resulting SHBCs are shown in Figure 4. It is apparent the joint SHBC does improve the overall spatial coverage, and especially provide a better contribution in modes where both systems contribute to, which is expected.

5 Conclusions

We have presented an extension to the *m*-mode formalism to allow for multiple systems to be used in a single imaging sweep. Not only does this formalism provide a means to generate a genuine all-sky map without the need for prior fitting or regularisation, but also shows better constraints are achieved on regions where systems overlap. This is significant, as this means the formalism is not only usefull to fill in missing information on the sky, but also allows for multiple systems to be used account for missing angular scales and aid in better resolving the sky; or improve overall sensitivity in general.

6 Acknowledgements

This scientific work makes use of the Murchison Radio-astronomy Observatory (MRO), operated by CSIRO. This research was supported by the Australian Research Council Centre of Excellence for All Sky Astrophysics in Three Dimensions (ASTRO3D), through project number CE170100013. We acknowledge the Wajarri Yamatji People as the traditional owners of the observatory site. We acknowledge the use of the Legacy Archive for Microwave Background Data Analysis (LAMDA), part of the High Energy Astrophysics Science Archive Center (HEASARC). HEASARC/LAMDA is a service of the Astrophysics Science Division at the NASA Goddard Space Flight Center.

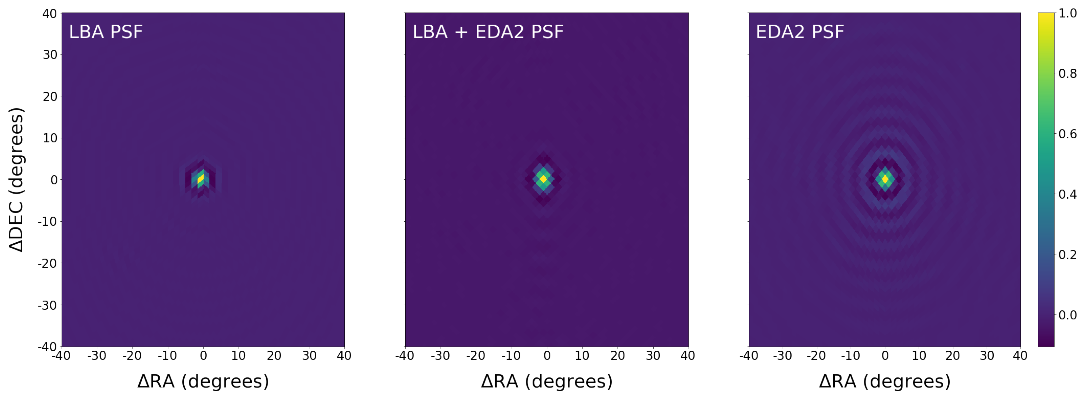

Figure 3. Point-spread functions at different declinations. Left: PSF at $\sim 55.9^\circ$ which is only a function on the LBA topology, Middle: PSF at $\sim 14.5^\circ$ which is jointly generated by both the LBA and EDA2 topology, Right: PSF at $\sim$ -26.7°, which is only a function of the EDA2 topology.

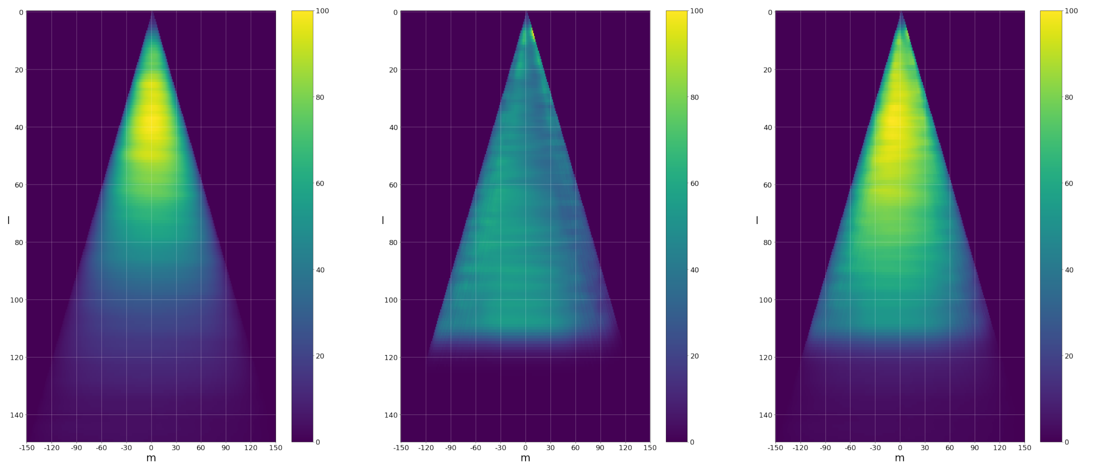

Figure 4. Percentage spherical harmonic beam coverage. Left: Lofar LBA topology SHBC, Middle: EDA2 topology SHBC, Right: Joint system SHBC.

References

[1] M. A. Kriele, R. B. Wayth, M. J. Bentum, B. Juswardy, and C. M. Trott, "Imaging the Southern Sky at 159MHz using Spherical Harmonics with the Engineering Development Array 2," *Publications of the Astronomical Society of Australia (PASA), accepted.*, 2022.

[2] R. Wayth *et al.*, "Engineering Development Array 2: design, performance, and lessons from an SKA-Low prototype station," *Journal of Astronomical Telescopes, Instruments, and Systems*, vol. 8, no. 1, pp. 1 – 14, 2021.

[3] J. R. Shaw, K. Sigurdson, U.-L. Pen, A. Stebbins, and M. Sitwell, "ALL-SKY INTERFEROMETRY WITH SPHERICAL HARMONIC TRANSIT TELESCOPES," *The Astrophysical Journal*, vol. 781, p. 57, jan 2014.

[4] M. W. Eastwood *et al.*, "The radio sky at meter wavelengths: m-mode analysis imaging with the OVRO-LWA," *The Astronomical Journal*, vol. 156, p. 32, jun 2018.

[5] M. P. van Haarlem *et al.*, "Lofar: The low-frequency array," *Astronomy & Astrophysics*, vol. 556, p. A2, Jul 2013.

[6] M. Remazeilles *et al.*, "An improved source-subtracted and destriped 408-mhz all-sky map," *Monthly Notices of the Royal Astronomical Society*, vol. 451, pp. 4311–4327, Aug. 2015.

[7] R. J. Sault, P. J. Teuben, and M. C. H. Wright, "A Retrospective View of MIRIAD," in *Astronomical Data Analysis Software and Systems IV* (R. A. Shaw, H. E. Payne, and J. J. E. Hayes, eds.), vol. 77 of *Astronomical Society of the Pacific Conference Series*, p. 433, Jan. 1995.

[8] K. M. Górski *et al.*, "HEALPix: A Framework for High-Resolution Discretization and Fast Analysis of Data Distributed on the Sphere," *The Astrophysical Journal*, vol. 622, pp. 759–771, Apr. 2005.