What is the temporal analogue of the spatial Brewster angle?

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Abstract

Temporal and spatiotemporally modulated media are becoming new paradigms for a full manipulation of electromagnetic wave propagation in four dimensions (*x,y,z,t*). Here we will give an overview of our recent efforts in exploiting temporal metamaterials where the permittivity of the medium where the wave travels is changed from isotropic to an anisotropic tensor. An in-depth discussion will be presented during the conference demonstrating how such temporal boundaries can be engineered to achieve the temporal version of the spatial Brewster angle.

1. Introduction

The Brewster angle can be considered as one of the fundamental discoveries for the control of polarization of light. Brewster demonstrated how *p*-polarized oblique incident light can be transmitted from a medium 1 to a medium 2 without exciting a reflected wave when $\theta_1 + \theta_2 = 90°$ (θ_1 as the incident/reflected angle and θ_2 as the angle of the refracted wave traveling within medium 2)[1], [2]. The implications of this discovery has been profound as it provided the scientific community with a powerful mechanism to generate polarized from unpolarized light[3].

The arbitrary manipulation of electromagnetic (EM) waves (in terms of phase, amplitude, polarization) has been indeed a hot topic for many years and the field of metamaterials has impacted research in this area significantly [4]. Metamaterials and metasurfaces have been demonstrated to allow a full control of light-matter interactions by enabling the design of artificial media with EM parameters not easily available in natural materials such as negative[5], [6] or near-zero values[7], [8]. They have been implemented in multiple scenarios such as lenses and antennas[9], [10], sensors[11], invisibility cloaking devices[12], [13], among others.

While mostly studied in the time-harmonic scenario (frequency domain) metamaterials and metasurfaces with temporally modulated EM parameters of permittivity (ε) and permeability (μ) have started to take off in recent years[14]. Temporal and spatiotemporal material platform were first explored by Morgenthaler [15] where wave propagation within an unbounded time-dependent medium was theoretically studied using $\varepsilon_r(t)$ rapidly changing (with rise/fall times smaller than the period of the incident wave) from ε_{r1} to ε_{r2} (both values larger than 1). This study showed that such isotropic-to-isotropic temporal change of $\varepsilon_r(t)$: i) generates a forward (FW) and a backward (BW) wave (temporal equivalent of transmission and reflection, respectively) traveling with the same angle as the angle of the incident wave θ_1, ii) the wavenumber $\boldsymbol{k}$ does not change and iii) the frequency is changed to $f_2 = f_1(\varepsilon_{r1}/\varepsilon_{r2})^{1/2}$. Recently, such temporal and spatiotemporal modulated media has been considered in exotic yet interesting applications such as meta-atoms[16], [17], antireflection temporal coatings[18], temporal modulation gain and loss[19], [20], effective medium in time domain[21], [22], temporal aiming[23], among others.

In this abstract we discuss our recent theoretical work[24] on how temporal metamaterials can be exploited to achieve

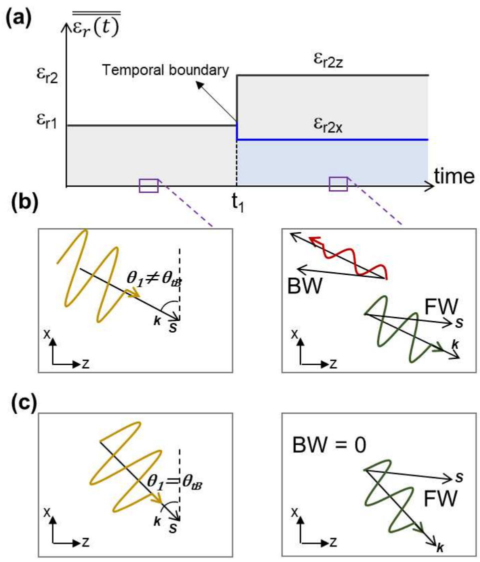

Figure 1. Schematic representation of (a) a temporal function of $\varepsilon_r(t)$ to induce an isotropic-to-anisotropic temporal boundary, (b) incident, FW and BW waves when $\theta_1 \neq \theta_{tB}$ and (b) when $\theta_1 = \theta_{tB}$.

the temporal equivalent of the Brewster angle. Here, we exploit isotropic-to-anisotropic changes of $\varepsilon_r(t)$ to show how such temporal boundaries can generate a FW and a BW wave with a direction of the energy propagation (*S*) depending on the incident angle θ_I of the EM wave traveling within an unbounded medium and the values of ε_r before (ε_{r1}) and after ($\overline{\overline{\varepsilon_{r2}}}$). Moreover, it will be discussed how with the proper choice of the incidence angle, a FW wave is excited without generating a BW wave. We call this angle $\theta_I = \theta_{tB}$ as the temporal equivalent/analogue of the Brewster angle demonstrating how it can be mathematically calculated using a closed equation[24] . In the next section, we briefly review one example of our results. More details can be found in [24] and will be presented in the conference.

2. Results and discussion

The schematic representation of a temporal boundary of the proposed technique to mimic the Brewster angle using temporal metamaterials is shown in Fig. 1a,b. Here, a *p*-polarized oblique incident wave is considered to travel within an unbounded medium with an angle θ_I for times $t < t_1$ (the permittivity of the medium is isotropic, ε_{r1}). At $t = t_1$, $\varepsilon_r(t)$ is rapidly changed to an anisotropic tensor $\overline{\overline{\varepsilon_{r2}}} = \{\varepsilon_{r2z}, \varepsilon_{r2x}\}$ (again with a rise/fall times smaller than the period of the incident wave). We consider nonmagnetic materials with $\mu_{r1} = \mu_{r2} = \mu_r$, however our approach can also be expanded to temporal variations of μ_r as shown in [23], [24]). It has recently been shown how isotropic-to-anisotropic temporal boundary generates a change of frequency (similar to isotropic-to-isotropic temporal boundaries) which depends on the values of $\varepsilon_r(t)$ and also the incident angle θ_I[25]. Interestingly, we have recently shown how for the temporal boundary shown in Fig. 1a, the wavenumber ***k*** does not change while the direction of the energy propagation (Poynting vector ***S***) is modified, a feature that we have exploited for beam steering in real time or *temporal aiming*[23].

However, as shown in Fig. 1b, such temporal boundary also excites a FW and a BW wave. Hence one can ask: how can we eliminate this BW wave? We will show during the conference an in-depth analysis of the performance of isotropic-to-anisotropic temporal boundaries showing how

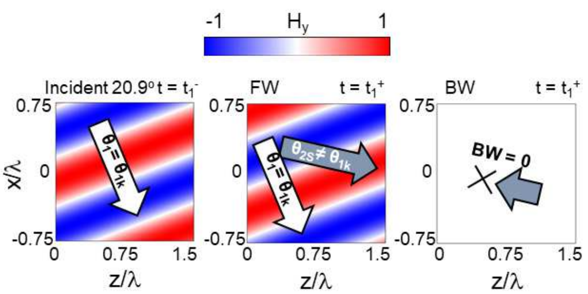

Figure 2. Example of our temporal equivalent of the Brewster angle technique showing the magnetic field distributions of the incident (left), FW (center) and BW=0 (right) waves.

indeed the direction of propagation of the EM waves traveling within such media can be steered in real time. Moreover, we will present how the BW can be completely eliminated by carefully engineering the incident angle of the incident EM wave such that it coincides with what we call the *temporal equivalent of the Brewster angle* ($\theta_I = \theta_{tB}$)[24] (see Fig. 1c for a schematic representation).

For completeness, an example of our *temporal equivalent of the Brewster angle* technique is shown in Fig. 2 where the analytical results of the magnetic field distribution for the incident (left panel, $t = t_I^-$), FW (center, $t = t_I^+$) and BW (right, $t = t_I^+$) waves are shown. Here $\theta_I = \theta_{tB} = 20.9°$ with $\varepsilon_{r1} = 5$ and $\overline{\overline{\varepsilon_{r2}}} = \{\varepsilon_{r2z} = 12, \varepsilon_{r2x} = 1\}$. As shown in Fig. 2 no BW wave is generated, as described above, and the angle of the energy propagation for the FW wave is modified from $\theta_I = \theta_{tB} = 20.9°$ to $\theta_2 = \theta_{2S} = 77.7°$.

3. Conclusions

We have discussed how temporal metamaterials that introduce isotropic-to-anisotropic temporal boundaries can be exploited to achieve real time beam steering of EM waves (*temporal aiming*). It has also been shown how the BW wave (temporal equivalent of reflection) can be eliminated by simply choosing the incident angle of the incident EM wave to coincide with that of our proposed *temporal equivalent of the Brewster angle*.

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