



Towards a Low-Frequency-Stabilized Electric Field Integral Equation on Simply Connected-Geometries for Arbitrary Excitations

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The formulation of electromagnetic scattering and radiation problems as integral equations in combination with the boundary element method (BEM) is an established methodology. For open and closed perfectly electrically conducting (PEC) structures, the electric field integral equation (EFIE) discretized by a Galerkin scheme employing Rao-Wilton-Glisson (RWG) basis and testing functions is known to yield accurate solutions. However, when the frequency becomes low, this accuracy deteriorates, foremost due to an increasing ill-conditioning of the discretized equation [1, 2]. This low-frequency breakdown can be remedied by leveraging quasi-Helmholtz decompositions of the induced surface current density using, for example, a loop-star/tree basis [3] or the more advantageous quasi-Helmholtz projectors proposed in [4]. In order to ensure accurate solutions, one has, in addition to curing the ill-conditioning of the discretized equation, also to compute the right-hand side (RHS) such that no numerical round-off errors occur [5–7].

For the case that not only plane waves but arbitrary excitations (such as dipoles, ring currents, or gap excitations) are allowed, yet another problem has to be overcome: the standard frequency normalization applied to the quasi-Helmholtz decomposed system does, in general, not ensure that the solenoidal and non-solenoidal current components are recovered with the same relative accuracy. This is, however, needed to obtain the correct scattered or radiated fields [5].

In this contribution, we propose a normalization scheme for simply-connected geometries which leads to a well-conditioned linear system of equations (LSE) and which recovers all current components with the same relative accuracy. The scheme is applicable to loop-star/tree decompositions as well as to quasi-Helmholtz projectors, enabling their usage also for excitations other than plane waves. Considering several excitations and geometries, we investigate the impact on the solution accuracy and the convergence of iterative solvers.

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