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Beam Frame Expansions in Plane Stratified Media

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In recent publications [1-3] we have introduced the concept of Beam-Frames (BF) (in the terminology of signal processing, a "frame" is an over-complete set of basis functions that provides a redundant, stable way of representing a signal). The BFs are an extension of the well-known windowed Fourier transform (WFT) frame formulations s (or the Gabor series formulations) that have been used extensively in the past to express radiation from apertures as a sum of beam propagators (see [4] and references theein). The BFs are phase-space sets of beam-waves that constitute "frames" not only over the aperture plane, where they reduce to the conventional WFT frames, but everywhere in the propagation domain. As such, the BFs provide the mathematic framework for a beam-based local analysis of wave interaction with the propagation environment, for example in propagation through randomly fluctuating medium [1,2] or local inverse scattering [5,6].

The BFs have also been scaled with frequency in a particular way termed isodiffracting (ID) such that they have the following key features: (*i*) The skeleton of beam-axes is frequency independent, and hence they need to be tracked only once and then used for all frequencies. (*ii*) The BF utilizes ID-Gaussian beam propagators (ID-GB), whose propagation parameters in inhomogeneous medium are frequency independent and therefore need to be calculated only once and then used for all frequencies. (*iii*) The beam parameter can be optimized to render the resulting frame "snuggest" for all frequencies, yielding local and stable expansions for all frequencies (unlike the conventional Gabor series which is notoriously nonlocal and unstable). (*iv*) These properties imply that the beam frame can be formulated either in the UWB frequency domain or directly in the time domain where the propagators are iso-diffracting pulsed beams (ID-PB) [3].

This paper extends the BF formulation for propagation in a plane stratified medium. We present the local expressions for the beam-waves, define the conditions under which they constitute frames, and derive the expressions for the dual-frame that is used to calculate the expansion coefficients. The applications considered are propagation through a randomly fluctuating medium in a plane stratified background, and local inverse scattering of targets that are submerged in a plane stratified background.

1. M. Leibovich and E. Heyman, "Beam summation scattering theory for waves in a fluctuating medium. Part I: The propagating beam frame and the beam-domain scattering matrix," *IEEE Trans. Antennas Propagat.*, **65**, pp. 5431-5442, Oct. 2017. (The 2018 R.W.P. King Best Paper Award).

2. M. Leibovich and E. Heyman, "Beam summation scattering theory for waves in a fluctuating medium. part II: Stochastic field representation," *IEEE Trans. Antennas Propagat.*,. **65**, pp. 5443-5452, Oct. 2017.

3. R. Tuvi, E Heyman and T. Melamed, "Beam Frame Representation for ultra wideband radiation from volume source distributions: Frequency-Domain and Time-Domain Formulations," *IEEE Trans. Antennas Propagat.*, **67**, pp. 1010-1024, Feb. 2019.

4. A. Shlivinski, E. Heyman, A. Boag and C. Letrou, "A phase-space beam summation formulation for wideband radiation," *IEEE Trans. Antennas Propagat.*, **52**, pp. 2042-2056, Aug. 2004.

5. R. Tuvi, E Heyman and T. Melamed, "UWB beam-based local diffraction tomography. Part I: Phase-space processing and physical interpretation," *IEEE Trans. Antennas Propagat.*,. **68**, pp. 7144–7157, Oct. 2020.

6. R. Tuvi, E Heyman and T. Melamed, "UWB beam-based local diffraction tomography. Part II: the inverse problem," *IEEE Trans. Antennas Propagat.*, **68**, pp. 7158–7169, Oct. 2020.