



## Energy Preserving Time Domain Boundary Element Methods

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### Extended Abstract

For the discretisation of the Maxwell wave equation, there are mainly two options: (i) finite element methods, which start from approximations of the fields as linear combinations of basis functions with a priori unknown coefficients, and (ii) finite integration methods, which start from degrees of freedom of the fields assigned to various geometric entities in the mesh.

The advantages of finite element methods are that (i) it is obvious how to deal with non-uniform and anisotropic permittivities and permeabilities, and (ii) it is possible to unambiguously compute the trace of the finite elements involved and so it is more straightforward to couple finite element methods to boundary element methods and volume integral equation methods governing the wave dynamics in neighbouring regions.

Among the most popular methods are the E-B method [2] which in combination with a conditionally or unconditionally stable time stepping scheme can be used to reconstruct the field-flux pair. Methods differ in the choice of fundamental unknowns and the discretisation of the curl and Hodge star operators.

In this contribution, we introduce an E-H method with a time stepping scheme that is orthogonal (i.e. perfectly norm preserving) with respect to the physical energy norm. The method is based on the use of dual Whitney 1-forms. These 1-forms are complementary to the 2-forms introduced in [3] Because the method is phrased intrinsically in the field-field pair E-H, it is a perfect candidate for coupling with boundary element methods, which are almost always formulated in terms of the traces of the electric field E and the magnetic field H. It can be shown that this finite element method is equivalent to a finite integration method similar to the discrete exterior calculus method as presented in [1].

### References

- [1] Shu C. Chen and Weng Cho Chew. Numerical electromagnetic frequency domain analysis with discrete exterior calculus. *Journal of Computational Physics*, 350:668–689, December 2017.
- [2] J. Kim and F. L. Teixeira. Parallel and Explicit Finite-Element Time-Domain Method for Maxwell's Equations. *IEEE Transactions on Antennas and Propagation*, 59(6):2350–2356, June 2011.
- [3] Rodney Voskamp, Aurele Adam, and Kristof Cools. Dual Spaces for 3D Finite Element Methods. In *URSI-GASS 2021*, September 2021.