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Improving the Numerical Stability of the Iterated Crank-Nicolson FDTD Method

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The FDTD method based on the Crank-Nicolson (CN) scheme [1] is known to be one of the most accurate implicit FDTD methods, since no operator splitting technique is applied [2]-[4]. Note, however, that the CN-FDTD method requires solving a large sparse matrix, resulting in a long computational time. To circumvent this problem, we have proposed an explicit FDTD method [5] based on the iterated CN (ICN) scheme [6]. Although the Courant-Friedrichs-Lewy (CFL) stability condition has been shown to be the same as that of the traditional explicit FDTD method, the numerical results tend to blow up with the time step close to the CFL condition. To improve the numerical stability of the ICN scheme, the θ-scheme has been proposed in [7]. In this work, we introduce the θ-scheme for improving the numerical stability of the ICN-FDTD method.

We apply the θ-scheme to the ICN-FDTD method with the two-iteration procedure as

$$\boldsymbol{\phi}_{1\mathrm{st}}^{n+1} = \boldsymbol{\phi}^n + \Delta t \boldsymbol{A} \boldsymbol{\phi}^n \tag{1}$$

where $\boldsymbol{\phi}$ is the field component and $\boldsymbol{A}$ is the matrix including spatial derivatives [5]. Eq. (1) corresponds to a first-order accurate differencing scheme. Using the result of (1), the intermediate field is estimated as

$$\boldsymbol{\phi}_{1\mathrm{st}}^{n+\frac{1}{2}} = \theta \boldsymbol{\phi}_{1\mathrm{st}}^{n+1} + (1-\theta)\boldsymbol{\phi}^n \tag{2}$$

Using the field of (2), the second iteration is carried out as

$$\boldsymbol{\phi}_{2\mathrm{nd}}^{n+1} = \boldsymbol{\phi}^n + \Delta t \boldsymbol{A} \boldsymbol{\phi}_{1\mathrm{st}}^{n+\frac{1}{2}} \tag{3}$$

The improved intermediate field is

$$\boldsymbol{\phi}_{2\mathrm{nd}}^{n+\frac{1}{2}} = \theta \boldsymbol{\phi}_{2\mathrm{nd}}^{n+1} + (1-\theta)\boldsymbol{\phi}^n \tag{4}$$

Finally, using the result of (4), the solution is obtained as

$$\boldsymbol{\phi}^{n+1} = \boldsymbol{\phi}^n + \Delta t \boldsymbol{A} \boldsymbol{\phi}_{2\mathrm{nd}}^{n+\frac{1}{2}} \tag{5}$$

For $\theta = 0.5$, the formulation corresponds to the original ICN-FDTD method [5]. At the presentation, we will discuss the improved numerical stability for the analysis of optical waveguide devices.

References

[1] Y. Yang, R. S. Chen, and E. K. N. Yung, "The unconditionally stable Crank Nicolson FDTD method for three-dimensional Maxwell's equations," *Microw. Opt. Tech. Lett.*, vol. 48, no. 8, pp. 1619-1622, 2006.

[2] T. Namiki, "A new FDTD algorithm based on alternating-direction implicit method," *IEEE Trans. Microw. Theory Tech.*, vol. 47, no. 10, pp. 2003-2007, 1999.

[3] F. H. Zheng, Z. Z. Chen, and J. Z. Zhang, "A finite-difference time-domain method without the Courant stability conditions," *IEEE Microw. Guided Wave Lett.*, vol. 9, no. 11, pp. 441-443, 1999.

[4] J. Shibayama, M. Muraki, J. Yamauchi, and H. Nakano, "Efficient implicit FDTD algorithm based on locally one-dimensional scheme," *Electron. Lett.*, vol. 41, no. 19, pp. 1046-1047, 2005.

[5] J. Shibayama, T. Nishio, J. Yamauchi, and H. Nakano, "Explicit FDTD method based on iterated Crank-Nicolson scheme," *Electron. Lett.*, vol. 58, no. 1, pp. 16-18, 2022.

[6] S. A. Teukolsky, "Stability of the iterated Crank-Nicholson method in numerical relativity," *Phys. Rev. D*, vol. 61, pp. 087501-1-2, 2000.

[7] G. Leiler and L. Rezzolla, "Iterated Crank-Nicolson method for hyperbolic and parabolic equations in numerical relativity," *Phys. Rev. D*, vol. 73, pp. 044001-1-7, 2006.