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Analysis of a Circular Loop Antenna Located on a Hyperbolic Metamaterial Cylinder

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Abstract

A study is made of the current distribution and input impedance of a circular loop antenna located on the surface of a cylinder filled with a uniaxial hyperbolic metamaterial and surrounded by an isotropic magnetodielectric. Integral equations for azimuthal harmonics of the antenna current excited by a given time-harmonic voltage are derived and solved. The behavior of the current distribution and input impedance of the antenna as functions of the metamaterial parameters is discussed.

1 Introduction

Over the past two decades, a great deal of attention has been paid to excitation and propagation of electromagnetic waves in the presence of metamaterials (see, e.g., [1] and references therein). The interest in the subject is stimulated by the progress in creating new artificial media whose properties differ significantly from those of materials that commonly serve as substrates of antenna devices. As is known, most promising metamaterials used in antenna systems usually have anisotropic properties. In this respect, of special interest are the so-called hyperbolic metamaterials, in which the refractive-index surface of at least one of the normal waves has the form of a one- or two-sheeted hyperboloid of revolution. It is evident that characteristics of the antennas located on the surface of such media should exhibit some unusual behaviors. This circumstance makes the development of the antenna theory very topical under such conditions.

It is the purpose of this paper to derive and solve integral equations for the current of a circular loop antenna located coaxially on the surface of a cylinder filled with a uniaxial anisotropic metamaterial. The emphasis is placed on the case where both normal waves of such a medium have hyperbolic dispersion. Based on the solution for the current distribution of the antenna, we also analyze its input impedance in this case.

2 Formulation of the Problem

Consider an antenna in the form of a perfectly conducting, infinitesimally thin, narrow strip of half-width d, which is coiled into a circular loop of radius a ($d \ll a$). The antenna is located coaxially on the surface of an infinitely long metamaterial cylinder aligned with the z axis of a cylindrical coordinate system (ρ, ϕ, z) and surrounded by a homogeneous magnetodielectric medium (see Fig. 1). The meta-

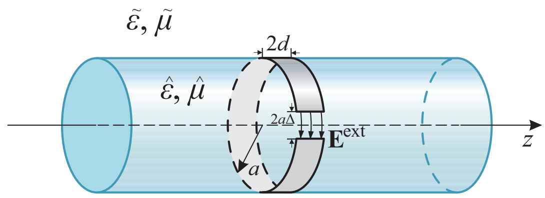

Figure 1. Geometry of the problem.

material inside the cylinder is assumed lossless and describable by a diagonal permittivity tensor $\hat{\varepsilon}$ with the elements $\varepsilon_{\rho\rho} = \varepsilon_{\phi\phi} = \varepsilon_0\varepsilon_\perp$ and $\varepsilon_{zz} = \varepsilon_0\varepsilon_\parallel$ and a diagonal permeability tensor $\hat{\mu}$ with the elements $\mu_{\rho\rho} = \mu_{\phi\phi} = \mu_0\mu_\perp$ and $\mu_{zz} = \mu_0\mu_\parallel$. Here, ε_0 and μ_0 are the permittivity and permeability of free space, respectively, and the elements of the tensors $\hat{\varepsilon}$ and $\hat{\mu}$ depend on a particular type of metamaterial [2]. The anisotropy axis of the metamaterial is parallel to the z axis. Recall that there are two normal waves, known as E and H waves, that exist in such a medium. In this paper, we focus on the special case where the following conditions are satisfied simultaneously for the elements of the tensors:

$$\operatorname{sgn}\varepsilon_\perp \neq \operatorname{sgn}\varepsilon_\parallel, \quad \operatorname{sgn}\mu_\perp \neq \operatorname{sgn}\mu_\parallel. \tag{1}$$

Note that the metamaterial is called hyperbolic if at least one of the above conditions is fulfilled. If both inequalities are satisfied simultaneously, the medium may be called doubly hyperbolic, which means that the refractive-index surfaces of both normal waves are hyperboloids of revolution and have the unbounded branches, which correspond to electrostatic and magnetostatic waves propagating in such a medium. The lossless medium outside the cylinder is isotropic and has the dielectric permittivity $\tilde{\varepsilon} = \varepsilon_0\varepsilon_a$ and the magnetic permeability $\tilde{\mu} = \mu_0\mu_a$, where ε_a and μ_a are positive constants.

The current in the antenna is excited by a time-harmonic voltage ($\sim \exp(i\omega t)$) that is applied across a narrow interval (gap) $|\phi - \phi_0| \leq \Delta \ll \pi$ and creates there an electric field with a single azimuthal component E_ϕ^{ext}, which can be written for $\rho = a$ and $|z| < d$ as

$$E_\phi^{\text{ext}}(a,\phi,z) = \frac{V_0}{2a\Delta}[U(\phi-\phi_0+\Delta) - U(\phi-\phi_0-\Delta)] \times [U(z+d) - U(z-d)]. \quad (2)$$

Here, V_0 is a constant amplitude of the given voltage supplied to the gap, Δ is the angular half-width of the gap centered at $\phi = \phi_0$, and U is a Heaviside function. The distribution of E_ϕ^{ext} for $|z| < d$ can be expanded in terms of azimuthal harmonics as

$$E_\phi^{\text{ext}} = \sum_{m=-\infty}^{\infty} A_m \exp(-im\phi), \quad (3)$$

where

$$A_m = \frac{V_0}{2\pi a}\frac{\sin(m\Delta)}{m\Delta}\exp(im\phi_0). \quad (4)$$

The surface density $I(\phi,z)$ of the electric current in the antenna, which is excited by the field (2), admits the analogous representation

$$I(\phi,z) = \sum_{m=-\infty}^{\infty} \mathscr{I}_m(z)\exp(-im\phi), \quad (5)$$

where the azimuthal harmonics $\mathscr{I}_m(z)$ are the desired functions of the coordinate across the strip. The antenna-excited field corresponding to these harmonics can be expanded in terms of vector eigenfunctions of the above-described open waveguide system with an eigenvalue spectrum containing the discrete and continuous parts. To obtain such a representation, the approach used in [3] can be applied. Then, allowing for the boundary conditions

$$E_\phi + E_\phi^{\text{ext}} = 0, \quad E_z = 0 \quad (6)$$

on the antenna surface (i.e., at $\rho = a$ and $|z| < d$), where E_ϕ and E_z are the tangential components of the antenna-excited electric field, integral equations for the azimuthal harmonics of the surface current density will be derived.

3 Integral Equations for the Antenna Current

We can represent the tangential components of the electric field excited by the current of the antenna on its surface as

$$\begin{bmatrix} E_\phi(a,\phi,z) \\ E_z(a,\phi,z) \end{bmatrix} = \sum_{m=-\infty}^{\infty} \exp(-im\phi) \times \int_{-d}^{d} \begin{bmatrix} K_m(z-z') \\ k_m(z-z') \end{bmatrix} \mathscr{I}_m(z')dz'. \quad (7)$$

Here, the quantities $K_m(\zeta)$ and $k_m(\zeta)$, where $\zeta = z - z'$, contain the discrete- and continuous-spectrum parts, hereafter denoted by the superscripts (ds) and (cs), respectively:

$$K_m(\zeta) = K_m^{(\text{ds})}(\zeta) + K_m^{(\text{cs})}(\zeta),$$
$$k_m(\zeta) = k_m^{(\text{ds})}(\zeta) + k_m^{(\text{cs})}(\zeta). \quad (8)$$

The quantities $K_m^{(\text{ds})}(\zeta)$ and $k_m^{(\text{ds})}(\zeta)$ correspond to the discrete-spectrum waves, i.e., eigenmodes guided by the anysotropic cylinder, and turn out to be

$$\begin{bmatrix} K_m^{(\text{ds})}(\zeta) \\ k_m^{(\text{ds})}(\zeta) \end{bmatrix} = \sum_n \frac{2\pi a}{N_{m,n}} \begin{bmatrix} E_{\phi;m,n}^2(a) \\ E_{\phi;m,n}(a)E_{z;m,n}(a)\operatorname{sgn}\zeta \end{bmatrix} \times \exp(-ik_0 p_{m,n}|\zeta|), \quad (9)$$

where m and n are the azimuthal and radial indices of eigenmodes ($n = 1,2,\ldots$), $N_{m,n}$ and $p_{m,n}$ are the norm and the dimensionless propagation constant of an eigenmode with the corresponding indices, respectively, the functions $E_{\phi;m,n}(\rho)$ and $E_{z;m,n}(\rho)$ describe the distributions of the azimuthal and longitudinal electric-field components of this eigenmode over the coordinate ρ, and k_0 is the wave number in free space. The quantities $K_m^{(\text{cs})}(\zeta)$ and $k_m^{(\text{cs})}(\zeta)$, which refer to the continuous-spectrum waves, are written as

$$\begin{bmatrix} K_m^{(\text{cs})}(\zeta) \\ k_m^{(\text{cs})}(\zeta) \end{bmatrix} = \frac{iZ_0k_0}{2\pi}\sum_{l=1}^{2}\int_0^\infty \begin{bmatrix} mp(q)Q_\perp^{-2}C_{l,m}(q) + D_{l,m}(q) \\ -(k_0a)^{-1}C_{l,m}(q)\operatorname{sgn}\zeta \end{bmatrix} \times \exp[-ik_0p(q)|\zeta|]\,dq. \quad (10)$$

Here, $p(q) = (\varepsilon_a\mu_a - q^2)^{1/2}$ is the dimensionless propagation constant of the normal mode in the background medium for the normalized (to k_0) transverse wave number q ($\operatorname{Im} p(q) < 0$), $Z_0 = (\mu_0/\varepsilon_0)^{1/2}$ is the free-space impedance, and the quantities $C_{l,m}(q)$ and $D_{l,m}(q)$ have the form

$$C_{l,m}(q) = \frac{m}{\varepsilon_a}\frac{(-1)^l}{Q\Delta_m^{(l)}}\left(\frac{\mu_\parallel}{\mu_a}J_H - \frac{Q^2}{Q_\perp^2}\mathscr{H}_m^{(l)}\right),$$
$$D_{l,m}(q) = \frac{(-1)^l Q}{p(q)\Delta_m^{(l)}}\mu_\parallel J_H\left[\left(\frac{\varepsilon_\parallel}{\varepsilon_a}J_E - \mathscr{H}_m^{(l)}\right)\mathscr{H}_m^{(l)} + \frac{p^2(q)}{\varepsilon_a\mu_a}\frac{m^2}{Q^4}\left(1 - \frac{Q^2}{Q_\perp^2}\right)\right]. \quad (11)$$

In the above, $Q = k_0aq$ and $Q_\perp = k_0a\left[\varepsilon_\perp\mu_\perp - p^2(q)\right]^{1/2}$. Other quantities in (10) and (11) are defined as follows:

$$J_{E,H} = \frac{J_{m+1}(Q_{E,H})}{Q_{E,H}J_m(Q_{E,H})} - \frac{m}{Q_{E,H}^2}, \quad \mathscr{H}_m^{(l)} = \frac{H_{m+1}^{(l)}(Q)}{QH_m^{(l)}(Q)} - \frac{m}{Q^2},$$
$$\Delta_m^{(l)} = \left(\frac{\varepsilon_\parallel}{\varepsilon_a}J_E - \mathscr{H}_m^{(l)}\right)\left(\frac{\mu_\parallel}{\mu_a}J_H - \mathscr{H}_m^{(l)}\right) - \frac{p^2(q)}{\varepsilon_a\mu_a}\frac{m^2}{Q^4}\left(1 - \frac{Q^2}{Q_\perp^2}\right)^2, \quad (12)$$

where J_m is a Bessel function of the first kind of order m, $H_m^{(l)}$ stands for Hankel functions of the first ($l = 1$) and the second ($l = 2$) kinds of order m, $Q_E = Q_\perp\left(\varepsilon_\parallel/\varepsilon_\perp\right)^{1/2}$, and $Q_H = Q_\perp\left(\mu_\parallel/\mu_\perp\right)^{1/2}$.

Substituting the field representation (7) into the boundary conditions (6), we arrive at the following integral equations for $\mathscr{I}_m(z)$:

$$\int_{-d}^{d}\begin{bmatrix} K_m(z-z') \\ k_m(z-z') \end{bmatrix}\mathscr{I}_m(z')dz' = \begin{bmatrix} -A_m \\ 0 \end{bmatrix}, \quad (13)$$

where $m = 0,\pm1,\pm2,\ldots$ and $|z| < d$.

4 Solution of the Integral Equations

Since the kernels $K_m(\zeta)$ and $k_m(\zeta)$ of integral equations (13) turn out to be singular at $\zeta \to 0$, we first transform these kernels so as to isolate their singularities explicitly.

To analyze the discrete-spectrum parts (9) of the kernels, one should find the eigenmode propagation constants $p_{m,n} = p(q_{m,n})$, which correspond to roots $q = q_{m,n}$ of the equation $\Delta_m^{(2)} = 0$ in the limit $|q_{m,n}| \to \infty$. Making this limiting transition in $\Delta_m^{(2)}$ (see (12)), we obtain the following dispersion relation for the higher-order eigenmodes:

$$\left(\frac{\varepsilon_\parallel}{\varepsilon_a} J_E - \mathscr{H}_m^{(2)}\right)\left(\frac{\mu_\parallel}{\mu_a} J_H - \mathscr{H}_m^{(2)}\right) = 0, \qquad (14)$$

which yields two infinite families of E and H eigenmodes with the propagation constants $p_{m,n}^{(\mathrm{e})}$ and $p_{m,n}^{(\mathrm{h})}$, respectively:

$$p_{m,n}^{(\mathrm{e})} = |\varepsilon_\perp/\varepsilon_\parallel|^{1/2}(k_0 a)^{-1}\left(\chi^{(\mathrm{e})} + \chi_{m,n}\,\mathrm{sgn}\,\varepsilon_\perp\right),$$
$$p_{m,n}^{(\mathrm{h})} = |\mu_\perp/\mu_\parallel|^{1/2}(k_0 a)^{-1}\left(\chi^{(\mathrm{h})} + \chi_{m,n}\,\mathrm{sgn}\,\mu_\perp\right). \qquad (15)$$

Here, $\chi_{m,n} = (m + 2n + 1/2)\pi/2$, $\chi^{(\mathrm{e})} = \arctan(\varepsilon_a/|\varepsilon_\perp\varepsilon_\parallel|^{1/2})$, and $\chi^{(\mathrm{h})} = \arctan(\mu_a/|\mu_\perp\mu_\parallel|^{1/2})$. When deriving (15), use was made of the large-argument approximations for cylindrical functions under the assumption that $k_0 a|\varepsilon_\parallel/\varepsilon_\perp|^{1/2} p_{m,n}^{(\mathrm{e})} \gg |m|$, $k_0 a|\mu_\parallel/\mu_\perp|^{1/2} p_{m,n}^{(\mathrm{h})} \gg |m|$, and $k_0 a p_{m,n}^{(\mathrm{e,h})} \gg |m|$. The summation in (9) can be performed analytically using the series in tables [4] if the antenna is so narrow that the conditions $d \ll 2a\Delta|\varepsilon_\parallel/\varepsilon_\perp|^{1/2} \ll a|\varepsilon_\parallel/\varepsilon_\perp|^{1/2}$ and $d \ll 2a\Delta|\mu_\parallel/\mu_\perp|^{1/2} \ll a|\mu_\parallel/\mu_\perp|^{1/2}$ take place. In this case, for $K_m^{(\mathrm{ds})}$ one obtains

$$K_m^{(\mathrm{ds})}(\zeta) = \frac{iZ_0}{\pi k_0 a^2}\left(\frac{m^2\varepsilon_u}{\varepsilon_a^2 - \varepsilon_u^2} + \frac{(k_0 a)^2\mu_u\mu_a^2}{\mu_a^2 - \mu_u^2}\right) \times \ln(|\zeta|/2a) + F_m^{(\mathrm{ds})}(\zeta). \qquad (16)$$

Here, ε_u and μ_u are introduced as

$$\varepsilon_u = -i|\varepsilon_\perp\varepsilon_\parallel|^{1/2}, \quad \mu_u = -i|\mu_\perp\mu_\parallel|^{1/2}, \qquad (17)$$

and $F_m^{(\mathrm{ds})}$ is the regular (nonsingular) term, which can be taken at $\zeta = 0$ for the narrow strip conductor of the antenna.

To isolate the singular part of the quantity $K_m^{(\mathrm{cs})}$, one should consider it in the limit $q \to \infty$. In this case, the approximate relations $Q_\perp = Q$, $Q_E = i|\varepsilon_\parallel/\varepsilon_\perp|^{1/2}Q$, and $Q_H = i|\mu_\parallel/\mu_\perp|^{1/2}Q$ take place, and the continuous-spectrum part of the kernel is expressed in terms of Legendre functions. Under the narrowness conditions of the strip conductor of the antenna, which are indicated above, the quantity $K_m^{(\mathrm{cs})}$ reduces to

$$K_m^{(\mathrm{cs})}(\zeta) = -\frac{iZ_0}{\pi k_0 a^2}\left(\frac{m^2\varepsilon_a}{\varepsilon_a^2 - \varepsilon_u^2} + \frac{(k_0 a)^2\mu_u^2\mu_a}{\mu_a^2 - \mu_u^2}\right) \times \ln(|\zeta|/2a) + F_m^{(\mathrm{cs})}(\zeta), \qquad (18)$$

where the regular term $F_m^{(\mathrm{cs})}(\zeta)$ can again be taken at $\zeta = 0$.

Then we substitute the sum of (16) and (18) into the first integral equation in (13) and reduce it to the form

$$\int_{-d}^{d} \mathscr{I}_m(z')\left(\ln\frac{|z - z'|}{2a} + S_m\right)dz' = -\frac{2\pi i\delta_m}{Z_0 k_0}A_m, \qquad (19)$$

where

$$\delta_m = (k_0 a)^2\varepsilon_{\mathrm{eff}}\left[m^2 - (k_0 a)^2\varepsilon_{\mathrm{eff}}\mu_{\mathrm{eff}}\right]^{-1},$$
$$S_m = \frac{2\pi i\delta_m}{Z_0 k_0}\left[F_m^{(\mathrm{ds})}(0) + F_m^{(\mathrm{cs})}(0)\right]. \qquad (20)$$

The quantities $\varepsilon_{\mathrm{eff}}$ and μ_{eff} in (20) are defined as

$$\varepsilon_{\mathrm{eff}} = (\varepsilon_a + \varepsilon_u)/2, \quad \mu_{\mathrm{eff}}^{-1} = (\mu_a^{-1} + \mu_u^{-1})/2. \qquad (21)$$

The discrete- and continuous-spectrum parts of the kernel $k_m(\zeta)$ can be analyzed in a similar way. It turns out that at $\zeta \to 0$, this kernel has the Cauchy singularity such that $k_m(\zeta) \sim m/\zeta$. Since the solution of the integral equation (19) with the logarithmic kernel automatically satisfies an integral equation with the Cauchy kernel, we can further restrict ourselves to consideration only of the integral equation (19).

As is known [5], the solution of the integral equation (19) has the form

$$\mathscr{I}_m(z) = \frac{2i}{Z_0 k_0\sqrt{d^2 - z^2}}\frac{A_m\delta_m}{\ln(4a/d) - S_m}. \qquad (22)$$

Allowing for (4) and (22), the current density (5) is then written as

$$I(\phi, z) = \frac{iV_0}{Z_0\pi k_0 a\sqrt{d^2 - z^2}}\sum_{m=-\infty}^{\infty}\frac{\sin(m\Delta)}{m\Delta}\frac{\delta_m}{\ln(4a/d) - S_m} \times \exp[-im(\phi - \phi_0)]. \qquad (23)$$

Integrating (23) over z between $-d$ and d, we can obtain the total current $I_\Sigma(\phi)$ in the cross section $\phi = \mathrm{const}$.

5 Current Distribution and Input Impedance

We now proceed to the analysis of the found solution for the antenna current. In the general case, the summation over m in (23) can be performed only numerically. A closed-form expression for the current distribution can be derived by neglecting S_m compared with the term $\ln(4a/d)$, which is possible for a sufficiently narrow strip conductor of the antenna. Neglecting S_m and making some algebra (see [5] for details), we deduce

$$I_\Sigma(\phi) = -\frac{iV_0\pi h}{\mu_{\mathrm{eff}}Z_0 k_0\ln(4a/d)}\frac{\cos[(\pi - \phi + \phi_0)ha]}{\sin(\pi ha)}, \qquad (24)$$

where $0 \le \phi - \phi_0 \le 2\pi$ and $h = k_0(\varepsilon_{\mathrm{eff}}\mu_{\mathrm{eff}})^{1/2}$ has the meaning of the complex-valued current propagation constant. Such behavior of h is related to the excitation of an infinite number of propagated electrostatic and magnetostatic eigenmodes in the doubly hyperbolic cylinder.

The closed-form expression (24) is convenient for analysis of the antenna current as a function of various parameters and gives results that coincide with those based on the rigorous formula (23) with graphical accuracy. Figure 2 presents the amplitude $|I_\Sigma(\phi)|$ (normalized to its maximum value) and phase $\theta(\phi) = \arctan(\mathrm{Im}\, I_\Sigma(\phi)/\mathrm{Re}\, I_\Sigma(\phi))$ of the antenna current as functions of the azimuthal angle ϕ for $k_0 a = 1.34$, $d/a = 0.02$, and $\phi_0 = 0$. For calculations, we take free

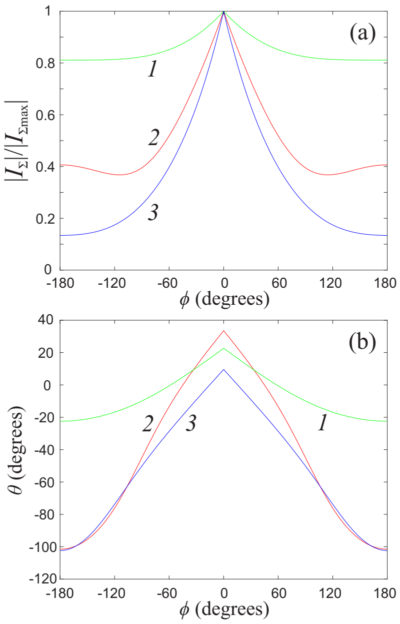

Figure 2. Normalized amplitude (a) and phase (b) of the antenna current as functions of azimuthal angle ϕ for $k_0 a = 1.34$ in the cases where $ha = 0.3 - i0.3$ (curves *1*), $ha = 0.7 - i0.5$ (curves *2*), and $ha = 0.6 - i0.9$ (curves *3*).

space as a medium surrounding the cylinder ($\varepsilon_a = \mu_a = 1$) and use different parameters of the medium inside the cylinder. For curves *1*, the elements of the tensors $\hat{\varepsilon}$ and $\hat{\mu}$ are taken as $\varepsilon_\perp = -\varepsilon_\parallel = 0.1$ and $\mu_\perp = -\mu_\parallel = 0.1$, when the dimensionless current propagation constant $ha = 0.3 - i0.3$. Remaining the elements of $\hat{\varepsilon}$ intact and increasing the elements of $\hat{\mu}$ up to $\mu_\perp = -\mu_\parallel = 0.5$ ($ha = 0.7 - i0.5$), we see that the current decays along the antenna and simultaneously undergoes oscillations, as is shown by curves *2* in the figure. For curves *3*, $\varepsilon_\perp = -\varepsilon_\parallel = 1$ and $\mu_\perp = -\mu_\parallel = 0.5$ ($ha = 0.6 - i0.9$), and the current decays at a higher rate, rather than oscillates, with distance from the excitation gap.

Upon calculation of $I_\Sigma(\phi)$, one can find the input impedance of the antenna by the formula $Z = V_0/I_\Sigma(\phi_0)$. Using the representation (24), we obtain

$$Z = iZ_0 k_0 (\pi h)^{-1} \mu_{\mathrm{eff}} \ln(4a/d) \tan(\pi h a). \qquad (25)$$

Note that in the case $\pi|\mathrm{Im}\, h|a \gg 1$, the antenna becomes electrically large and the input impedance (25) is written as

$$Z = Z_0 \frac{k_0 \mu_{\mathrm{eff}}}{\pi h} \ln \frac{4a}{d} = Z_0 \frac{h}{\pi k_0 \varepsilon_{\mathrm{eff}}} \ln \frac{4a}{d}, \qquad (26)$$

which coincides with the result of [6] for an infinitely long strip conductor at the plane interface of the same media, as expected. In the opposite case $\pi|\mathrm{Im}\, h|a \ll 1$, the input impedance (25) reduces to the inductive impedance $Z = iZ_0 k_0 a \mu_{\mathrm{eff}} \ln(4a/d)$ of an electrically small loop antenna located at a cylindrical interface of such media.

6 Conclusion

In this paper, we have obtained the solution to the problem of the current distribution of a circular loop antenna located on the surface of a cylinder filled with a hyperbolic metamaterial and surrounded by an isotropic magnetodielectric. Using the rigorous approach based on the integral equation method, the current distribution and input impedance of the antenna are found in closed form. The performed analysis makes it possible to study the electrodynamic characteristics of the antenna as functions of its parameters as well as the parameters of the cylinder and the surrounding medium.

7 Acknowledgements

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